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RESEARCH ARTICLE

EFFECTS OF CRACK ON MODAL FREQUENCY OF CANTILEVER BEAM

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ABSTRACT

Finite Element Method is used to determine the natural frequency of beam whereas the experimentation is performed using Fast Fourier Transform analyser. In experimentation, cantilever beam with crack are considered. The experimental results are validated with the results of FEM software. This formulation can be extended for various boundary conditions as well as varying cross sectional areas. A crack in a structural member introduces local flexibility that would affect vibration response of the structure. i.e., a crack causes a reduction in the stiffness and an increase in the damping of the structure. These changes of physical properties cause a reduction in the natural frequencies and a deviation in the mode shape. Therefore it is possible to predict the crack depth and crack location by measuring changes in the vibration parameters. Changes in the natural frequencies are more often considered than deviation of mode shapes, since frequencies can be measured more easily than mode shapes, and they are less seriously affected by experimental errors. This property may be used to detect existence of a crack together with its location and depth in the structural member. In this analysis the natural frequencies obtained from experimental analysis and finite element analysis are used to obtain the crack location and crack depth.

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INTRODUCTION

Many structural applications worldwide have been in use form many years. Their failure could lead to tragic consequences and therefore structures have regular costly inspections. During the last decades vibration based damage detection methods have attracted the most attention due to their simplicity for implementation. The presence of crack in structure changes its dynamic characteristics (Besselink *et al*., 2013). The change is characterized by change in modal parameters like modal frequencies, modal value and mode shapes associated with each modal frequency (Baviskar and Vinod B. Tungikar, 2013). It also alters the structural parameters like mass, damping matrix, stiffness matrix and flexibility matrix of structure (Baviskar and Vinod B. Tungikar, 2013). The vibration technique utilizes one or more of these parameters for crack detection. The frequency reduction in cracked beam is not due to removal of mass from beam, indeed the reduction in mass would increase natural frequency (Baviskar and Vinod B. Tungikar, 2013). But reduction in natural frequency is observed due to removal of material which carries significant stresses when defect is a narrow crack or notch. It reduces the stiffness of structure and natural frequency (Esfandiari *et al*., 2013). Due to presence of crack there is local influence which results from reduction area of cross section where it is located (Zuhir A. Jassim, 2010). Finite Element Analysis is powerful tool which gives the reasonably accurate results for complicated structure. The present study is based on observation of changes in natural frequency.

Experimental set up

The instruments used for experimental analysis are Fast Fourier Transform (FFT) analyser, accelerometer, impact hammer and related accessories. The accelerometer is mounted on the beam using mounting clips. The accelerometer is mounted near the crack to capture the correct signal. The impact hammer is used to excite the beam whose frequency response function has to be captured. For every test, the location of impact of impact hammer is kept constant. The beam is tapped gently with the impact hammer. The experiments are performed on mild steel beams with cantilever boundary conditions with crack of different depths at different locations. The properties of mild steel are, Young's modulus (E) 2.1 e11 N/ m^2, density (ρ) 7450 N/ m^3 and Poisson's ratio 0.3. Specimen beams under consideration have circular cross section area. For pipe cantilever beam the cross sectional outer diameter is 38 mm

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and thickness is 2 mm. The geometry of beams is as shown in Figure 1. Crack depth is represented in terms of (a/d) ratio where a is depth of crack and d is diameter of beam and crack location is represented in terms of (e) where (e) is ratio of location of crack at distance L1 from the fixed support to the length of the beam L. The experimental setup is as shown in Figure 2. The aim of experimental analysis is to verify the practical applicability of the theoretical method developed. For the beam with single cracks, transverse and open cracks are considered. Initially, the natural frequency of uncrack beam is found out. Hairline crack is generated to simulate the actual crack in the working components. There after depth of crack is increased. The change in natural frequency due to the crack is monitored. Table I shows the natural frequencies of simply supported beam with single crack.

