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RESEARCH ARTICLE

BEHAVIOR OF BULK VISCOUS BIANCHI V COSMOLOGICAL MODEL IN *f(R)* THEORY OF GRAVITY

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ABSTRACT

In this paper, modified Einstein's field equations are solved in the presence of bulk viscous fluid for spatially homogeneous and anisotropic Bianchi Type-V cosmological model by using the average scale factor as $a(t) = t - \frac{1}{t}$, which leads to the time varying deceleration parameter. We also use the barotropic equation of state for pressure and density. The physical aspects of this model have been discussed.

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INTRODUCTION

Alternative theories of gravitation proposed by Einstein is a generalization of Einstein's General theory of relativity. In the last decade, as an alternative to general relativity, scalar tensor theories and modified theories of gravitation have been proposed. Brans and Dicke, Saez and Ballester have much importance amongst the scalar-tensor theory gravity and *f(R,T)* theory of gravitation. Recently, *f(R)* gravity theory (Bertolami *et al*., 2007; Nojiri and Odintsov, 2006; Nojiri and Odintsov, 2007) has much importance amongst the modified theories of gravity because this theory is supposed to provide natural gravitation alternatives to dark energy and explain current acceleration expansion of the universe. From the cosmological observations, it is known that the energy composition of the universe has 76% dark energy, 20% dark matter and 4% Baryon matter. This is confirmed by the high red shift supernovae experiments (Perlmutter *et al*., 1997; Perlmutter *et al*., 1998) and cosmic microwave background radiation observations (Reiss *et al*., 1998; Perlmutter *et al*., 1999). Many authors have studied *f(R)* theory of gravitation in different context (Antonio De Felice, 2010; Adhav, 2006; Hollenstein and Lobo, 2008; Sharif *et al*., 2009; Sharif and RizwanaKausar, 1967). Cosmic microwave background radiation suggests that the matter behaves like a viscous fluid at the early stages of the evolution of the universe. There are several processes which are expected to give rise to viscous effect. The strong dissipation due to the neutrino viscosity may considerably reduce the anisotropy of the black-body radiation studied by Misner (1967, 1968). Murphy (1973) shows that the combination of cosmic fluid with bulk deceptive pressure can generate the accelerated expansion of universe. The interesting feature of the solutions is that the big-bang singularity appears in the infinite past. Fabris *et al*. (2006) discussed that the bulk viscosity leading to an accelerated phase of the universe. Coley (1990) have studied Bianchi-V viscous fluid cosmological models for a barotropic fluid distribution. Padmanabhan and Chitre (1987) have investigated the effect of bulk viscosity on the evolution of the universe at large stage. They showed that bulk viscosity leads to inflationary like solutions. Moreover, Singh and Baghel (2010) examined Bianchi type-V cosmological models with bulk viscosity.

The bulk viscosity plays an important role in the early phase evolution of the universe. Many authors have studied modified theory of gravitation by using source of matter is bulk viscous fluid. Naidu *et al*. (2013) have examined FRW viscous fluid cosmological model in $f(R,T)$ gravity. The Non-existence of

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Bianchi type-III bulk viscous string cosmological model in $f(R,T)$ gravity have studied by Kiran *et al*. (2013). Ghate *et al*. (2014) studied Bianchi type-IX viscous string cosmological model in $f(R,T)$ gravity with the special form of declaration parameter. Reddy *et al*. (2013) have discussed Kaluza-Klein universe with cosmic strings and bulk viscosity in $f(R,T)$ gravity. Mahanta *et al*. (2014) have investigated bulk viscous cosmological model in $f(R,T)$ gravity. Reddy *et al*. (Reddy *et al*., 2014) have investigated Kantowski-Sachs bulk viscous string cosmological model in $f(R,T)$ gravity. Same author (Reddy *et al*., 2013) have studied LRS Bianchi type-II universe with cosmic strings and bulk viscosity in a modified theory of gravity. Naidu *et al*. (2013) have studied Bianchi type-V bulk viscous string cosmological model in $f(R,T)$ gravity. Reddy *et al*. (Naidu *et al*., 2012; BhaskaraRao *et al*., 2015; Naidu *et al*., 2013) have investigated bulk viscous string cosmological model in different modified theory.

