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INTERNATIONAL JOURNAL OF CURRENT RESEARCH

International Journal of Current Research
Vol. 5, Issue, 5, pp.1168-1171, May, 2013

ISSN: 0975-833X

RESEARCH ARTICLE

NUMERICAL SOLUTION OF TYPICAL INITIAL VALUE PROBLEMS USING HAAR WAVELET TRANSFORM METHOD

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ARTICLE INFO

Article History:
Received 02nd March, 2013
Received in revised form
20th April, 2013
Accepted 27th April, 2013
Published online 12th May, 2013

Key words:
Typical Initial value problems;
Haar wavelet transform method;
Taylor's series method;
R. K. Method; singular non-linear.

ABSTRACT

Wavelet analysis is a recently developed mathematical tool for many problems. In this paper, we apply Haar wavelet transform method to solve typical Initial value problems. Numerical examples are shown which including first, second, higher order differential equations with constant and variable coefficients, singular non-linear initial value problems. The results show that the haar wavelet transform method is quite reasonable when compare to R. K. Method and exact. R. K. Methods they are distinguished by their orders in the sense that they agree with Taylor's series solution up to terms of h^r, where r is the order of the method. These methods do not demand prior computation of higher derivatives of $y(t)$ as in Taylor's series method. Fourth-order Runge –Kutta methods are widely used for finding the numerical solutions of linear or non-linear ordinary differential equations, the development of which is complicated algebraically. Also, Numerical accuracy of the R. K. Method is not quite good as compared to exact and haar wavelet transforms method in case of higher order and typical initial value problems.

INTRODUCTION

Haar wavelet transform is the simplest wavelet. Haar transform or Haar wavelet transform has been used as an earliest example for orthonormal wavelet transform with compact support. The Haar wavelet transform is the first known wavelet and was proposed in 1910 by Alfred Haar. They are step functions (piecewise constant functions) on the real line that can take only three values. Haar wavelets, like the well-known Walsh functions (Rao 1983), form an orthogonal and complete set of functions representing discretized functions and piecewise constant functions. A function is said to be piecewise constant if it is locally constant in connected regions. The Haar transform is one of the earliest examples of what is known now as a compact, dyadic, orthonormal wavelet transform. The Haar function, being an odd rectangular pulse pair, is the simplest and oldest orthonormal wavelet with compact support. In the mean time, several definitions of the Haar functions and various generalizations have been published and used. They were intended to adopt this concept to some practical applications as well as to extend its applications to different classes of signals. Haar functions appear very attractive in many applications as for example, image coding, edge extraction, and binary logic design.

After discretizing the differential equations in a conventional way like the finite difference approximation, wavelets an be used for algebraic manipulations in the system of equations obtained which lead to better condition number of the resulting system. The previous work in system analysis via Haar wavelet transform was led by Chen and Hsiao[1], who first derived a Haar operational matrix for the integrals of the Haar function vector and put the application for the Haar analysis into the dynamical systems. Then, the pioneer work in state analysis of linear time delayed systems via Haar wavelet transform was laid down by Hsiao[2], who first proposed a Haar product matrix and a coefficient matrix. Hsiao and Wang proposed a key idea to transform the time varying function and its product with states into a Haar product matrix. Meanwhile in numerical analysis, wavelet trans form based algorithms have become an important tools because of the properties of localization. One of the popular families of wavelet is Haar wavelet transform. Due to its simplicity, Haar wavelet transform had become an effective tool for solving many problems, among that are higher order as well as nonlinear ODEs by using R. K. Method and Wavelet transform method. Operator or matrix representation is expanded in a wavelet transform basis. Sometime, the works for writing up the operational matrices are quite tedious especially when ones intend to perform the calculation in high resolution [3-6].

The purpose of this paper is to investigate the effect of the proposed matrix P for solving linear or nonlinear ODEs with initial conditions. In section 2, Haar wavelet transform preliminaries with a simplified mathematical formulation of the proposed matrix P. In section 3, the Haar wavelet transform method for solving ODEs with initial conditions is presented. Initially, singular nonlinear IVPs and then linear IVPs with variable coefficients of different orders are solved using Haar wavelet transform method is given in section 4. In section 5, Conclusion of the numerical findings. In the subsequent sections results of these problems are presented in the form of graphs and Tables.