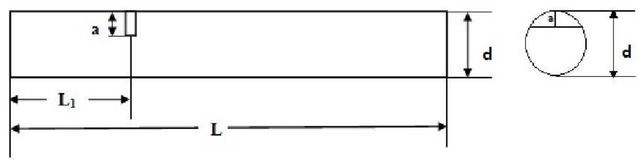

Figure 1. Geometry of beam with single crack

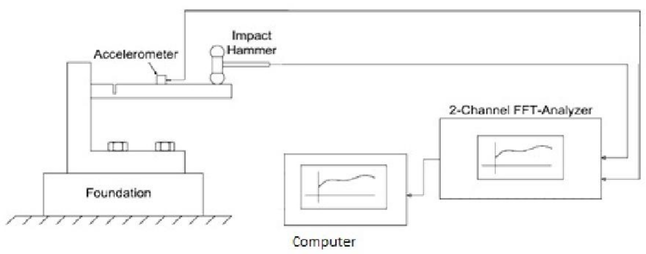

Figure 2. Experimental setup

Table 1. Natural Frequencies of Cantilever Beam by experiments

Crack size & location (mm)			Natural Frequency by Experiments (Hz)				
Sr. No.	a/d	e	1	2	3	4	5
1	0	0	1078	2766	4945	7453	10030
2	0.1052	0.2188	1055	2719	4875	7336	9984
3	0.1052	0.4267	984.4	2695	4922	7242	9750
4	0.1052	0.6565	960.9	2578	4852	7289	9820
5	0.1052	0.8753	937.5	2742	4898	7359	10010
6	0.2105	0.2188	1031	2531	4523	7313	9636
7	0.2105	0.4267	914.1	2555	4547	7289	9609
8	0.2105	0.6565	937.5	2508	4641	7172	9750
9	0.2105	0.8753	960.9	2578	4828	7195	9938
10	0.3158	0.2188	1008	2742	4641	7078	9188
11	0.3158	0.4267	773.4	2719	4594	7102	9867
12	0.3158	0.6565	820.3	2531	4852	7008	9609
13	0.3158	0.8753	890.6	2555	4617	7031	9867
14	0.4211	0.2188	984.4	2695	4594	7242	9164
15	0.4211	0.4267	843.8	2578	4547	7078	9750
16	0.4211	0.6565	867.2	2531	4828	6867	9539
17	0.4211	0.8753	890.6	2414	4008	6305	9703
18	0.5263	0.2188	914.1	2484	4570	7125	9141
19	0.5263	0.4267	796.9	2438	4430	6914	9680
20	0.5263	0.6565	843.8	2320	4641	6703	9422
21	0.5263	0.8753	1008	2297	3586	6047	9656

3. Computational work

The pipe cantilever beam of the cross sectional outer diameter is 38 mm and thickness is 2 mm. was prepared from mild steel. The Finite Element Method (FEM) of an artificially cracked and un-cracked beam had carried out with the help of ANSYS 14.5 package, the cracked beam is modelled as a pipe beam. The mesh generation, analysis, display of results, are all performed within ANSYS Workbench 14.5. For the natural frequencies by FEM software refer Table II and all mode shape are as shown in Fig. 5-9.

Table 2. Natural Frequencies of Cantilever Beam by FEM software

Crack size and location (mm)			Natural Frequency by FEM (Hz)				
Sr. No.	a/d	e	1	2	3	4	5
1	0	0	1067	2763	4931.3	7393.9	10017
2	0.1052	0.2188	1070.5	2730.7	4875.8	7392.1	9977.8
3	0.1052	0.4267	1054.2	2784.5	4909.2	7406.5	10054
4	0.1052	0.6565	1045.6	2711.4	4841.4	7254.2	9864
5	0.1052	0.8753	1059.1	2740.1	4888.8	7338.9	9959.2
6	0.2105	0.2188	1062.7	2727.1	4743.8	7346.4	9626.5
7	0.2105	0.4267	997.96	2767.4	4759	7298.8	9934.2
8	0.2105	0.6565	1040.2	2693.6	4822	7159.3	9799.1
9	0.2105	0.8753	1057.4	2721.9	4828.3	7270.5	9919
10	0.3158	0.2188	1043.4	2724.3	4649	7290	9155.3
11	0.3158	0.4267	929.53	2723.1	4625.9	7154.8	9833
12	0.3158	0.6565	1027.9	2651.3	4792.7	6991.2	9595.9
13	0.3158	0.8753	1053.4	2678.9	4602.9	6944.4	9838.9
14	0.4211	0.2188	996.13	2680	4593.8	7223.9	9106.3
15	0.4211	0.4267	858.37	2662	4540.6	7020.2	9773.8
16	0.4211	0.6565	1004.3	2577.1	4745.2	6847.6	9523
17	0.4211	0.8753	1045	2572.8	4021	6279.1	9725.2
18	0.5263	0.2188	911.05	2629.6	4573.2	7157.3	9102.4
19	0.5263	0.4267	790.82	2610.5	4475.7	6897.1	9691
20	0.5263	0.6565	969.42	2466.3	4657.8	6703.2	9447.9
21	0.5263	0.8753	1030.1	2392.4	3584.3	6046	9664.2

RESULTS

The crack is generated at known location in mild steel beam. The changes in natural frequencies for the un cracked and cracked beams are measured. The predicted values are determined by theoretical and experimental technique. Table I shows the natural frequency values extracted for cantilever beam with single crack determined by using experimentation. Table II shows the natural frequency values extracted for cantilever beam with single crack determined by using FEM software. The results for crack location (e), crack size (a/d) are computed. Table III shows the % reduction in natural frequencies for crack location (e), crack size (a/d) between cracked and un cracked values for cantilever beam with single crack by using experimentation.

