ISSN: 0975-833X

Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 7, Issue, 01, pp.11942-11953, January, 2015

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

RESEARCH ARTICLE

STUDY ON POSSIBISLITIC MULTIOBJECTIVE SOLID TRANSPORTATION PROBLEMS

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ARTICLE INFO

Article History:
Received 05th October, 2014
Received in revised form
05th November, 2014
Accepted 27th December, 2014
Published online 31st January, 2015

Key words:
Multiobjective Solid Transportation Problems,
Possibilistic Variables,
α-Possibly Efficient solution,
α-Possibly Optimal Solution,
Stability Notions.

ABSTRACT

The Solid Transportation Problem (STP) arises when bounds are given on three item properties. Usually, these properties are supply, demand, and type of product or mode of transport (conveyance). In this paper, the efficient solutions and stability of multiobjective solid transportation problem (Poss MOSTP) with possibilitic coefficients $\tilde{c}^{r}_{ijk}$ and / or possibilistic supply quantities $\tilde{a}_i$ and / or possibilistic demand quantities $\tilde{b}_j$ and / or possibilistic conveyances $\tilde{e}_k$ are investigated. We consider the problem by incorporating possibilistic data into the objective functions coefficients, supplies, demands and conveyances. The concept of α-possibly efficient is specified in which the ordinary efficient solution is extended based on the α-cut of a possibilistic variables. A solution of the weighting problem of Poss MOSTP is deduced. A necessary and sufficient condition for such a solution is established. The basic notions like the solvability set and the stability set of the first kind are defined and characterized that's to characterize the parametric optimal solution for the auxiliary problem. An algorithm for the determination of the stability set is proposed. Finally, a numerical example is given to illustrate the aspects of the developed results.

INTRODUCTION

The solid transportation problem (STP) is a generalization of the well-known transportation problem (TP) in which three item properties (supply, demand and conveyance) are taken into account in the constraint set instead of two (supply and demand), and its of great use in public distribution systems. The STP was first stated by Shell (1955). Although STP was forgotten for long time, because of the new existing solution methodologies, recently it is receiving the attention and the interest of the researchers in this field (Ida *et al*. 1995; and Bit *et al*. 1993). Haley (1962) introduced the solution procedure of STP which is an extension of the modified distribution method. Patel and Tripathy (1989)developed a computationally superior method for a STP with mixed constraints. Vajda (1988) proposed an algorithm for a multi-index transportation problem which is an extension of the modified distribution method. Basu *et al*. (1994) provided an algorithm for finding the optimum solution of solid fixed charge linear transportation problem. Li *et al*. (1997) designed a neural network approach for multicriteria STP. Jimenez and Verdegay (1996) developed a parametric approach for solving fuzzy STPs by an evolutionary algorithm (EA). Pandian and Natarajan (2010) are introduced the zeropoint method for finding an optimal solution to a classical TP. Pandian and Auradha, (2010) proposed a new method using the principle of zero point method introduced by Pandian and Natarajan (2010) for finding an optimal solution of STP. Ojha *et al*. (2010) formulated a STP with discounted costs, fixed charges and vechicle costs as a linear programming problem. Qualitative analysis of some basic notions such as the set of feasible parameters, the solvability set and the stability set of the first kind and the stability set of the second kind are introduced by Osman (1977). Luhandjula (1987) deals with multi-objective programming problems with possibilistic coefficients. Hussein (1998) introduced the complete solutions of multi-objective transportation problems with possibilistic coefficients. Sakawa and Yano (1989) provided the concept of -Pareto optimality of fuzzy parametric programs. Kassem (1998) introduced the multiobjective nonlinear programming problems with possibilistic variables and without differentiability for the considered objective functions. Ammar and Youness (2005) introduced the solution of multi-objective TP with fuzzy objectives, fuzzy sources, and fuzzy destinations.In this paper, we deal with a multi-objective solid transportation problem (Poss MOSTP) with possibilistic coefficients, possibilistic supply values, possibilistic demand values, and possibilistic conveyances.

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Ammar and Khalifa (2014) studied the multiobjective solid transportation problem with fuzzy numbers.

The concept of α-possibly efficient and α-parametric efficient solution are introduced. The relation between the above two concept of solutions is established. A parametric analysis is introduced to characterize the set of all α-parametric efficient solutions. An algorithm to determine the stability set of the first kind corresponding to one parametric efficient solution of Poss MOSTP is presented. A numerical example is given to illustrate and clarify the obtained results.

Preliminaries

In this section, we recall some definitions and lemma needed through the paper (Zadeh, (1970)).

Definition 1. A possibilistic variable y on $T \subseteq R^n$ is a variable characterized by a possibility distribution on $\pi_y(t)$.