HAAR WAVELET TRANSFORM PRELIMINARIES

The haar wavelet family for is defined as follows.

$$h_i(t)\begin{cases} 1 \quad for \quad t\epsilon\left[\frac{k}{m},\frac{k+0.5}{m}\right[\\ -1 \quad for \quad t\epsilon\left[\frac{k+0.5}{m},\frac{k+1}{m}\right[\\ 0 \quad\quad elsewhere \end{cases} \tag{1.1}$$

Integer $m = 2^j (j = 0,1,2,.....J)$ indicates the level of the wavelet; $k = 0,1,2,....,m-1$ is the translation parameter. Maximal level of resolution is J. The index i is calculated

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according to the formula $i = m + k + 1;$ in the case of minimal values $m = 1, k = 0,$ we have $i = 2,$ the maximal value of i is $2M = 2^{(J+1)}.$ It is assumed that the value $i = 1$ corresponds to the scaling function for which $h_1 \equiv 1$ in [0,1].

The haar function can be described as a step function as a step function $\psi(t)$ as follows:

$$\psi(t) = \begin{cases} 1 & 0 \le t \le 1/2 \\ -1 & 1/2 \le t \le 1 \\ 0 & elsewhere \end{cases} \tag{1.2}$$

In order to perform wavelet transform, Haar wavelet uses translations and dilations of the function, i.e. the transform make use of following function

$$\psi(t) = \psi(2^j t - k) \tag{1.3}$$

Translation / shifting $\psi(t) = \psi(t-k)$

Dilation / scaling $\psi(t) = \psi(2^j t)$

Where this is the basic works for wavelet transform expansion

With the dilation and translation process as in Eqn. (1.3), ones can easily obtain father wavelet, daughter wavelet, granddaughter wavelet and so on as in the matrix form, the haar matrix for resolution up to 3 levels is given by

$$h_i(t) = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{bmatrix}$$

The pattern of the Haar wavelet transform undergo translation and dilation process and its matrix pattern are observed when the Haar wavelet transform been used in solving ordinary differential equations.

The pattern of design of the matrices P is similar to Haar wavelet transform. For calculation of P matrices, we focus on the elements need to be counted as following formulas.

$$\frac{1}{\alpha!}\frac{1}{(2m)^\alpha}\left[(C(\frac{m}{2^L} + 2l - 1))^\alpha - 2(C(2l-1))^\alpha\right]$$

$$\alpha = n - v$$

$$L = 0, 1, 2, ..., j$$

$l = 1, 2, 3, ..., \dfrac{m}{2(2^L)}$, and where $C = B - A$

Using these formulas we get the matrix P is shown by

$$\begin{bmatrix}
1^\alpha & 3^\alpha & 5^\alpha & 7^\alpha & 9^\alpha & 11^\alpha & 13^\alpha & 15^\alpha & 17^\alpha & 19^\alpha & 21^\alpha & 23^\alpha & 25^\alpha & 27^\alpha & 29^\alpha & 31^\alpha \\
1^\alpha & 3^\alpha & 5^\alpha & 7^\alpha & 9^\alpha & 11^\alpha & 13^\alpha & 15^\alpha & 287 & 343 & 391 & 431 & 463 & 487 & 503 & 511 \\
1^\alpha & 3^\alpha & 5^\alpha & 7^\alpha & 79 & 103 & 119 & 127 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1^\alpha & 3^\alpha & 5^\alpha & 7^\alpha & 79 & 103 & 119 & 127 \\
1^\alpha & 3^\alpha & 23 & 31 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1^\alpha & 3^\alpha & 23 & 31 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1^\alpha & 3^\alpha & 23 & 31 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1^\alpha & 3^\alpha & 23 & 31 \\
1^\alpha & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1^\alpha & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1^\alpha & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1^\alpha & 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1^\alpha & 7 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1^\alpha & 7 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1^\alpha & 7 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1^\alpha & 7
\end{bmatrix}$$

In the matrix P we factored out the common factor $\dfrac{1}{\alpha!}\dfrac{C^\alpha}{(2m)^\alpha}$ for all the elements in the matrix, P [1, 2, 7].

