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REVIEW ARTICLE

ON SEMIGAMMA BI- IDEALS IN Γ- SEMINEAR-RINGS

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ABSTRACT

The concept of bi-ideals for near- rings was introduced by T.Tamizh Chelvam and N.Ganesan [6]. Subsequently the notion of quasi ideals and bi-ideals in Γ-near-rings was introduced by T.Tamizh Chelvam and N.Meenakumari [7]. An interesting special case of bi-ideals is given by the quasi-ideals of Steinfeld [5]. The concept of Γ-seminear-rings was introduced by Kyung Ho Kim [3]. In this paper we introduce the notion of semigamma quasi-ideals, semigamma bi-ideals and b-simple Γ- seminear-rings. Using the notion of semigamma bi-ideals, we show that the set of all semigamma bi-ideals of a Γ- seminear-ring form a moore system. Also we proved that the intersection of a semigamma bi-ideals of Γ- seminear-ring M and sub- Γ- seminear-ring S is again a semigamma bi- ideal of S. We define b-simple Γ- seminear-ring and prove certain equivalent conditions of Γ- seminear-field. Throughout this paper, by a Γ- seminear-ring M we shall mean a zero symmetric Γ- seminear-ring.

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INTRODUCTION

Preliminaries: An algebraic structure (N, +, .) is said to be a seminear-ring if i) (N, +) is a semigroup ii) (N, .) is a semigroup. iii) (a + b)c = ac + bc for all a, b, c ∈ N. Let M be an additive semigroup and Γ a nonempty set. Then M is called a right Γ-seminear-ring if there exists a mapping M × Γ × M → M satisfying the following conditions: i) $(a + b)\gamma c = a\gamma c + b\gamma c$ ii) $(a\gamma b)\beta c = a\gamma(b\beta c)$ for all a, b, c ∈ M and γ, β ∈ Γ. Let M be a Γ-seminear-ring under the mapping f : M × Γ × M → M. A Subsemigroup A of M is called a sub Γ-seminear-ring of M if A is a Γ-seminear-ring under the restriction of f to A × Γ × A → A. Let S and Γ be two nonempty sets. Then S is called a Γ- Semigroup if there exists a mapping from S × Γ × S → S which maps (a, α, b)→aαb satisfying the condition: $(a\gamma b)\beta c = a\gamma(b\beta c)$ for all a, b, c ∈ S and γ, β ∈ Γ. A nonempty subset A of a Γ-semigroup S is called a Γ-subsemigroup of S if AΓA ⊆ A. A right Γ- seminear-ring M is said to have an absorbing zero '0' if i) a + 0 = 0 + a = a ii) $a\gamma 0 = 0\gamma a = 0$, hold for all a ∈ M and $\gamma \in \Gamma$. (M, +, .) is a Γ-seminear-field if i) (M, +) is a semigroup ii) (M*, Γ) is a group (M* is M without addition zero, if it has one) iii) $(a + b)\gamma c = a\gamma c + b\gamma c$ for all $a, b, c \in M$ and $\gamma \in \Gamma$. $M_0 = \{m \in M / m\gamma 0 = 0 \text{ for all } \gamma \in \Gamma\}$ is called the zero-symmetric part of M. A Γ-seminear-ring M is called zero-symmetric, if M = M_0.

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In a Γ-seminear-ring M if there exists an element 'e' such that $a\gamma e = e\gamma a = a$ for all a ∈ M then M is called a Γ-seminear-ring with identity element. An element a ∈ M is said to be idempotent if aγa = a ∀ γ ∈Γ.

Semigamma bi- ideals in Γ-seminear-ring

In this section, we introduce the notion of semigamma bi-ideals in Γ-seminear-ring. Also we study the properties of semigamma bi-ideals.

Definition 2.1

A Γ-subsemigroup Q of (M, +) is said to be a semigamma quasi-ideal of M if

(QΓM) ∩ (MΓQ) ⊆ Q.

Definition 2.2

A Γ-subsemigroup B of (M, +) is said to be a semigamma bi-ideal of M if BΓMΓB ⊆ B.

Example 2.3

Consider the Γ-seminear-ring defined by the Klien's four group $\{0, a, b, c\}$ with Γ= $\{\gamma_1, \gamma_2\}$ where γ_1, γ_2 are given by the schemes 7: (0,7,11,1) and 12: (0,7,0,7) (see p.408, Pilz [4]).