This means that, if y is a variable taking values in T, then a possibility distribution π_y associated with y may be viewed as a fuzzy constraint on the values that may be assigned to y. such a distribution is characterized by a possibility distribution function $\pi_y : T \to [0, 1]$ which associated with each $t \in T$ the degree of compatibility of the variable y with the realization $t \in T$.

If T is a Cartesian product of $T_1, T_2, ..., T_n$, then $\pi_y(t_1, t_2, ..., t_n)$ is an n-array possibility distribution, i.e., $\pi_y(t) = (\pi_{y_1}(t_1), \pi_{y_2}(t_2), ..., \pi_{y_n}(t_n))$.

Definition 2. The α-cut of a possibilistic variable y is $y_\alpha = \{t \in T : \pi_y(t) \geq \alpha\}$.

Definition 3. A possibility distribution π_y on T is said to be convex if

$$\pi_y(\lambda t^1 + (1-\lambda)t^2) \geq \min(\pi_y(t^1), \pi_y(t^2)), \qquad \forall\ t^1, t^2 \in T,\ \lambda \in [0, 1].$$

It is appropriate to define the support of a possibilistic variable y in the case of $T \subseteq R^n$.

Definition 4. The support of a possibilistic variable y is

$$\sup(y) = \left\{ t \in T : \sup_{u \in N_\varepsilon(t)} (\pi_y(u)) > 0 \text{ for all } \varepsilon > 0 \right\},$$

where, $N_\varepsilon(t) = \{u \in T : \| u - t \| < \varepsilon\}$.

Lemma 1. $\operatorname{Sup}(y)$ is a closed set in T.

Proof. See, Hussein (1998).

Problem formulation

Consider the general possibilistic multiobjective solid transportation problem with possibilistic coefficients in the objective functions, possibilistic supplies, possibilistic demands, and possibilistic conveyances (Poss MOSTP)

(Poss MOSTP) $\min Z_r(x, \tilde{c}^r) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{\ell} \tilde{c}^r_{ijk} x_{ijk};\ r = 1, ..., q$

subject to

$$x \in G(\tilde{a}, \tilde{b}, \tilde{e}) = \left\{ x \in R^{m \times n \times \ell} : \sum_{j=1}^{n} \sum_{k=1}^{\ell} x_{ijk} = \tilde{a}_i;\ i = 1, ..., m; \right.$$

$\sum_{i=1}^{m}\sum_{k=1}^{\ell} x_{ijk} = \tilde{b}_j$, $j = 1, ..., n;$ $\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = \tilde{e}_k$,

$k = 1, ..., \ell;$ $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j = \sum_{k=1}^{\ell} \tilde{e}_k$, and

$x_{ijk} \ge 0,\ i = 1, ..., m;\ j = 1, ..., n;$

$k = 1, ..., \ell \Big\}$,

where, $\tilde{c}^r_{ijk}\ (r = 1, ..., q),\ \tilde{a}_i,\ i = 1, ..., m;\ \tilde{b}_j,\ j = 1, ..., n;\ \tilde{e}_k,\ k = 1, ..., \ell$ are possibilistic variables on R characterized by possibility distributions $\pi_{\tilde{c}^r_{ijk}}$, $\pi_{\tilde{a}_i}$, $\pi_{\tilde{b}_j}$ and $\pi_{\tilde{e}_k}$, respectively. For all outcomes it is assumed that all possibility distributions involved in Poss MOSTP are convex ones with compact (i.e., bounded and closed) supports and $y_0 = \sup(y)$ (see, Definitions 2-4).

Efficiency aspects

Definition 5. (α-possibly feasible actions): Let $\beta_1 = (\beta_{11}, \beta_{12}, ..., \beta_{1m})$, $B_{1i} \in [0, 1]$, $i = 1, 2, ..., m;$ $\beta_2 = (\beta_{21}, \beta_{22}, ..., \beta_{2n})$, $\beta_{2j} \in [0, 1]$, $j = 1, 2, ..., n$; and $\beta_3 = (\beta_{31}, \beta_{32}, ..., \beta_{3\ell}), \beta_{3k} \in [0, 1]$, $k = 1, 2, ..., \ell$. Then $x \in G(\tilde{a}, \tilde{b}, \tilde{e}) = \{x \in R^{m\times n\times \ell} : x_{ijk} \ge 0,\ i = 1, 2, ..., m;\ j = 1, 2, ..., n;\ k = 1, 2, ..., \ell\}$ is said to be α-possibly feasibly actions for problem (Poss MOSTP) if:

$$\text{Poss}\left(\sum_{i=1}^{m}\sum_{k=1}^{\ell} x_{ijk} = \tilde{b}_j\right) = \sup(\pi_{\tilde{b}_j}(b_j)) \ge \beta_{2j},\ \sum_{i=1}^{m}\sum_{k=1}^{\ell} x_{ijk} = b_j,$$
$j = 1, 2, ..., n$;

and

$$\text{Poss}\left(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = \tilde{e}_k\right) = \sup(\pi_{\tilde{e}k}(e_k)) \ge \beta_{3k},\ \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k,$$
$k = 1, 2, ..., \ell$,

where Poss denotes possibility.