HAAR WAVELET TRANSFORM METHOD

For solving typical initial value problem with higher order, say

$$a_1 y^{(n)}(t) + a_2 y^{(n-1)}(t) + ... + a_n y(t) = f(t), \tag{3.1}$$

where $t \in [A, B]$ and initial conditions $y^{(n-1)}(A), y^{(n-2)}(A), ..., y(A)$ are known.

We follow the work done by Lepik [8, 9]. Say intend to do until j^{th} level of resolution, hence we let $m = 2(2^j).$ The interval $[A, B]$ will be divided into m subintervals, hence $\Delta x = \dfrac{B-A}{m}$

And the matrices are in the dimension of $m \times m.$

Here we suggest the step the step procedures for easy understanding. Mainly, there are 7 step as shown in the procedure as follows.

Procedure

Step 1: We take $y^{(n)}(t) = \sum_{i=1}^{m} x_i h_i(t)$ where h is haar transform matrix, x_i is wavelet transform coefficients n is Highest order of the given ODEs and m is order of matrices.

Step 2: Obtain appropriate v order of $y(t)$ by using

$$y^{(v)}(t) = \sum_{i=1}^{m} x_i P_{n-v,i}(t) + \sum_{\sigma=0}^{n-v-1} \frac{1}{\sigma!}(t-A)^\sigma y_0^{(v+\sigma)}$$

Where v is the lowest order of the given ODEs, y_0 is the given initial conditions

Step 3: Replace $y^{(n)}(t)$ and all the value of $y^{(v)}(t)$ into the given problem.

Step 4: We get the system of Algebraic equations.

Step 5: Compute the wavelet transform coefficients, x_i.

Step 6: Obtain the numerical Solution by using $y(t)=\sum_{i=1}^{m} x_i P_{n-v,i}(t)$.

Step 7: Compare the numerical solution with R. K. Method.

NUMERICAL EXAMPLES

In this section, we illustrate the initially two singular nonlinear IVPs and two IVPs with variable coefficients of different orders and are solved using Haar wavelet transform method.

Example 1: First consider the Singular IVP [10]

$$(t^{1/2}y')' = 5t^{3/2}-1, t\in(0,1) \quad (4.1)$$

with initial conditions $y(0)=y'(0)=0$

The exact solution is given by

$$y(t)=\frac{2}{3}(t^3-t^{3/2}). \quad (4.2)$$

Using above explained procedure of Haar Wavelet transform method of solution is as follows,

Step 1: $y''(t)=\sum_{i=1}^{m} a_i h_i(t)$.

Step 2:

$$y'(t)=\sum_{i=1}^{m} a_i P_{1,i}(t)+\sum_{\sigma=0}^{2-1-1}\frac{1}{\sigma!}(t-0)^{\sigma} y_0^{\sigma}=\sum_{i=1}^{m} a_i P_{1,i}(t).$$

Step 3: Substitute $y''(t)$ and $y'(t)$ in Eqn. (4.1), then we get step 4.

$$\sum_{i=1}^{m} a_i\left[t^{1/2}h_i(t)+\frac{t^{-1/2}}{2}P_{1,i}(t)\right]=5t^{3/2}-1$$

Step 5: Compute the wavelet transform coefficients, a_i.

Step 6: Obtain the numerical Solution by using $y(t)=\sum_{i=1}^{m} a_i P_{n-v,i}(t)$.

Step 7: Compare the numerical solution with R. K. Method the computational results are presented in Table 1.

Example 2: We solve the differential equation [10]

$$(t^{\alpha}y')' = \beta t^{\alpha+\beta-2}((\alpha+\beta-1)+\beta t^{\beta})y, \quad 0<t<1 \quad (4.3)$$

with initial conditions $y(0)=y'(0)=0$

The exact solution is given by

$$y(t)=e^{t^{\beta}}.$$

The experimental results are given $\alpha=1/2\ \&\ \beta=3$.