The modified Einstein's field equations are the system of highly non-linear differential equations. Berman (1983) has proposed a law of variation for Hubble's parameter to obtain solutions of the field equations. Interesting feature of the Hubble law is that yields a constant value of deceleration parameter. In earlier literature cosmological models with a constant deceleration parameter have been studied by Berman and Gomide (1988), Johri and Desikan (1994), Singh and Desikan (1997), Maharaj and Naidoo (1993), Pradhan *et al*. (2002). The universe is decelerating for the value of q within the range $0 < q < 1$. But today's situation quite different because of the observations of Type Ia Supernovae (SNe) provide the evidence of expansion history of the universe. The result of these observations is that the expansion of the universe is accelerating. So we can consider the cosmological models with time variable deceleration parameter.

Bianchi type models are important because these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. From the theoretical point of view anisotropic universe has a greater generality than isotropic models. Motivating by the above investigations of bulk viscous Bianchi type models in modified theories of gravitation, in this paper, we have developed spatially homogeneous and anisotropic Bianchi Type-V cosmological model in the presence of Bulk viscous fluid in $f(R)$ gravity. The modified Einstein's field equations are solved by using the average scale factor $a(t)=t-1/t$, which leads to the time varying deceleration parameter. We also use the barotropic equation of state for pressure and energy density. The physical aspects of this model have been discussed.

f(R) Gravity Formalism

The action for the $f(R)$ gravity is given by

$$S=\int\left[\frac{1}{16\pi G}f(R)+L_m\right]\sqrt{-g}\ d^4x, \tag{1}$$

where $f(R)$ is a general function of Ricci scalar R, g is the determinant of the metric g_{ij} and L_m is the metric Lagrangian on that depends on g_{ij}.

The field equations resulting for this action are the following form

$$F(R)R_{ij}-\frac{1}{2}f(R)g_{ij}-\nabla_i\nabla_j F(R)+g_{ij}\,\Box F(R)=kT_{ij}, \tag{2}$$

$$\text{where } F(R)\equiv\frac{df(R)}{dR},\ \Box\equiv\nabla_i\nabla_j, \tag{3}$$

Here, ∇_i is the covariant derivative and T_{ij} is the standard matter energy-momentum tensor derived from the Lagrangian L_m and $k=\frac{8\pi G}{c^4}$.

The contraction of equation (2) gives

$$F(R)-2f(R)+3\,\Box F(R)=kT, \tag{4}$$

Thus we have calculated $f(R)$ from this equation.
Using equations (4) and (2), the field equations can be written as

$$F(R)R_{ij}-\nabla_i\nabla_j F(\mathrm{R})-\mathrm{k}T_{ij}=\frac{1}{4}[F(R)R-\Box F(R)-kT]g_{ij}, \tag{5}$$

Bianchi type-V metric

We consider the Bianchi type –V space time.

$$ds^2=dt^2-A^2dx^2-e^{2mx}[B^2dy^2+C^2dz^2], \tag{6}$$

where A, B and C are metric coefficients and are the functions of cosmic time t, $m\neq 0$ is a any arbitrary constant.

The corresponding Ricci scalar becomes

$$R=2\left[\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{3m^2}{A^2}\right], \tag{7}$$

where overhead dot represents derivative with respect to time t.

The energy momentum tensor T_{ij} for a bulk Viscous fluid distribution is given by

$$T_{ij}=(\rho+\overline{p})u_iu_j-\overline{p}g_{ij}, \tag{8}$$

$$\text{and } \overline{p}=p-\xi\,u^i_{;i}, \tag{9}$$

where ρ is the energy density, p is the effective pressure, $\overline{p}$ the isotropic pressure, ξ is the coefficient of bulk viscosity

and $u_i = \sqrt{g_{00}}(1,0,0,0)$ is the four velocity vector in co-moving co-ordinates.
With the help of equation (5) the Einstein's field equations are written as

$$-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{A}\dot{F}}{AF}-\frac{\ddot{F}}{F}-\frac{2m^2}{A^2}=\frac{k}{F}(\rho+\overline{p}), \quad (10)$$

$$-\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{B}\dot{F}}{BF}-\frac{\ddot{F}}{F}-\frac{2m^2}{A^2}=\frac{k}{F}(\rho+\overline{p}), \quad (11)$$

$$-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{C}\dot{F}}{CF}-\frac{\ddot{F}}{F}-\frac{2m^2}{A^2}=\frac{k}{F}(\rho+\overline{p}), \quad (12)$$