Definition 6. (α-possibly efficient): A point $x^*(\tilde{c}) \in G(\tilde{a}, \tilde{b}, \tilde{e})$ is said to be α-possibly efficient to Poss MOSTP if there is no $x(\tilde{c}) \in G(\tilde{a}, \tilde{b}, \tilde{e})$ such that:

$$\text{Poss}(z_1(x, \tilde{c}^1) \le z_1(x^*, \tilde{c}^1), ..., z_{r-1}(x, \tilde{c}^{r-1}) \le z_{r-1}(x^*, \tilde{c}^{r-1}),$$
$$z_r(x, \tilde{c}^r) < z_r(x^*, \tilde{c}^r), z_{r+1}(x, \tilde{c}^{r+1}) \le z_{r+1}(x^*, \tilde{c}^{r+1}), ...,$$
$$z_q(x, \tilde{c}^q) \le z_q(x^*, \tilde{c}^q)) \ge \alpha, \quad \text{……………………(1)}$$

On account of the extension principle,

$$\text{Poss}(z_1(x, \tilde{c}^1) \le z_1(x^*, \tilde{c}^1), ..., z_{r-1}(x, \tilde{c}^{r-1}) \le z_{r-1}(x^*, \tilde{c}^{r-1}),$$
$$z_r(x, \tilde{c}^r) < z_r(x^*, \tilde{c}^r), z_{r+1}(x, \tilde{c}^{r+1}) \le z_{r+1}(x^*, \tilde{c}^{r+1}), ..., z_q(x, \tilde{c}^q)$$
$$\le z_q(x^*, \tilde{c}^q)) = \sup_{(c^1, c^2, ..., c^q) \in C} \min(\pi_{\tilde{c}^1}(c^1), \pi_{\tilde{c}^2}(c^2), ..., \pi_{\tilde{c}^{r-1}}(c^{r-1}),$$
$$\pi_{\tilde{c}^r}(c^r), \pi_{\tilde{c}^{r+1}}(c^{r+1}), ..., \pi_{\tilde{c}^q}(c^q)), \quad \text{……………………(2)}$$

where,

$$C = \{(c^1, c^2, ..., c^q) \in R^{q(m\times n\times \ell)} : z_1(x, c^1) \le z_1(x^*, c^1), ..., z_{r-1}(x, c^{r-1})$$
$$\le z_{r-1}(x^*, c^{r-1}), z_r(x, c^r) < z_r(x^*, c^r), z_{r+1}(x, c^{r+1}) \le z_{r+1}(x^*, c^{r+1}),$$
$$..., z_q(x, c^q) \le z_q(x^*, c^q)\} \quad (3)$$

For characterizing the α -possibly efficient solution for Poss MOSTP, let us consider the following α -parametric multi objective solid transportation problem (α -PMOSTP)

$$(\alpha\text{-PMOSTP}) \min z_r(x, c^r) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{\ell} c^r_{ijk}\, x_{ijk},\ r = 1, ..., q \quad (4)$$

subject to

$$x \in G(a, , e),\ c^r_{ijk} \in (\tilde{c}^r_{ijk})_\alpha,\ i = 1, ..., m;\ j = 1, ..., n;\ k = 1, ..., \ell;$$
$$r = 1, ..., q;\ a_i \in (\tilde{a}_i)_\alpha, i = 1, ..., m;\ b_j \in (\tilde{b}_j)_\alpha,\ j = 1, ..., n;$$
$$e_k \in (\tilde{e}_k)_\alpha, k = 1, ..., \ell, \sum_{i=1}^{m}(a_i)_\alpha = \sum_{j=1}^{n}(\tilde{b}_j)_\alpha = \sum_{k=1}^{\ell}(\tilde{b}_j)_\alpha$$
$$= \sum_{k=1}^{\ell}(\tilde{e}_k)_\alpha, \text{ and}$$
$$x_{ijk} \ge 0, i = 1, ..., m;\ j = 1, ..., n;\ k = 1, ..., \ell, \quad (5)$$

where $(\tilde{c}^r_{ijk})_\alpha, (\tilde{a}_i)_\alpha, (\tilde{b}_j)_\alpha$ and $(\tilde{e}_k)_\alpha$ denote the α -cut of the possibilistic variables $\tilde{c}^r_{ijk}$, $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{e}_k$, respectively. By the convexity assumption, $\pi_{\tilde{c}^r_{ijk}}(c^r_{ijk}), r = 1, 2, ..., q;\ i = 1, ..., m;\ j = 1, ..., n;\ k = 1, ..., \ell; \pi_{\tilde{a}_i}(a_i)$, $i = 1, ..., m;$ $\pi_{\tilde{b}_j}(b_j), j = 1, ..., n;\ \pi_{\tilde{e}_k}(e_k),\ k = 1, ..., \ell$ are real intervals that will be denoted as $[c^{r-}_{ijk}(\alpha), c^{r+}_{ijk}(\alpha)],\ [a^{r-}_i(\alpha), a^{r+}_i(\alpha)],\ [b^{r-}_j(\alpha), b^{r+}_j(\alpha)],\ [e^{r-}_k(\alpha), e^{r+}_k(\alpha)]$.