γ_1	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

γ_2	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	0	0
c	0	a	0	a

In this Γ-seminear-ring, {0,a} and {0, b} are semigamma bi-ideals.

Proposition 2.4

The set of all semigamma bi-ideals of a Γ-seminear-ring M form a Moore system on M.

Proof

Let $\{B_i\}_{i\in I}$ be a set of semigamma bi-ideals in M. Let B = $\cap_{i\in I} B_i$. Then BΓMΓB ⊆ B_iΓMΓB_i, ∀ i. Therefore B is a semigamma bi-ideal of M.

Remark 2.5

Every semigamma quasi-ideal is a semigamma bi- ideal.

Proof

For, if Q is a semigamma quasi-ideal, then (QΓM)∩(MΓQ) ⊆ Q. Now, QΓMΓQ = QΓ(M∩ M)ΓQ = (QΓM) ∩ (MΓQ) ⊆ Q. Therefore Q is a semigamma bi- ideal. But the converse is not true.

Example 2.6

Consider the Γ-seminear-ring M defined by the Klien's four group {0, a, b, c} with Γ= $\{\gamma_1, \gamma_2\}$ where γ_1, γ_2 are given by the schemes 1: (0, 13,0,13) (see p.408, Pilz [4]) and (0,13,0,0)

γ_1	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

γ_2	0	a	b	c
0	0	0	0	0
a	0	b	0	0
b	0	0	0	0
c	0	b	0	0

In this Γ-seminear-ring M, {0,a} and {0, c} are semigamma bi-ideal but not semigamma quasi-ideal.

Proposition 2.7

Let M be a Γ-seminear-ring in which every semigamma quasi-ideal is idempotent. Then for left Γ-subsemigroup L and right Γ-subsemigroup R of N, RΓL = R∩L ⊆ $L\Gamma R$ is true.

Proof:

Let A and B be two semigamma quasi-ideals in M. Then by proposition 2.4, A∩B is also a semigamma quasi-ideal. By the assumption on semigamma quasi-ideals we have A∩ B = (A ∩ B)Γ(A ∩ B) ⊆ (AΓB) ∩ (BΓA). On the other hand, we have (AΓB) ∩ (BΓA) ⊆ (AΓM) ∩ (MΓA) ⊆ A and analogously, (AΓB) ∩ (BΓA) ⊆ B. Hence (AΓB) ∩ (BΓA) ⊆ A ∩ B. Hence A ∩ B = (AΓB) ∩ (BΓA). Since one sided Γ-subsemigroup are always semigamma quasi-ideals, we have R∩ L = (RΓL) ∩ (LΓR) ⊆ RΓL for a left Γ-subsemigroup L and right Γ-subsemigroup of M. Trivially RΓL ⊆ R∩ L and so RΓL = R∩ L = LΓR

Proposition 2.8

Let R and L be respectively right and left Γ-subsemigroups of M. Then any subsemigroup B of M such that RΓL ⊆ B ⊆ R∩L is a semigamma bi- ideal of M.

Proof:

For a subsemigroup B of (M, +) with RΓL ⊆ B ⊆ R∩L, we have BΓMΓB ⊆ (R∩L)ΓMΓ(R∩L) ⊆ RΓMΓL ⊆ RΓ L ⊆ B and so B is a semigamma bi- ideal of M.

Proposition 2.9

If B is a semigamma bi-ideal of M and S is a sub Γ-seminear-ring of M, then B∩S is a semigamma bi- ideal of S.

Proof:

Since B is a semigamma bi- ideal of M, BΓMΓB ⊆ B. Let C = B∩S. Now, CΓSΓC = (B∩S)ΓSΓ(B∩S) ⊆ (BΓSΓB)∩S ⊆ B∩S = C. Hence C is a semigamma bi-ideal of S.

Proposition 2.10

Let M be a Γ-seminear-ring. If B is a semigamma bi- ideal of M, then Bγn and n'γB for all $\gamma \in \Gamma$ are semigamma bi- ideal of M where n, n'∈ M and n' is distributive.