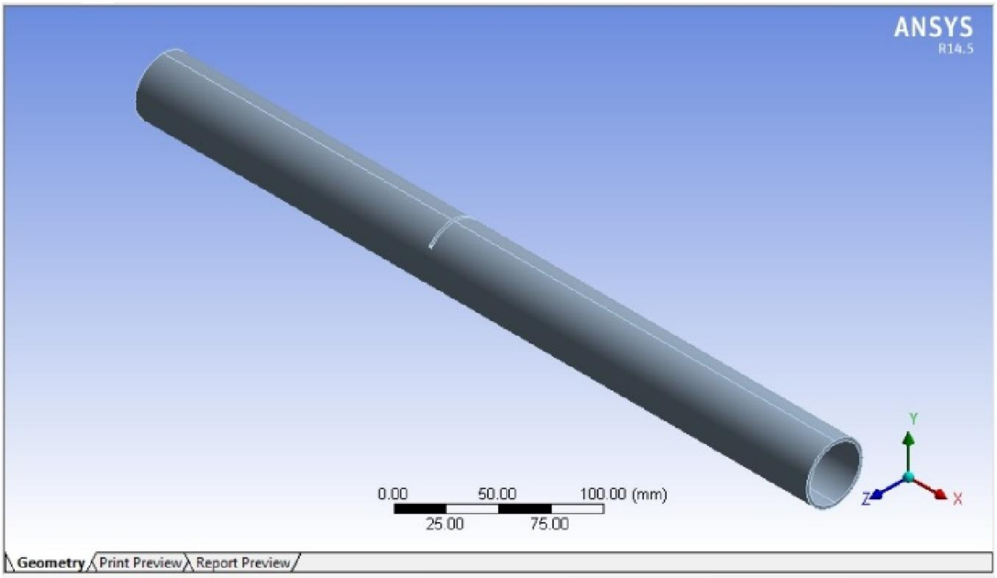

Figure 3. Cracked beam 8mm depth at 200mm from the fixed end

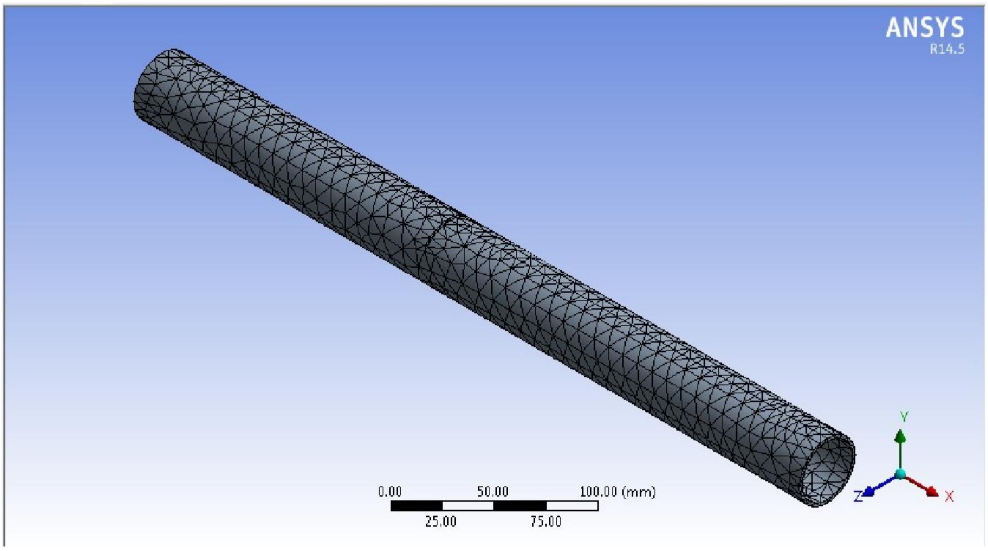

Figure 4. Meshing of cracked beam

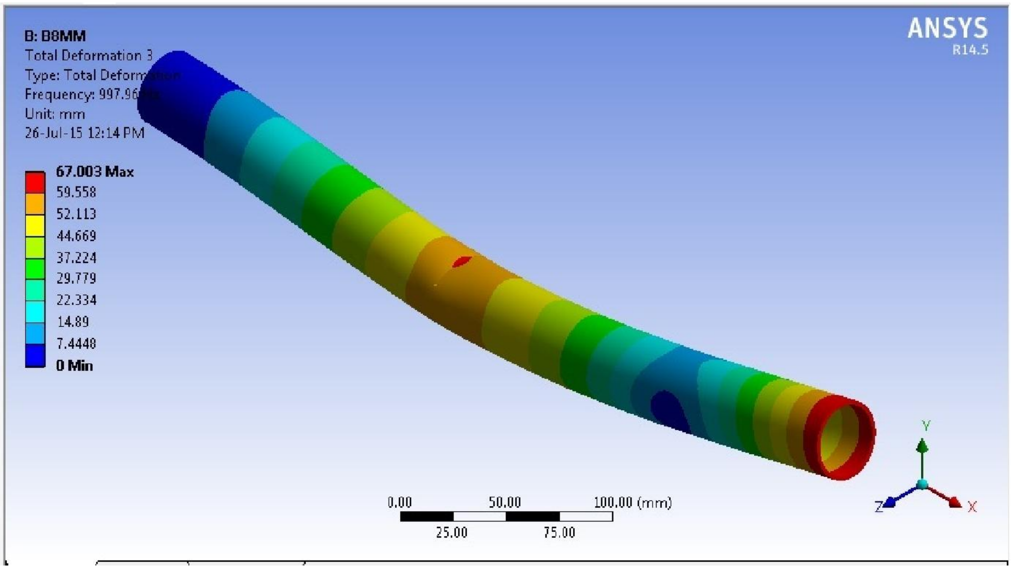

Figure 5. First mode shape of cracked beam

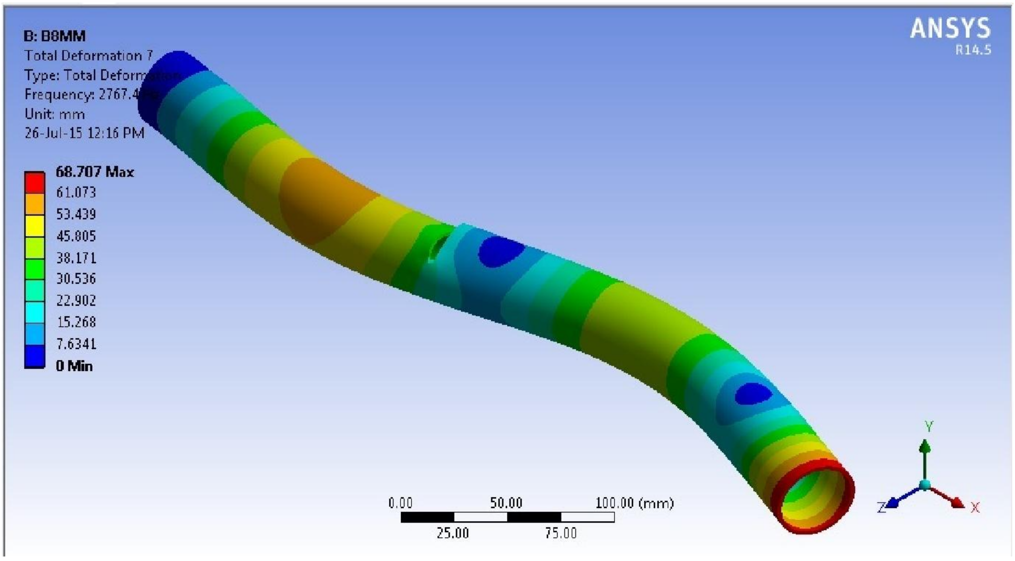

Figure 6. Second mode shape of cracked beam

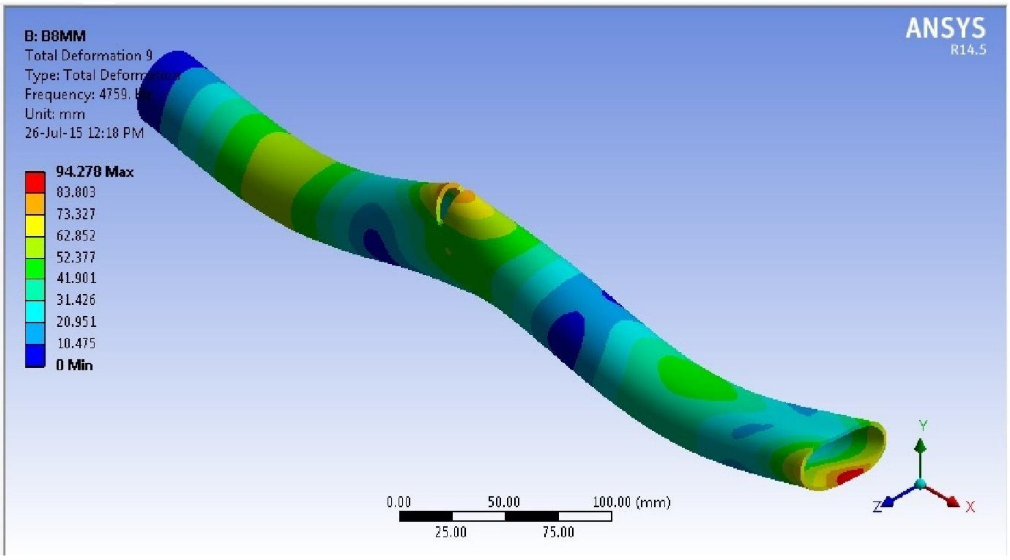

Figure 7. Third mode shape of cracked beam