The 01- component can be written as by using equation (2) in the following form

$$\frac{2\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0, \quad (13)$$

Integrating equation (13), we obtain

$$A^2 = k_1 BC, \quad (14)$$

where k_1 is a constant of integration.

For the sake of simplicity, we take $k_1 = 1,$ so that the equation (14) becomes

$$A^2 = BC, \quad (15)$$

The conservation equation for energy momentum $T^{ij}_{;j} = 0$ leads to

$$\dot{\rho}+(\rho+\overline{p})\left[\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right]=0, \quad (16)$$

The physical quantities of observational interest in cosmology are volume scale V, average scale factor a, average Hubble parameter H, expansion scalar θ, average anisotropy parameter $\overline{A}$ and the deceleration parameter q.

The volume scale factor V and the average scale factor a are defined as

$$V = ABC, \quad (17)$$

$$a = (ABC)^{\frac{1}{3}}, \quad (18)$$

The average Hubble's parameter is given in the form

$$H = \frac{1}{3}\left[H_x + H_y + H_z\right], \quad (19)$$

where $H_x = \frac{\dot{A}}{A},\ H_y = \frac{\dot{B}}{B},\ H_z = \frac{\dot{C}}{C}$ are the directional Hubble parameters along x, y and z axes respectively.

Using equation (18) and (19), we obtain

$$H = \frac{\dot{V}}{3V} = \frac{\dot{a}}{a} = \frac{1}{3}\left[H_x + H_y + H_z\right], \quad (20)$$

The expansion scalar (θ) and shear scalar (σ^2) are defined as follows

$$\theta = u^i_{;i} = \frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}, \quad (21)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{\theta^2}{3} - \left[\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}\right], \quad (22)$$

where $\sigma_{ij} = \frac{1}{2}(\nabla_j u_i + \nabla_i u_j) - \frac{1}{3} g_{ij}\theta$.

To examine whether expansion of the universe is anisotropic or not, we define anisotropy parameter $\overline{A}$

$$\overline{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = 6\left(\frac{\sigma}{\theta}\right)^2, \quad (23)$$

The important observational quantity deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{d}{dt}\left(\frac{1}{H}\right), \quad (24)$$

Here the deceleration parameter q measures the rate of expansion of the universe and $q > 0$ indicates inflation of the universe $q < 0$ denotes deflation of the universe while $q = 0$ shows expansion with constant velocity.

Solution of Field Equations

We have four independent field equations with seven unknowns $A, B, C, \rho, \overline{p}, p, \xi$. We should adopt additional assumption to solve these field equations. For a Baratropic fluid, the combined effect of the proper pressure and the Baratropic bulk viscous pressure can be written as

$$\overline{p} = \gamma\rho, \quad (25)$$

where $p = \gamma_0\rho,\ \ 0 \le \gamma \le 1$ (26)

Subtracting equations (10), (11), (12) from (11), (12), (10) respectively, we get

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\left(\frac{\dot{C}}{C}+\frac{\dot{F}}{F}\right)=0, \tag{27}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{F}}{F}\right)=0, \tag{28}$$

$$\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}+\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)\left(\frac{\dot{B}}{B}+\frac{\dot{F}}{F}\right)=0, \tag{29}$$