The computational results are presented in Table 2 and Figure 2.

Example 3: We consider the higher order with variable coefficient differential equation [11]

$$y^{IV}(t)+ty(t)=16\sin 2t+t\sin 2t,\ t\in[0,1] \quad (4.4)$$

with initial conditions $y(0)=0, y'(0)=2, y''(0)=0, y'''(0)=-8$

The exact solution is given by

$$y(t)=\sin 2t.$$

The computational results are presented in Table 3 and Figure 3.

Example 4: We consider the first order with variable coefficient differential equation [11]

$$y'(t)+e^{t}y(t)=t^2,\ t\in[0,1] \quad (4.5)$$

with initial conditions $y(0)=4.$

The series solution is given by

$$y(t)=4-4t+t^3+\frac{1}{12}t^4$$ (up to first four nonzero terms only)

The computational results are presented in Table 4 and Figure 4.

Table 1. Numerical results for Example-1

x	Wavelet Solution	R. K. Solution	Exact Solution
0.0313	0.0005	-0.0053	-0.0037
0.0938	0.0071	-0.0218	-0.0186
0.1563	0.0044	-0.0429	-0.0386
0.2188	-0.0011	-0.0665	-0.0612
0.2813	-0.0212	-0.0908	-0.0846
0.3438	-0.0350	-0.1144	-0.1073
0.4063	-0.0683	-0.1359	-0.1279
0.4688	-0.1038	-0.1541	-0.1453
0.5313	-0.1479	-0.1680	-0.1582
0.5938	-0.1689	-0.1761	-0.1655
0.6563	-0.2007	-0.1776	-0.1660
0.7188	-0.2307	-0.1713	-0.1587
0.7813	-0.2743	-0.1594	-0.1425
0.8438	-0.2943	-0.1373	-0.1162
0.9063	-0.3219	-0.1041	-0.0790
0.9688	-0.3342	-0.0587	-0.0296

Table 2. Numerical results for Example-2

x	Wavelet Solution	R. K. Solution	Exact Solution
0.0313	1.0062	1.0321	1.0000
0.0938	1.0507	1.0435	1.0008
0.1563	1.0849	1.0541	1.0038
0.2188	1.1325	1.0672	1.0105
0.2813	1.1454	1.0851	1.0225
0.3438	1.2031	1.1099	1.0415
0.4063	1.2175	1.1438	1.0693
0.4688	1.2373	1.1894	1.1085
0.5313	1.2009	1.2162	1.1618
0.5938	1.2820	1.2324	1.2328
0.6563	1.3311	1.2738	1.3266
0.7188	1.3874	1.3455	1.4496
0.7813	1.4312	1.4857	1.6110
0.8438	1.5274	1.6730	1.8234
0.9063	1.5960	1.9233	2.1050
0.9688	1.7069	2.2601	2.4822

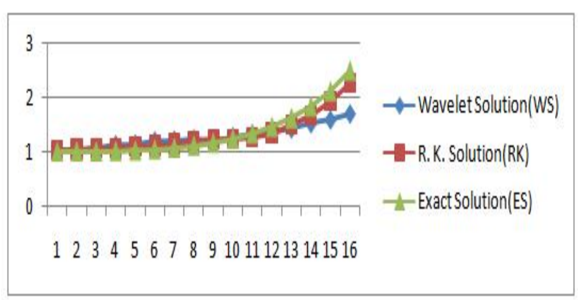

Fig. 2. Comparison of Numerical Example-2

Table 3. Numerical results for Example-3

x	Wavelet Solution	R. K. Solution	Exact Solution
0.0313	0.0625	0.0845	0.0625
0.0938	0.1864	0.1965	0.1864
0.1563	0.3075	0.3685	0.3074
0.2188	0.4237	0.4733	0.4237
0.2813	0.5339	0.5631	0.5333
0.3438	0.6356	0.6556	0.6346
0.4063	0.7276	0.7566	0.7260
0.4688	0.8086	0.8196	0.8061
0.5313	0.8870	0.8997	0.8736
0.5938	0.9465	0.9860	0.9274
0.6563	0.9927	1.0003	0.9668
0.7188	1.0250	1.0952	0.9911
0.7813	1.0430	1.1232	1.0000
0.8438	1.0465	1.1463	0.9932
0.9063	1.0357	1.2560	0.9709
0.9688	1.0110	1.2844	0.9335