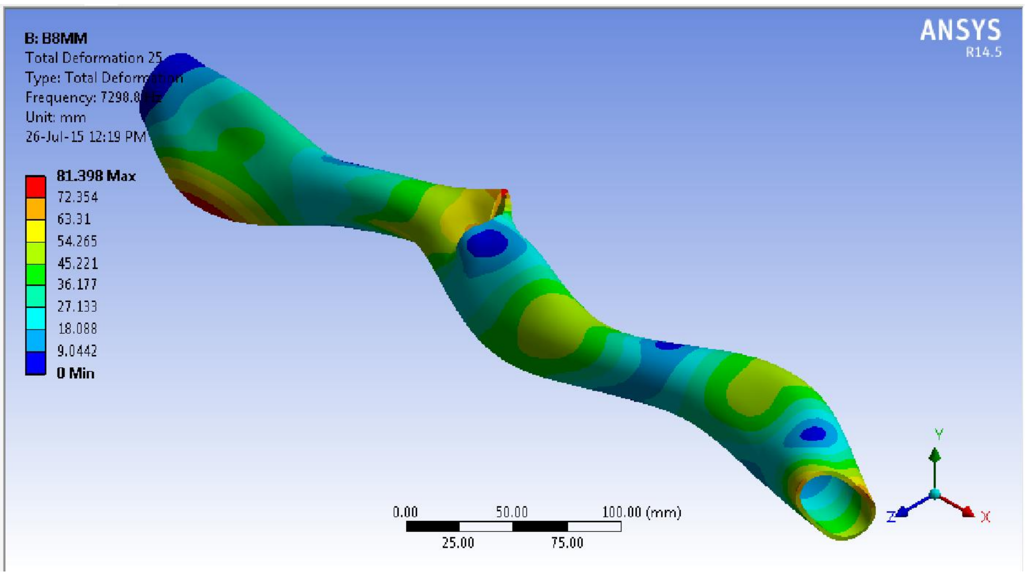

Figure 8. Fourth mode shape of cracked beam

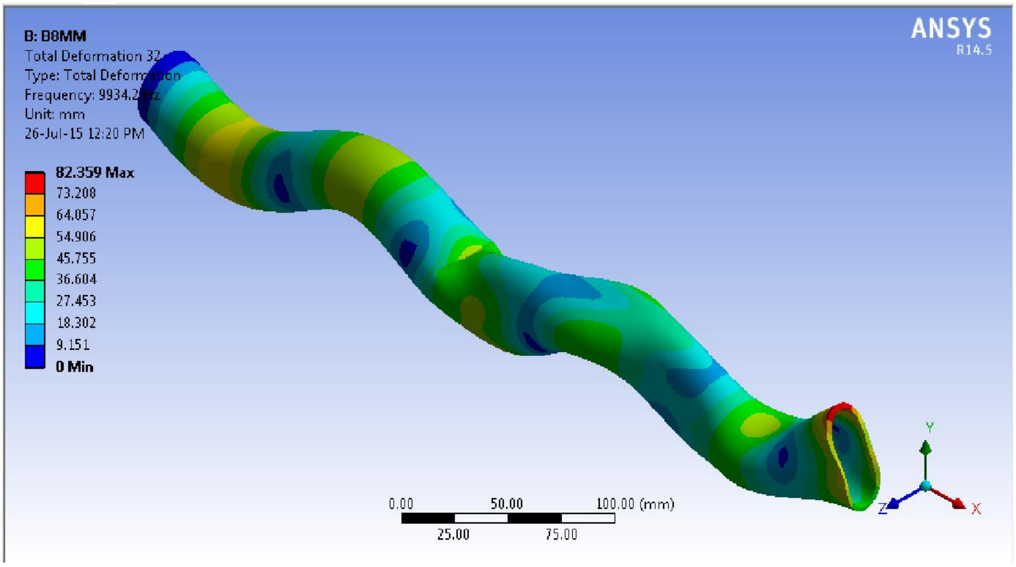

Figure 9. Fifth mode shape of cracked beam

Table IV shows the % reduction in natural frequencies for crack location (e), crack size (a/d) between cracked and un cracked values for cantilever beam with single crack by using FEM software. It is observed that experimental results have some deviation from the results obtained by FEM as model of structure generated by Finite Element Analysis differs from actual structure. Hence the response of structure in practice differs. The results are close to the actual for finding the crack locations. These results approach to the actual results found by FEM as compared to the experimental findings. The results of crack depth findings are close to the actual depth for large (a/d) ratio as compared to small (a/d) ratio.