Definition 7.A point $x^*(c) \in G(a, b, e)$ is said to be α -parametric efficient solution for α -PMOSTP found only if there $x \in G(a, b, e)$ and $c^r_{ijk} \in (c^r_{ijk})_\alpha$ such that $z_r(x, c^r) \le z_r(x^*, c^r)$ for all $r = 1, ..., q$ and strict inequality holds for at least one *r*.

Theorem 1. A point $x^*(c) \in G(a, b, e)$ is an α -possibly efficient solution for Poss MOSTP if and only if $x^*(c) \in G(a, b, e)$ is an α -parametric efficient solution for α -PMOSTP.

Proof: (Necessity). Let $x^*(c) \in G(a, b, e)$ be an α -possibly efficient solution for Poss MOSTP and $x^*(c) \in G(a, b, e)$ be not an α -parametric efficient solution for α -PMOSTP. Then there are $x^1(c) \in G(a, b, e),\ t^r \in (\tilde{c}^r_{ijk})_\alpha,\ r = 1, ..., q;\ i = 1, ..., m;\ j = 1, 1, ..., n;\ k = 1, ..., \ell$ such that
$z_s(x^1, t^s) \le z_s(x^*, t^s)$, for all $s \in \{1, ..., q\}$ and $r \in \{1, ..., q\}$,

such that
$z_r(x^1, t^r) < z_r(x^*, t^r)$.

As $t^r \in [\tilde{c}^r_{ijk}]_\alpha$, $r = 1, ..., q$, we have

Poss $(z_1(x^1, \tilde{c}^1) \le z_1(x^*, \tilde{c}^1), ..., z_{r-1}(x^1, \tilde{c}^{r-1}) \le z_{r-1}(x^*, \tilde{c}^{r-1}),$
$z_r(x^1, \tilde{c}^r) < z_r(x^*, \tilde{c}^r), z_{r+1}(x^1, \tilde{c}^{r+1}) \le z_{r+1}(x^*, \tilde{c}^{r+1}), ...,$
$z_q(x^1, \tilde{c}^q) \le z_q(x^*, \tilde{c}^q) \ge \alpha, \ \alpha \in [0, 1].$

This contradicts the α -possibly efficient of $x^*(c) \in G(a, b, e)$ for Poss MOSTP and the necessity part is established.

Sufficiency: Let $x^*(c) \in G(a, b, e)$ be α -parametric efficient solution for α -PMOSTP and $x^*(c) \in G(a, b, e)$ be not an α -possibly efficient solution for Poss MOSTP. Then there are $x^2 \in G(a, b, e)$ and $r = 1, ..., q$ such that

Poss$(z_1(x^2, \tilde{c}^1) \le z_1(x^*, \tilde{c}^1), ..., z_{r-1}(x^2, \tilde{c}^{r-1}) \le z_{r-1}(x^*, \tilde{c}^{r-1}),$
$z_r(x^2, \tilde{c}^r) < z_r(x^*, \tilde{c}^r), z_{r+1}(x^2, \tilde{c}^{r+1}) \le z_{r+1}(x^*, \tilde{c}^{r+1}), ...,$
$z_q(x^2, \tilde{c}^q) \le z_q(x^*, \tilde{c}^q) \ge \alpha,$

i.e.,

$$\sup_{(c^1, ..., c^q) \in \overline{C}} \min(\pi_{\tilde{c}^1}(c^1), ..., \pi_{\tilde{c}^{r-1}}(c^{r-1}), \pi_{\tilde{c}^r}(c^r), \pi_{\tilde{c}^{r+1}}(c^{r+1}), ...,$$
$$\pi_{\tilde{c}^q}(c^q)) \ge \alpha, \quad \text{......................(6)}$$

where

Poss$\overline{C} = \{(c^1, ..., c^q) \in R^{q(m \times n \times \ell)} : z_1(x^2, c^1) \le z_1(x^*, c^1), z_{r-1}(x^2, c^{r-1}) \le,$
$z_{r-1}(x^*, c^{r-1}), z_r(x^2, c^r) < z_r(x^*, c^r), z_{r+1}(x^2, c^{r+1}) \le,$
$z_{r+1}(x^*, c^{r+1}), ..., z_q(x^2, \tilde{c}^q) \le z_q(x^*, c^q)\}.$