Using the equations (27) –(29), we get

$$\frac{A}{B}=d_1\exp\left[c_1\int\frac{1}{a^3F}dt\right], \tag{30}$$

$$\frac{B}{C}=d_2\exp\left[c_2\int\frac{1}{a^3F}dt\right], \tag{31}$$

$$\frac{A}{C}=d_3\exp\left[c_3\int\frac{1}{a^3F}dt\right], \tag{32}$$

Where c_i and $d_i\,(i=1,2,3)$ are constants of integration which satisfy the relation

$$c_1+c_2+c_3=0, \tag{33}$$

$$d_1d_2d_3=1. \tag{34}$$

Using equations (15). (30),(31),(32), we can write the metric function as

$A=a,$
(35)

$$B=Pa\exp\left[Q\int\frac{1}{a^3F}dt\right], \tag{36}$$

$$C=\frac{1}{P}a\exp\left[-Q\int\frac{1}{a^3F}dt\right], \tag{37}$$

where P and Q are arbitrary constants.
Using the power law relation between F and a, we have

$$F=la^b, \tag{38}$$

where l is a constant of proportionality and b is any integer (here taken as -2)

Now we take the following average scale factor a, as increasing function of time as

$$a(t)=t-\frac{1}{t}, \tag{39}$$

Using equations (24) and (39), the deceleration parameter becomes

$$q=2\frac{(t^2-1)}{(t^2+1)^2} \tag{40}$$

The above choice (39) of scale factor yields a time dependent form of deceleration parameter.

Using equation (38), equation (38) leads to

$$F=l\left(t-\frac{1}{t}\right)^{-2}, \tag{41}$$

The metric coefficient A, B and C turn out to be

$$A=t-\frac{1}{t}, \tag{42}$$

$$B=P\left(t-\frac{1}{t}\right)(t^2-1)^{\frac{Q}{2l}}, \tag{43}$$

$$C=\frac{1}{P}\left(t-\frac{1}{t}\right)(t^2-1)^{-\frac{Q}{2l}}, \tag{44}$$

The volume scale factor V becomes

$$V=\left(t-\frac{1}{t}\right)^3, \tag{45}$$

The directional Hubble parameter are calculated as

$$H_x=\frac{t^2+1}{t(t^2-1)}, \tag{46}$$

$$H_y=\frac{Qt}{l(t^2-1)}+\frac{t^2+1}{t(t^2-1)}, \tag{47}$$

$$H_z=\frac{-Qt}{l(t^2-1)}+\frac{t^2+1}{t(t^2-1)}, \tag{48}$$

The generalized mean Hubble parameter H and expansion scalar θ are found to be

$$H=\frac{t^2+1}{t(t^2-1)}, \tag{49}$$

$$\theta=\frac{3(t^2+1)}{t(t^2-1)}, \tag{50}$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{Q^2 t^2}{l^2 (t^2-1)^2}, \qquad (51)$$

The average anisotropy parameter $\overline{A}$ becomes

$$\overline{A} = \frac{2Q^2 t^4}{3l^2 (t^2+1)^2}, \qquad (52)$$

From the equations (16), (20) and (25), we get

$$\rho = c\,a^{-3(1+\gamma)}, \qquad (53)$$

where c is a constant of integration.

Using equation (39) in equation (53), the energy density is given by

$$\rho = c\left[t - \frac{1}{t}\right]^{-3(1+\gamma)}, \qquad (54)$$

Using equation (25) and (54), we have

$$\bar{p} = c\gamma\left[t - \frac{1}{t}\right]^{-3(1+\gamma)}, \qquad (55)$$

From equations (26) and (53), we get

$$p = c\gamma_0\left[t - \frac{1}{t}\right]^{-3(1+\gamma)}, \qquad (56)$$

The coefficient of Bulk Viscosity becomes

$$\xi = \frac{1}{3}(\gamma - \gamma_0)\left[\frac{t^2+1}{t^2}\right]\left[t - \frac{1}{t}\right]^{-(3\gamma+4)}, \qquad (57)$$

The Ricci scalar becomes

$$R = -\frac{2}{t^2(t^2+1)^2}\left[t^4\left(\frac{Q^2}{l^2} - 3m^2 + 3\right) + 9\right]. \qquad (58)$$

The function of Ricci scalar, $f(R)$ is

$$f(R) = \frac{l}{(t^2-1)^4}\left\{\begin{array}{l}\frac{6}{t^2-1}\left[2t^2 + \frac{(t^2+1)^2(5-3t)}{t^{10}(t^2-1)}\right] \\ -\frac{t^2}{(t^2+1)^2}\left[9 + t^4\left(\frac{Q^2}{l^2} - 45\right)\right]\end{array}\right\} - \frac{1}{2}kc(1-3\gamma)\left(\frac{t}{t-1}\right)^{3(1+\gamma)}, \qquad (59)$$