Table 3. % reduction in Natural Frequencies of Cantilever Beam by experiments

Crack size and location (mm)			% Reduction in natural frequency by experiments (Hz)				
Sr. No.	a/d	e	1	2	3	4	5
1	0	0	1078	2766	4945	7453	10030
2	0.1052	0.2188	2.1336	1.6992	1.4156	1.5698	0.4586
3	0.1052	0.4267	8.6828	2.5669	0.4651	2.8311	2.7916
4	0.1052	0.6565	10.8627	6.7968	1.8807	2.2005	2.0937
5	0.1052	0.8753	13.0334	0.8677	0.9505	1.2612	0.1994
6	0.2105	0.2188	4.3599	8.4960	8.5339	1.8784	3.9282
7	0.2105	0.4267	15.2040	7.6283	8.0485	2.2005	4.1974
8	0.2105	0.6565	13.0334	9.3276	6.1476	3.7703	2.7916
9	0.2105	0.8753	10.8627	6.7968	2.3660	3.4617	0.9173
10	0.3158	0.2188	6.4935	0.8677	6.1476	5.0315	8.3948
11	0.3158	0.4267	28.2560	1.6992	7.0981	4.7095	1.6251
12	0.3158	0.6565	23.9054	8.4960	1.8807	5.9708	4.1975
13	0.3158	0.8753	17.3841	7.6283	6.6330	5.6622	1.6251
14	0.4211	0.2188	8.6828	2.5669	7.0981	2.8312	8.6341
15	0.4211	0.4267	21.7254	6.7968	8.0485	5.0315	2.7916
16	0.4211	0.6565	19.5547	8.4960	2.3660	7.8626	4.8953
17	0.4211	0.8753	17.3840	12.7260	18.948	15.403	3.2602
18	0.5263	0.2188	15.2041	10.1952	7.5834	4.4009	8.8634
19	0.5263	0.4267	26.0761	11.8583	10.414	7.2320	3.4895
20	0.5263	0.6565	21.7254	16.1244	6.1476	10.063	6.0618
21	0.5263	0.8753	6.4935	16.9559	27.482	18.864	3.7288

It is observed because for small (a/d) ratio, the reduction in the stiffness of beam is less as compared to large (a/d) ratio. Due to the high stiffness, the vibrations are damped and natural frequency does not reduce. The readings obtained are used as database for finding the crack location and size from fixed end. Effect of crack depth (a/d) and position (e) on the natural frequency are as shown in Fig.10-19.

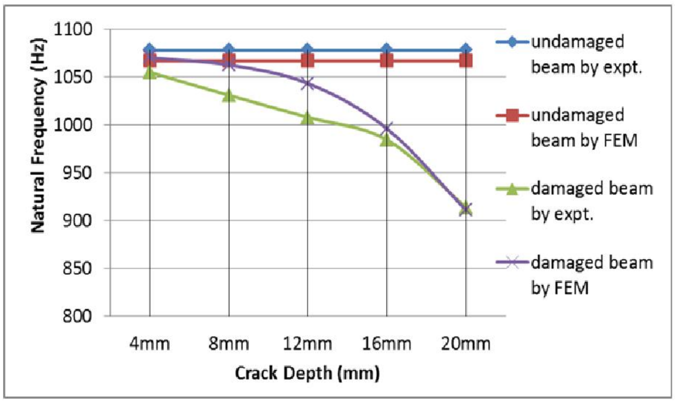

Figure 10. Effect of crack depth on the natural frequency of the first mode shape of a damaged cantilever beam at 100 mm from the fixed end

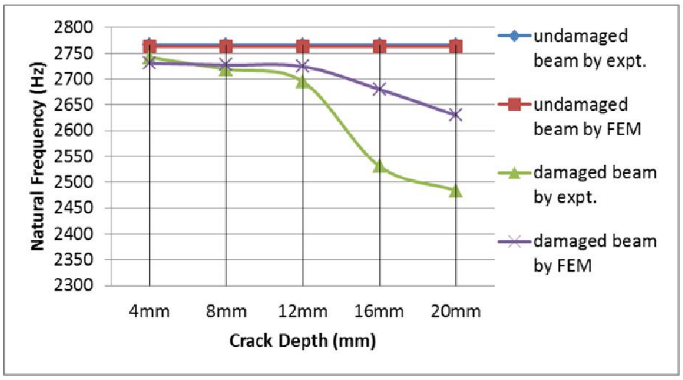

Figure 11. Effect of crack depth on the natural frequency of the second mode shape of a damaged cantilever beam at 100 mm from the fixed end