For the supremum to be existed, there is $(P^1, P^2, ..., P^q) \in \overline{C}$ with
$\min(\pi_{\tilde{P}^1}(P^1), \pi_{\tilde{P}^2}(P^2), \ \pi_{\tilde{P}^q}(P^q)) < \alpha$, then
$$\sup_{(P^1, P^2, ..., P^q) \in \overline{C}} \min(\pi_{\tilde{P}^1}(P^1), \pi_{\tilde{P}^2}(P^2), \ \pi_{\tilde{P}^q}(P^q)) < \alpha.$$

This contradicts (8). Then there is $(P^1, P^2, ..., P^q) \in \overline{C}$ satisfying
$\min(\pi_{\tilde{P}^1}(P^1), \pi_{\tilde{P}^2}(P^2), \ \pi_{\tilde{P}^q}(P^q)) \ge \alpha$ (5)

i.e.,

$$P^r \in (\tilde{c}^r_{ijk})_\alpha, \ r = 1, ..., q; \ i = 1, ..., m; \ j = 1, ..., n; \ k = 1, ..., \ell$$

From (4) and (6) we arrive to the contradiction of the efficiency of $x^*(c)$ for α -PMOSTP at certain $\alpha \in [0, 1]$.

Problem (α -PMOSTP) will be treated using the weighting approach (see, Chanas *et al.* (1984), i.e., defining the following problem (STP (w))

$$(\mathrm{STP}(w)) \quad \min \sum_{r=1}^{q} w_r z_r(x, c^r)$$

subject to

$$x(c) \in G(a,b,e), c^r_{ijk} \in (\tilde{c}^r_{ijk})_\alpha, \ r=1,...,q; \ i=1,...,m;$$
$$j=1,...,n; \ k=1,...,\ell; a_i \in (\tilde{a}_i)_\alpha, i=1,...,m; \ b_j \in (\tilde{b}_j)_\alpha,$$
$$j=1,...,n; \ e_k \in (\tilde{e}_k)_\alpha, \ k=1,...,\ell, \ \sum_{i=1}^{m}(a_i)_\alpha = \sum_{j=1}^{n}(b_j)_\alpha =$$
$$\sum_{k=1}^{\ell}(e_k)_\alpha, \ w \in W = \left\{ w \in R: \ \sum_{r=1}^{q} w_r = 1, \ w_r \ge 0 \right\},$$
$$i=1,...,m; \ j=1,...,n; k=1,...,\ell$$

It is clear that problem $\mathrm{STP}(w)$ is a single objective solid transportation problem which is considered as a parametric nonlinear programming problem since the possibilistic parameters will be treated as decision variables.

Definition 7. The set of α -parametric optimal solutions of $\mathrm{STP}(w)$ is defined as:

$E(w) = \{x^* \in R^{q(m\times n\times \ell)} : \sum_{r=1}^{q} w_r z_r(x^*, c^{*r}) \le \sum_{r=1}^{q} w_r z_r(x, c^{*r})$, for each

$x(c^*) \in G(a^*, b^*, e^*), \ c^* \in (\tilde{c})_\alpha, \ a^* \in (\tilde{a})_\alpha, \ b^* \in (\tilde{b})_\alpha, \ e^* \in (\tilde{e})_\alpha$ and $w_r > 0, \ r=1,...,q, \ \sum_{r=1}^{q} w_r = 1\}$.

Remark 1.A point x^* is said to be a proper α -parametric efficient solution of α -PMOSTP problem if and only if there exists $w^* > 0$ such that $x^* \in E(w^*)$.

Parametric analysis

Definition 8. The solvability set of α -PMOSTP problem is defined by:

$B = \{w \in R^q$: there exists α -parametric efficient solution x^* of α -PMOSTP problem, $x^* \in E(w)\}$.

Definition 9. Suppose that $w^* \in B$ with a corresponding α - parametric optimal solution $x^* \in E(w^*)$, then the stability set of the first kind of α -PMOSTP problem corresponding to x^*, denoted by $S(x^*)$ and is defined by:

$S(x^*) = \{w \in B : x^* \in E(w^*)$ is an α -parametric optimal solution of $\mathrm{STP}(w)$ problem $\}$.

.

For the sake of parametric analysis and for $\alpha = 0$, $\mathrm{STP}(w)$ problem can be written as in the following form (Bazara *et al.* (1990) and Steuer (1986))

$$(\mathrm{STP}(w))' \min \sum_{r=1}^{q} w_r z_r(x, c^r) = \min \sum_{r=1}^{q}\sum_{i=1}^{m}\sum_{m=1}^{n}\sum_{k=1}^{\ell} w_r (c^{rL}_{ijk}(0) + \theta c^{ru}_{ijk}(0)) x_{ijk}$$

subject to

$$g_i(x, a_i) = \sum_{j=1}^{n}\sum_{k=1}^{\ell} x_{ijk} - (a^L_i(0) + \theta a^U_i(0)) = 0, \ i=1,...,m;$$
$$h_j(x, b_j) = \sum_{i=1}^{m}\sum_{k=1}^{\ell} x_{ijk} - (b^L_j(0) + \theta b^U_j(0)) = 0, \ j=1,...,n;$$