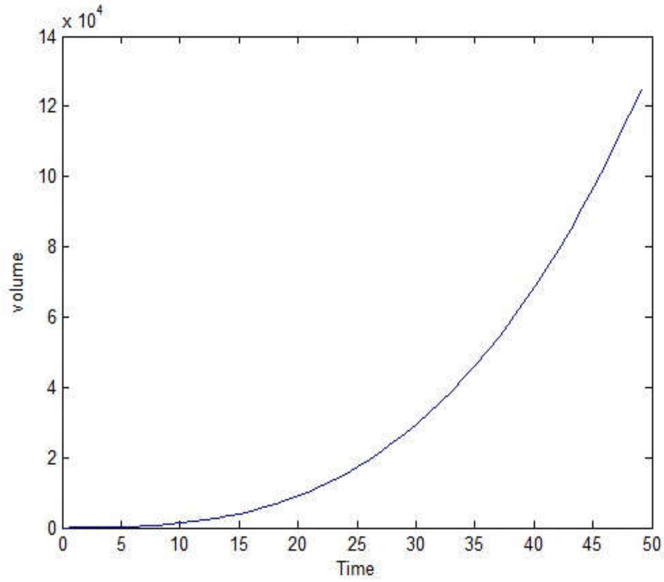

Fig. 1. Volume Vs time

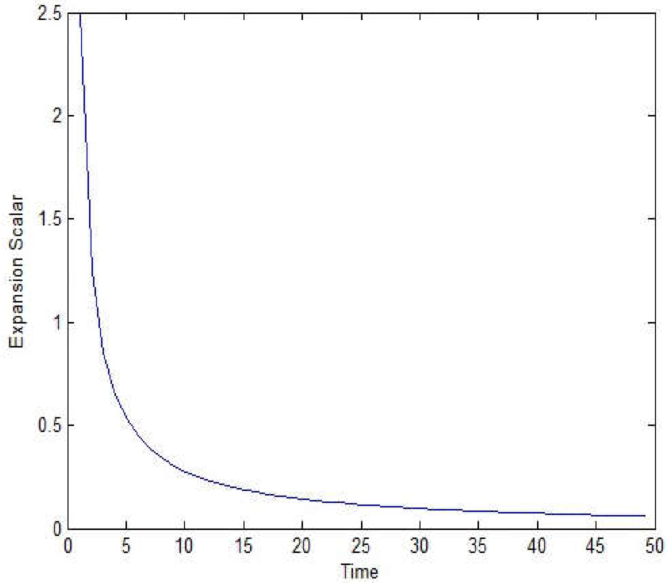

Fig. 2. Expansion scalar Vs time

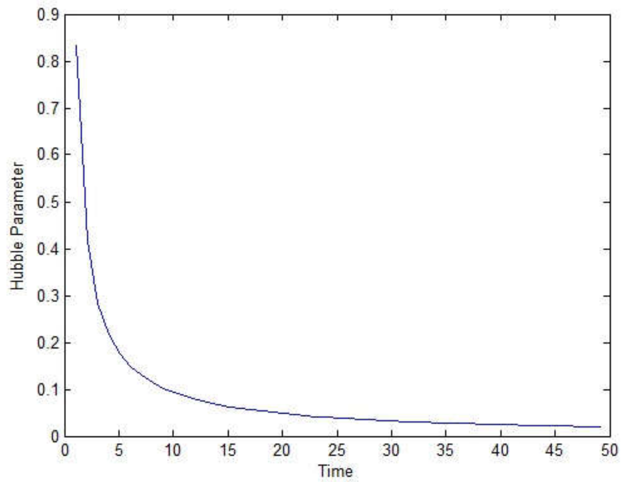

Fig.3. Hubble parameter Vs time

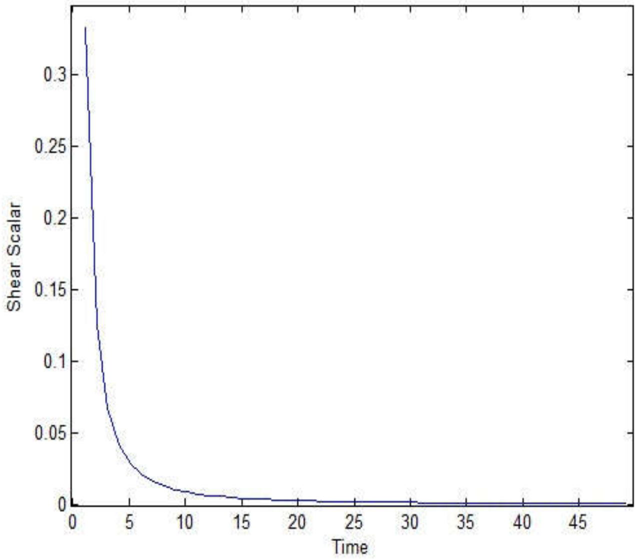

Fig. 4. Sheare scalar Vs time

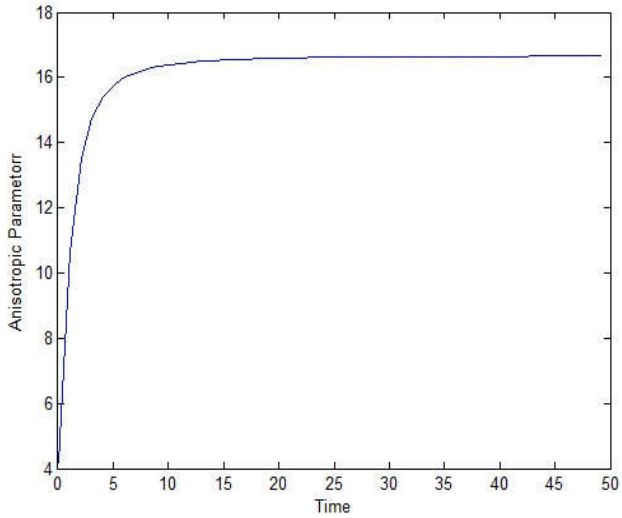

Fig. 5. Anisotropic Parameter vs time

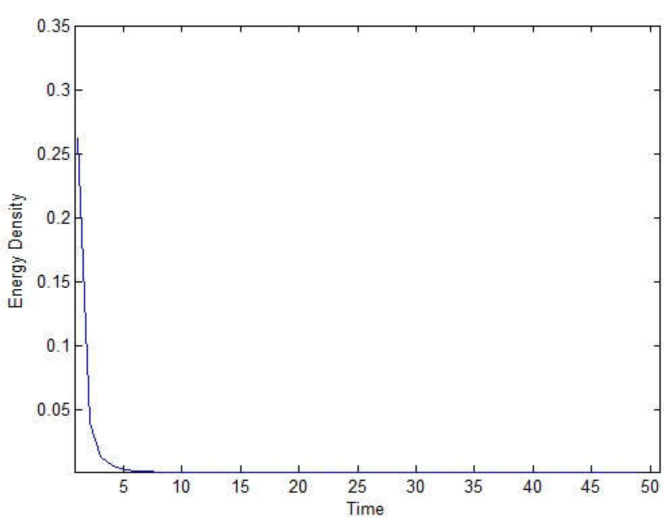

Fig.6. Energy density vs time

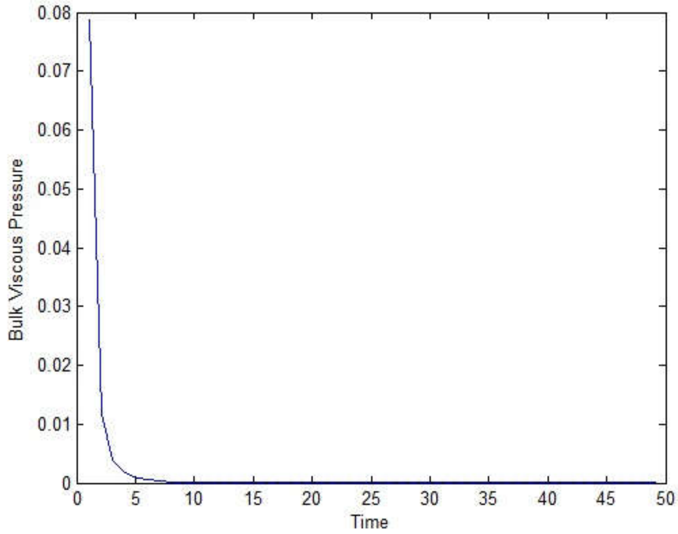

Fig. 7. Bulk viscous pressure vs time

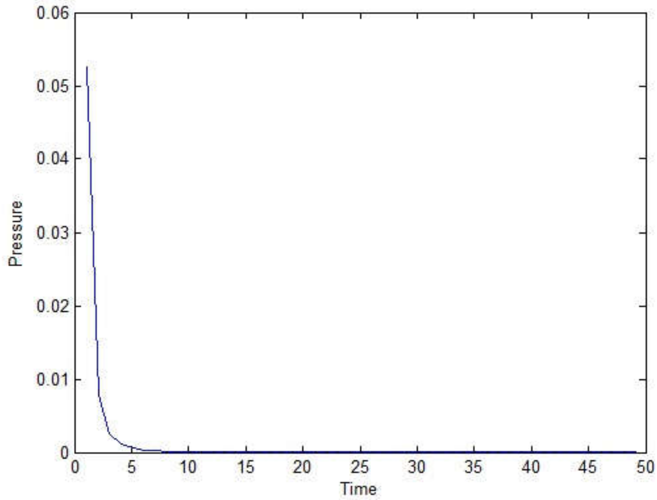

Fig. 8. Pressure vs time

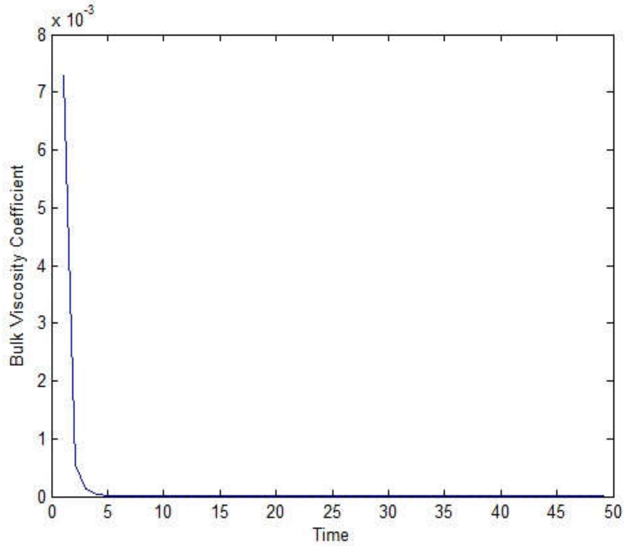

Fig.9. Bulk viscous Coefficient vs time

Conclusion