Proof:

Clearly Bγn is a subsemigroup of (M, +) ∀ γ ∈Γ. Also (Bγn)ΓMΓ(Bγn) ⊆ BΓMΓ(Bγn) ⊆ Bγn and so we get that Bγn is a semigamma bi- ideal of M. Since n' is distributive, n'γB is a subsemigroup of (M, +) for all γ ∈Γ and hence n'γB is a semigamma bi- ideal of M.

Corollary 2.11

If B is a semigamma bi- ideal of M and b is a distributive element in M, then bγBγc is a semigamma bi- ideal of M for c ∈ M and for all $\gamma \in \Gamma$.

Proposition 2.12

If B is a semigamma bi- ideal and sub Γ-seminear-ring of a Γ-seminear-ring M and C is a semigamma bi- ideal of the Γ-seminear-ring B such that $C^2 = C$, then C is a semigamma bi-ideal of the Γ-seminear-ring M.

Proof:

Since C is a semigamma bi- ideal of the Γ-seminear-ring B we have, CΓBΓC ⊆ C Now, CΓMΓC = C^2ΓMΓC^2=CΓ(CΓMΓC)ΓC ⊆ CΓ(BΓMΓB)ΓC ⊆ CΓBΓC ⊆ C. Hence C is a semigamma bi- ideal of the Γ-seminear-ring M.

Theorem 2.13

Let M be a Γ-seminear-ring. Let B be a semigamma bi- ideal of the Γ-seminear-ring M and A be a non-empty subset of M, then following are true.

- BΓA is a semigamma bi- ideal of the Γ-seminear-ring M.
- AΓB is a semigamma bi- ideal of the Γ-seminear-ring M.

Proof

- We see that (BΓA)Γ(BΓA) = (BΓAΓB)ΓA and (BΓA)ΓMΓ(BΓA) = (BΓAΓMΓB)ΓA. Since B is a semigamma bi-ideal of the Γ-seminear-ring M, (BΓA)Γ(BΓA) = (BΓAΓB)ΓA ⊆ BΓA and (BΓA)ΓMΓ(BΓA) = (BΓAΓMΓB)ΓA ⊆ (BΓMΓB)ΓA ⊆ BΓA. Therefore BΓA is a semigamma bi- ideal of the Γ-seminear-ring M.
- Similar to i).

b-simple Γ-seminear-ring

Definition 3.1

A Γ-seminear-ring M is said to be b-simple, if it has no proper semigamma bi- ideals.

Example 3.2

Consider the Γ-seminear-ring $(Z_5,+,\Gamma)$ under $\Gamma = \{\gamma_1,\gamma_2\}$ where γ_1,γ_2 are given by the schemes 2:(0,1,0,0,0) and 3:(0,1,1,0,0).(see p.408, Pilz [4])

γ_1	0	1	2	3	4
0	0	0	0	0	0
1	0	1	0	0	0
2	0	2	0	0	0
3	0	3	0	0	0
4	0	4	0	0	0

γ_2	0	1	2	3	4
0	0	0	0	0	0
1	0	1	1	0	0
2	0	2	2	0	0
3	0	3	3	0	0
4	0	4	4	0	0

Since $(Z_5,+)$ has no proper Γ-subsemigroup, the above Γ-seminear-ring is b-simple.

Lemma 3.3

Let M be a Γ-seminear-ring. Then the following conditions are equivalent.

- M is a Γ-seminear-field
- $M_d \neq \{0\}$ and for all m ∈ M*, Mγm = M for all γ ∈Γ.

Proof:

i)⇒ii) Since M is a Γ-seminear-field, it contains the identity element, which is also distributive. Clearly Mγm ⊆ M. For m ∈ M, m = eγm ∈ Mγm implies M ⊆ Mγm. Thus M = Mγm.

ii)⇒i) For all a, b ∈ M*, there exists a', b' ∈ M* such that b'γ_1b = a and a'γ_2a = b' for all γ_1,γ_2 of Γ. Thus a'γ_2(aγ_1b) = (a'γ_2a) γ_1b = b'γ_1b = a ≠ 0 so aγ_1b ≠ 0. Hence M contains no zero divisors. Let d ≠ 0 ∈ M be a distributive element. Then there exists e ∈ M such that eγd = d ∀ γ ∈Γ. Now $(d\gamma e - d)\gamma$d = 0⇒ dγe = d. Let n ∈ M*. Then dγ(eγn-n) = (dγe)γn-dγn = dγn-dγn = 0. This implies eγn = n ∀γ ∈Γ. Similarly nγe = n ∀ γ ∈Γ. Therefore 'e' becomes two-sided identity in M. Finally for all n ∈ M*, there exists n' ∈ M* such that n'γn = e. Hence M is a Γ-seminear-field.