$f_k(x, e_k) = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} - (e_k^L(0) + \theta e_k^U(0)) = 0,\ k = 1, ..., \ell;$

$\sum_{i=1}^{m} (a_i^L + \theta a_i^u)_{\alpha=0} = \sum_{j=1}^{n} (b_j^L + \theta b_j^u)_{\alpha=0} = \sum_{k=1}^{\ell} (e_k^L + \theta e_k^u)_{\alpha=0}$

$x_{ijk} \geq 0,\ i = 1, ..., m;\ j = 1, ..., n;\ k = 1, ..., \ell;$

$w \in W$ and $\theta \in [0, 1]$.

Let $w^* \in B$ with x^* is an α-parametric efficient solution of α-PMOSTP problem, then the Kuhn-Rucker necessary optimality conditions corresponding to (STPw))$'$ problem will take the form:

$$\sum_{r=1}^{q} w_r \frac{\partial z_r}{\partial x_\eta} - \left(\sum_{i=1}^{m} \delta_i \frac{\partial g_i}{\delta x_\eta} + \sum_{j=1}^{n} \gamma_j \frac{\partial h_j}{\delta x_\eta} + \sum_{k=1}^{\ell} \xi_k \frac{\partial f_u}{\delta x_\eta} \right) = 0, \quad \text{…………………(7)}$$

$$\sum_{r=1}^{\theta} w_r \frac{\partial z_r}{\partial \theta} - \left(\sum_{i=1}^{m} \delta_i \frac{\partial g_i}{\delta \theta} + \sum_{j=1}^{n} \gamma_j \frac{\partial h_j}{\delta \theta} + \sum_{k=1}^{\ell} \xi_k \frac{\partial f_u}{\delta \theta} \right) = 0, \quad \text{………………...(8)}$$

$$\sum_{j=1}^{n} \sum_{k=1}^{\ell} x_{ijk} = a_i^L(0) + \theta a_i^U(0), \qquad i = 1, ..., m, \quad \text{………..………(9)}$$

$$\sum_{i=1}^{m} \sum_{k=1}^{\ell} x_{ijk} = b_j^L(0) + \theta b_j^U(0), \qquad j = 1, ..., n, \quad \text{……………….…(10)}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = e_k^L(0) + \theta e_k^U(0), \qquad k = 1, ..., \ell, \quad \text{……………… (11)}$$

$$\sum_{i=1}^{m} (a_i^L + \theta a_i^U)_{\alpha=0} = \sum_{j=1}^{n} (b_j^L + \theta b_j^U)_{\alpha=0} = \sum_{k=1}^{\ell} (e_k^L + \theta e_k^U)_{\alpha=0}, \quad \text{……………….. (12)}$$

$$\delta_i\, g_i(x, a_i) = 0, \qquad i = 1, ..., m, \quad \text{…………….. (13)}$$

$$\gamma_j\, h_j(x, b_j) = 0, \qquad j = 1, ..., n, \quad \text{……………(14)}$$

$$\xi_k\, f_k(x, e_k) = 0, \qquad k = 1, ..., \ell, \quad \text{………….. (15)}$$

δ_i, γ_j and , $\zeta_k \geq 0$, (16)

where $\delta_i\ (i = 1, ..., m)$, $\gamma_j\ (j = 1, ..., n)$, and $\xi_k\ (k = 1, ..., \ell)$ are the Lagrange multipliers and the above expressions are evaluated at x_{ijk}^*, c_{ijk}^{r*}, a_i^*, b_j^* and e_k^*. According to whether any of the variables $\delta_i\ (i = 1, ..., m)$, $\gamma_j\ (j = 1, ..., n)$, and $\xi_k\ (k = 1, ..., \ell)$ are zero or positive, the stability of the first kind $S(x^*)$ of $(\text{STP}(w))'$ problem can be determined.

Proposed algorithm

The steps of the proposed algorithm to determine the stability set of the first kind $S(x^*)$ can be summarized as in the following steps:

Step 1: Start with an initial degree of $\alpha = 0$.

Step 2: Constrict (STP$(w))'$ problem.

Step 3: Ask the DM to specify the initial value of $\theta (0 \leq \theta \leq 1)$.

Step 4: Choose a certain $w^* \in B$ and solve (STP$(w))'$ problem using any available computer package (say, WINQSB Package). Let x^* be 0- parametric optimal solution of (STP$(w))'$ problem with the corresponding parameters $(c^{rL*}, c^{ru*}, a^{L*}, a^{U*}, b^{L*}, b^{U*}, e^{L*}, e^{U*})$.