Table 4. % reduction in Natural Frequencies of Cantilever Beam by FEM software

Crack size and location (mm)			Natural Frequency by FEM (Hz)				
Sr. No.	a/d	e	1	2	3	4	5
1	0	0	1067	2763	4931.3	7393.9	10017
2	0.1052	0.2188	0.328022	1.1690	1.1255	0.0243	0.3913
3	0.1052	0.4267	1.1996	0.7781	0.4481	0.1704	0.3694
4	0.1052	0.6565	2.0056	1.8675	1.8230	1.8894	1.5274
5	0.1052	0.8753	0.7404	0.8288	0.8618	0.7439	0.5770
6	0.2105	0.2188	0.4029	1.2993	3.8022	0.6424	3.8983
7	0.2105	0.4267	6.4704	0.1592	3.4940	1.2862	0.8266
8	0.2105	0.6565	2.5117	2.5118	2.2165	3.1728	2.1753
9	0.2105	0.8753	0.8997	1.4875	2.0886	1.6689	0.9784
10	0.3158	0.2188	2.2118	1.4006	5.7246	1.4052	8.6023
11	0.3158	0.4267	12.8838	1.4441	6.1931	3.2337	1.8369
12	0.3158	0.6565	3.6645	4.0427	2.8106	5.4463	4.2038
13	0.3158	0.8753	1.2746	3.0438	6.6595	6.0793	1.7779
14	0.4211	0.2188	6.6419	3.0039	6.8440	2.2991	9.0915
15	0.4211	0.4267	19.5529	3.6555	7.9229	5.0541	2.4279
16	0.4211	0.6565	5.8763	6.7282	3.7739	7.3885	4.9316
17	0.4211	0.8753	2.0619	6.8838	18.4596	15.077	2.9130
18	0.5263	0.2188	14.6157	4.8281	7.2618	3.1999	9.1305
19	0.5263	0.4267	25.8838	5.5194	9.2389	6.7191	3.2545
20	0.5263	0.6565	9.1453	10.7383	5.5462	9.3414	5.6813
21	0.5263	0.8753	3.4583	13.4129	27.31531	18.2299	3.5220

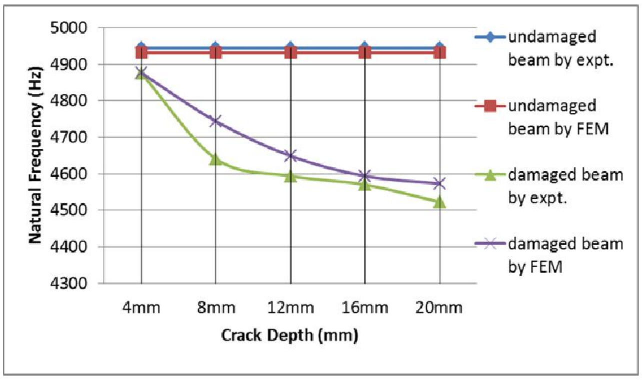

Figure 12. Effect of crack depth on the natural frequency of the third mode shape of a damaged cantilever beam at 100 mm from the fixed end

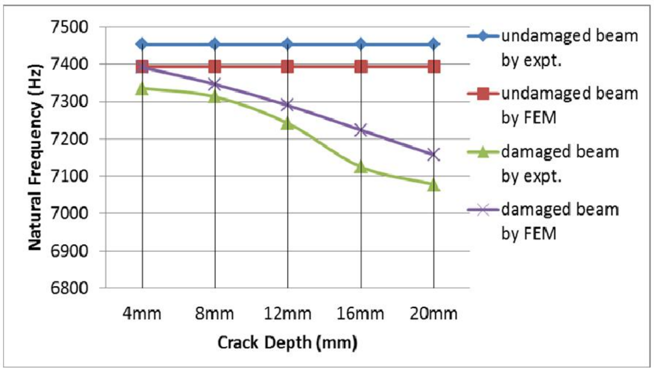

Figure 13. Effect of crack depth on the natural frequency of the fourth mode shape of a damaged cantilever beam at 100 mm from the fixed end

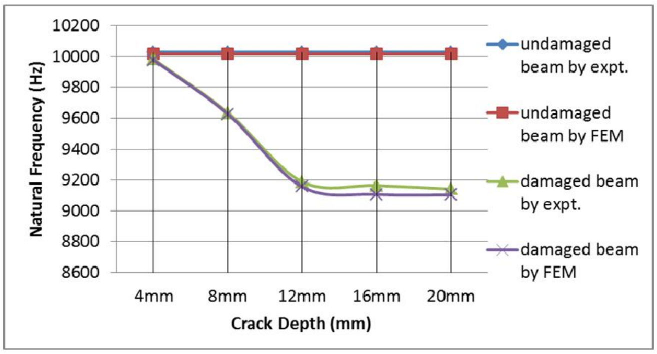

Figure 14. Effect of crack depth on the natural frequency of the fifth mode shape of a damaged cantilever beam at 100 mm from the fixed end

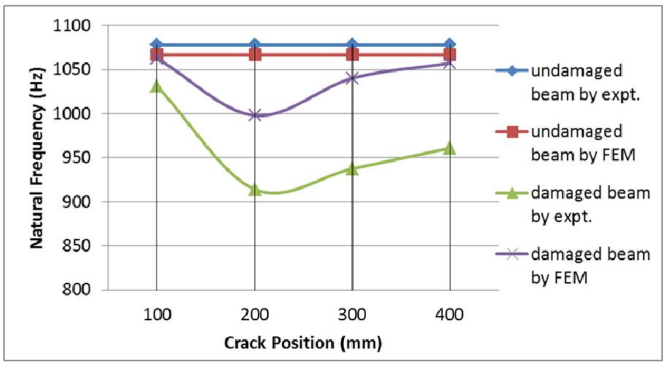

Figure 15. Effect of crack position on the natural frequency of the first mode shape of damaged cantilever beam with 8 mm crack depth from the fixed end