Remark 3.4

It is clear that any Γ-seminear-field is b-simple. However, any b-simple Γ-seminear-ring is not in general a Γ-seminear-field and in the following theorem we obtain the necessary and sufficient condition for a b-simple Γ-seminear-ring to a Γ-seminear-field.

Lemma 3.5

Let M be a Γ-seminear-ring with more than one element and an absorbing zero. Then the following conditions are equivalent.

- M is a Γ-seminear-field
- M is b-simple, $M_d \neq \{0\}$ and for 0 ≠ n ∈ M, there exists n' ∈ M such that n'γn ≠ 0 for every γ ∈ Γ

Proof

i) ⇒ii) If M is a Γ-seminear-field, then {0} and M are the only semigamma bi-ideals of M. For if {0} ≠ B is a semigamma bi-ideal of M, clearly MΓb ⊆ M. On other hand, n ∈ M, n = nγe = nγ(b'γb) = (nγb')γb ∈ MΓb implies M ⊆ MΓb. Hence M = MΓb. Similarly, M = bΓM. Now, M = M^2 = MΓM = (bΓM)Γ(MΓb) ⊆ bΓMΓb ⊆ B. Therefore M = B. Hence M is b-simple and the identity in M satisfies the required conditions.

ii)⇒i) Since $M_d \neq \{0\}$, there exists d ∈ M_d and dγd' = dγ(d' + 0) = dγd' + dγ0. This implies that dγ0 = 0. We know that M_0 is a semigamma bi-ideal of M and since M is b-simple, we get M = M_0. Let 0 ≠ n ∈ M, then by proposition 2.10, Mγn is a semigamma bi-ideal of M and 0 ≠ n'γn ∈ Mγn for some n' ∈ M. Since M is b-simple, M = Mγn. Therefore by lemma 3.3, M is a Γ-seminear-field.

Theorem 3.6

Let M be a Γ-seminear-ring then M is b-simple Γ-seminear-ring iff M = mΓMΓm ∀ m ∈ M.

Proof:

Let M be a b-simple Γ-seminear-ring. Let m ∈ M. Then by theorem 2.13, mΓMΓm is a semigamma bi-ideal of M. Then M = mΓMΓm. Let B be a semigamma bi-ideal of M. Let b ∈ M, then M = bΓMΓb ⊆ BΓMΓB ⊆ B ⇒ M ⊆ B ⇒ M = B. Therefore M is a b-simple Γ-seminear-ring.

REFERENCES

Chinnaraj, P., Perumal, R. 2016. A Note on P(r, m) Γ-seminear-rings, *International Journal of Pure and Applied Mathematics,* Vol 106, No.6,85-92.

Kaushik, J. P. and Moin Khan, 2008. On bi-Γ-Ideal in Γ-Semirings, *Int. J. Contemp. Math. Sciences,* Vol.3, no.26, 1255-1260.

Kyung Ho Kim, 2001. On prime and Semi prime ideals in Gamma-seminear-rings, Scientiae Mathematicae Japonicae Online, Vol.4., 885-889.

O. Steinfeld, *Acta.Sci. Math.* (Szeged)17 (1956), 170-80

Pilz Gunter, Near-Rings, North Holland, Amsterdam, 1983.

Tamizh Chelvam, T. and N. Meenakumari, 2004. Bi-ideals of Gamma Near-Rings, *Southeast Asian Bulletin of Mathematics,* 27: 983-988.

Tamizh Chelvam, T. and N. Ganesan 1987. On Bi-ideals of Near-Rings, *Indian J. Pure appl. Math.* 18(11),1002-1005 Nov.

Volety V. S. 2011. Ramachandram, Commutativity of Seminear-rings, *Journal of Science and arts,* year 11, No.4 (17), pp.367-368.

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