Step 5: Substituting with x^*, c^{rL*}, c^{ru*}, a^{L*}, a^{U*}, b^{L*}, b^{U*}, e^{L*}, e^{U*} into the Kuhn-Tucker necessary optimality conditions $(7)-(16)$, and solve the resulted system.

Step 6: Determine the stability set of the first kind $S(x^*)$ according to the values of the Lagrange multipliers δ_i, γ_i and ξ_k.

Step 7: Set $\theta = (\theta^* + \varepsilon) \in [0, 1]$ and go to step 2.

Repeat the above procedure until the interval $[0, 1]$ is fully exhausted. Then stop.

The flow chart of the proposed algorithm can be characterized as in the following figure:

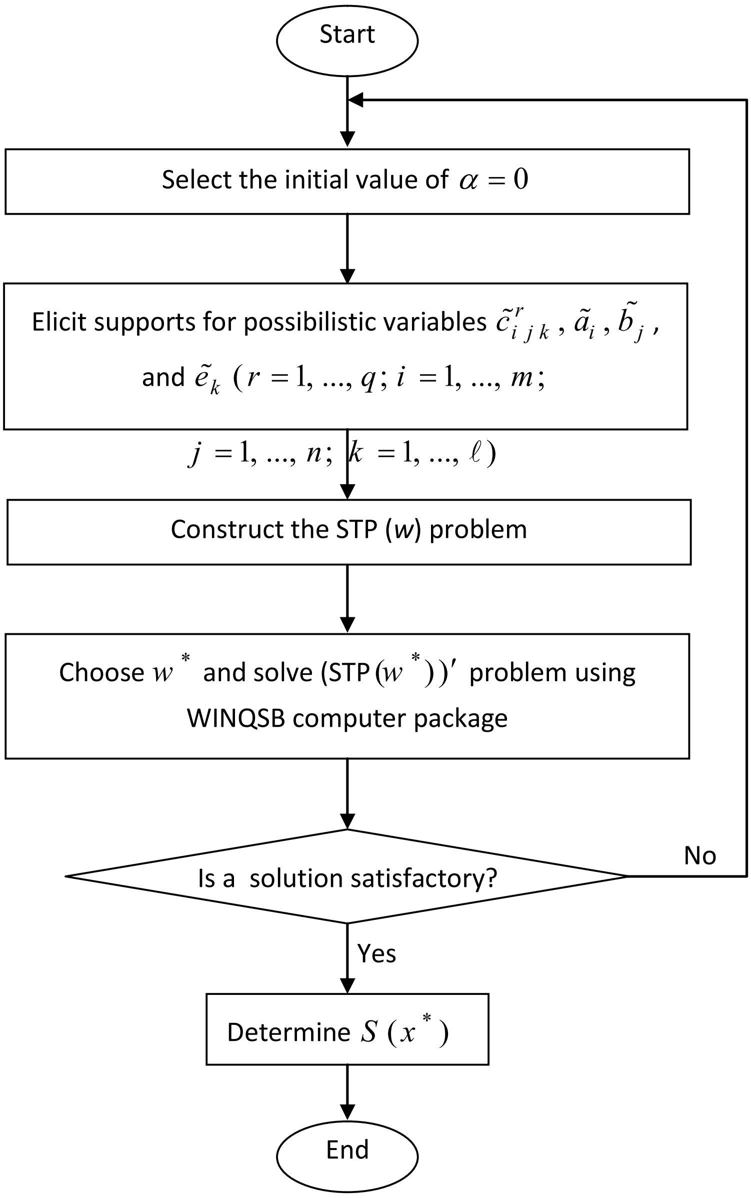

Fig. 1. Flowchart for proposed algorithm

Numerical example

Consider the following

(Poss MOSTP) $\min\ \tilde{z}_1 = \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{2} \tilde{c}^1_{ijk}\, x_{ijk}$,

$$\min\ \tilde{z}_2 = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} \tilde{c}^{2}_{ijk}\, x_{ijk}$$

subject to

$$x \in G = \Bigg\{ x, \in R^{12} : \sum_{j=1}^{3}\sum_{k=1}^{2} x_{1jk} = \tilde{a}_1,\ \sum_{j=1}^{3}\sum_{k=1}^{2} x_{2jk} = \tilde{a}_2,$$

$$\sum_{i=1}^{2}\sum_{k=1}^{2} x_{i1k} = \tilde{b}_1,\ \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i2k} = \tilde{b}_2,\ \sum_{i=1}^{2}\sum_{k=1}^{2} x_{i3k} = \tilde{b}_3,$$

$$\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ij1} = \tilde{e}_1,\ \sum_{i=1}^{2}\sum_{j=1}^{3} x_{ij2} = \tilde{e}_2,$$

$$x_{ijk} \ge 0,\ i = 1, 2;\ j = 1, 2, 3;\ k = 1, 2 \Bigg\}$$

In this example, two objective are simultaneously satisfied: the first and second are the minimization of possibilistic transportation costs.