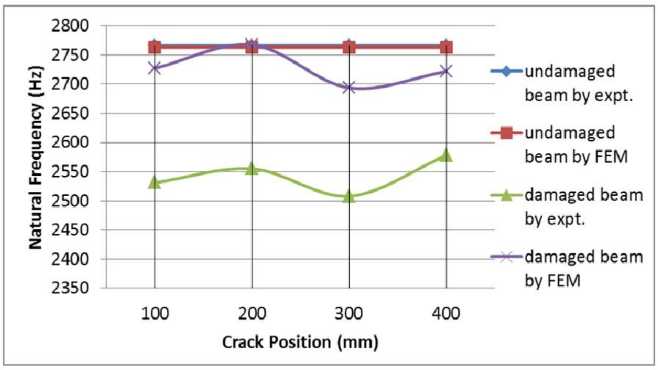

Figure 16. Effect of crack position on the natural frequency of the second mode shape of damaged cantilever beam with 8 mm crack depth from the fixed end

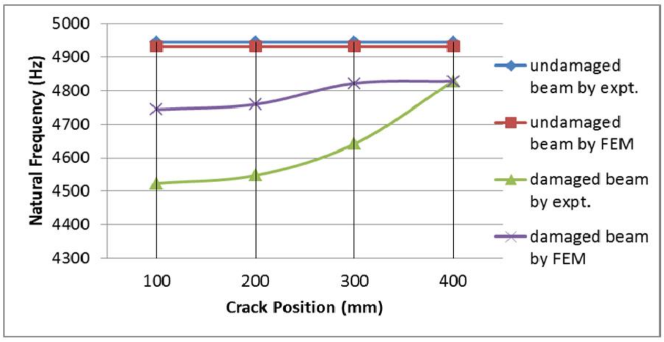

Figure 17. Effect of crack position on the natural frequency of the third mode shape of damaged cantilever beam with 8 mm crack depth from the fixed end

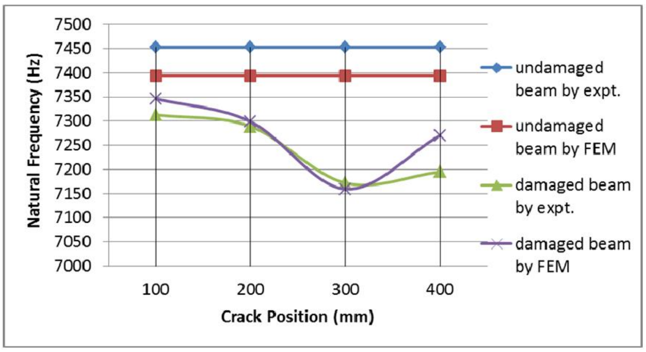

Figure 18. Effect of crack position on the natural frequency of the fourth mode shape of damaged cantilever beam with 8 mm crack depth from the fixed end

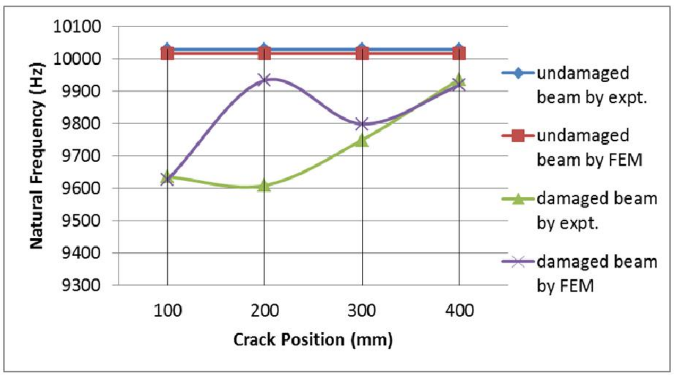

Figure 19. Effect of crack position on the natural frequency of the fifth mode shape of damaged cantilever beam with 8 mm crack depth from the fixed end

Conclusion

This work attempts to establish a systematic method of prediction of crack characteristics from measurement of natural frequencies using experimental study, following conclusions can be drawn

1. Natural frequency of the of cracked beam decreases as the crack depth increases and the crack location is constant.
2. Natural frequency of the cracked beam increases as the crack location increases from fixed end and the crack depth is constant.
3. The results of Finite Element Analysis and experimental analysis are compared and they are in good agreement.
4. The study showed small crack depth ratios had small effect on the sensitivity of the natural frequencies. It was also observed that the changes became more significant as the crack grew deeper.
5. The effect of crack is more pronounced when the cracks are near to the fixed end than at free end.
6. Experimental tests results can be used to detect and monitor fatigue cracks in beams, shafts or rotating machine element, from which the health of the element could be recorded at various stages of fatigue damage.
7. In the present study, the beams under consideration have uniform cross section but this method can be extended to components with varying cross section, different geometry and any boundary condition.
8. The proposed method can be extended for fault diagnosis in beams, shafts or rotating machine element.

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