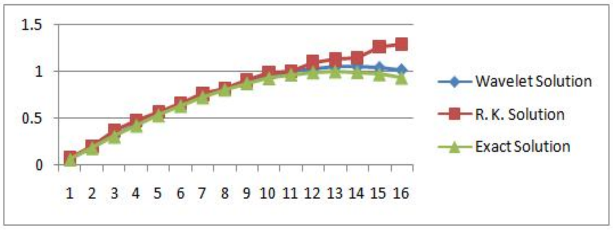

Fig. 3. Comparison of Numerical Example-3

Table 4. Numerical results for Example-4

x	Wavelet Solution	R. K. Solution	Series Solution
0.0156	3.9390	3.9710	3.9375
0.0469	3.8125	3.8232	3.8126
0.0781	3.6796	3.6801	3.6880
0.1094	3.5425	3.5819	3.5638

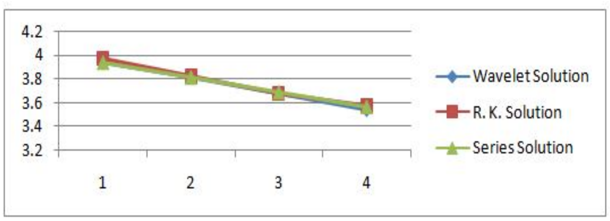

Fig. 4 Comparison of Numerical Example-4

Conclusion

The main goal of this paper is to demonstrate that Haar wavelet transform method is a powerful tool for solving typical initial value problems. The algorithm and procedure have been applied to use Haar wavelet transform method in solving typical initial value problems. The results is comparable to the R.K. Method and exact solution.

Acknowledgments

The authors wish to thank The University Grants Commission, New Delhi, Govt. of India for the grant of the Project No. F. 38-102/2009 (SR) under which the work was carried out.

REFERENCES

Chen, C.F. and Hsiao C.H., (1997), "Haar wavelet method for solving lumped and distributed-parameter systems", IEE Proc. Control Theory Appl. Vol 144, pp. 87-94.

Hsiao C.H., (2004), "Haar wavelet approach to linear stiff systems." Mathematics and Computers in Simulation vol. 4, pp. 561-567.

Potter, M. C., Goldberg, J. L. and Aboufadel, E., *Advanced Engineering Mathematics*. 3rd Ed. New York: Oxford University Press, pp.670-699.

Aboufadel, E. and Schlicker, S., (2003) "Wavelets: Wavelets for undergraduates." [Online] Available: http://www.gvsu.edu/math/wavelets/undergrad.htm

Aboufadel , E. and Schlicker, S., (1999), *Discovering Wavelets*, New York: John Wiley & sons, Inc, pp. 12-18.

O'Neil, P.V., (2003), *Advanced Engineering Mathematics*. 5th Ed. Thomson Brooks/Cole, pp.841-854.

Chen, C.F. and Hsiao C.H., (1999), "Wavelet approach to optimizing dynamic systems." IEE Proc. Control Theory, Appl. Vol 146, pp. 213-219.

Lepik, U., (2008), "Haar wavelet method for solving higher order differential equations", Vol. 1, No. 08.

Lepik, U., (2005), "Numerical solution of differential equations using Haar wavelets." Mathematics and Computers in Simulation vol 68, pp. 127-143.

Bhowmik, S. K., and Banu, S., (Jan-2004), Shooting method for solving singular BVP of ODE, Dhaka, Univ. J. Sci., 52-2, 241-247.

Zhongdi C., (March-2007), "Numerical method for a class of singular non-linear boundary value problems using Green's Functions" International Journal of Computer Mathematics vol.84,No.3, 403-410.

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