In this paper, we have studied the behavior of Bianchi Type-V cosmological model filled with bulk viscous fluid as a source of matter in $f(R)$ theory of gravity. The proposed law of variation of scale factor as increasing function of time which yields a time dependent form of deceleration parameter which indicates model of the universe is expanding from initial decelerated phase to present accelerating phase (as shown in Fig.(1). This is a good agreement with recent observations. The Fig.(2) indicates that the expansion scalar θ tends to infinity for $t \to 0$ and vanishes for $t \to \infty$. This model has point type singularity at $t = 1$. The all metric coefficients and volume scale factor are zero at this singularity and the physical parameters H, θ and σ^2 are all infinite at this point of singularity and decrease with time t as shown in Figure (2),(3),(4) respectively. Since $\frac{\sigma}{\theta} \neq 0 \ as \ t \to \infty$, the model approaches to anisotropy. Moreover anisotropic nature of the model is shown in Figure (5). The Figures (6),(7),(8),(9) show that, the parameters $\rho, \overline{p}, p, \xi$ are well behaved and are decreasing functions of time. Also shear scalar tends to infinity for $t \to \infty$ and vanishes for $t \to 0$. Hence the model describes a shearing and expanding universe. We hope that our model will be useful in the study of structure formation and accelerating expansion of the universe at present.

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