Also, the possibilistic variables $\tilde{c}^{1}_{ijk}$, $\tilde{c}^{2}_{ijk}$, $\tilde{a}_i$, $\tilde{b}_j$, and $\tilde{e}_k$ are characterized by possibility distributions $\pi_{\tilde{c}^{1}_{ijk}}(\cdot)$, $\pi_{\tilde{c}^{2}_{ijk}}(\cdot)$, $\pi_{\tilde{a}_i}(\cdot)$, $\pi_{\tilde{b}_j}(\cdot)$ and $\pi_{\tilde{e}_k}(\cdot)$, respectively.

The supports of the possibilistic variables $\tilde{c}^{1}_{ijk}$, $\tilde{c}^{2}_{ijk}$, $\tilde{a}_i$, $\tilde{b}_j$, and $\tilde{e}_k$ are $[9, 13]$, $[10, 14]$, $[20, 24]$, $[15, 19]$ and $[22, 29]$, respectively. it is appropriate to characterize these supports by parametric functions beginning by the points of maximum possibility of $\tilde{c}^{1}_{ijk}$ and $\tilde{c}^{2}_{ijk}$. Hence, the parametric functions of θ to the supports for $0 \le \theta \le 1$ are:

Table 1. The supports of $\tilde{c}^{1}_{ijk}$ and the corresponding possibly distributions

Support ($\tilde{c}^{1}_{ijk}$)	$\pi \tilde{c}^{1}_{ijk}(\cdot)$
$\sup(\tilde{c}^{1}_{111}) = 9 + 4\theta$	$\pi_{\tilde{c}^{1}_{111}}(9) = \pi_{\tilde{c}^{1}_{111}}(13) = 0$
$\sup(\tilde{c}^{1}_{211}) = 14 - 4\theta$	$\pi_{\tilde{c}^{1}_{211}}(14) = \pi_{\tilde{c}^{1}_{211}}(10) = 0$
$\sup(\tilde{c}^{1}_{121}) = 18 + 4\theta$	$\pi_{\tilde{c}^{1}_{121}}(18) = \pi_{\tilde{c}^{1}_{121}}(22) = 0$
$\sup(\tilde{c}^{1}_{221}) = 9 - 4\theta$	$\pi_{\tilde{c}^{1}_{221}}(9) = \pi_{\tilde{c}^{1}_{221}}(5) = 0$
$\sup(\tilde{c}^{1}_{131}) = 13 + 4\theta$	$\pi_{\tilde{c}^{1}_{131}}(13) = \pi_{\tilde{c}^{1}_{131}}(17) = 0$
$\sup(\tilde{c}^{1}_{231}) = 7 + 4\theta$	$\pi_{\tilde{c}^{1}_{231}}(7) = \pi_{\tilde{c}^{1}_{231}}(11) = 0$
$\sup(\tilde{c}^{1}_{112}) = 10 - 4\theta$	$\pi_{\tilde{c}^{1}_{112}}(10) = \pi_{\tilde{c}^{1}_{112}}(6) = 0$
$\sup(\tilde{c}^{1}_{212}) = 7 - 4\theta$	$\pi_{\tilde{c}^{1}_{212}}(7) = \pi_{\tilde{c}^{1}_{212}}(3) = 0$
$\sup(\tilde{c}^{1}_{122}) = 11 + 4\theta$	$\pi_{\tilde{c}^{1}_{122}}(11) = \pi_{\tilde{c}^{1}_{122}}(15) = 0$
$\sup(\tilde{c}^{1}_{222}) = 18 - 4\theta$	$\pi_{\tilde{c}^{1}_{222}}(18) = \pi_{\tilde{c}^{1}_{222}}(14) = 0$
$\sup(\tilde{c}^{1}_{132}) = 5 + 4\theta$	$\pi_{\tilde{c}^{1}_{132}}(15) = \pi_{\tilde{c}^{1}_{132}}(9) = 0$
$\sup(\tilde{c}^{1}_{232}) = 2 + 4\theta$	$\pi_{\tilde{c}^{1}_{232}}(2) = \pi_{\tilde{c}^{1}_{232}}(6) = 0$

Table 2. The supports of $\tilde{c}^{2}_{ijk}$ and the corresponding possibly distributions

Support ($\tilde{c}^{2}_{ijk}$)	$\pi \tilde{c}^{2}_{ijk}(\cdot)$
$\sup(\tilde{c}^{2}_{111}) = 14 - 4\theta$	$\pi_{\tilde{c}^{2}_{111}}(14) = \pi_{\tilde{c}^{2}_{111}}(10) = 0$
$\sup(\tilde{c}^{2}_{211}) = 5 + 4\theta$	$\pi_{\tilde{c}^{2}_{211}}(5) = \pi_{\tilde{c}^{2}_{211}}(9) = 0$
$\sup(\tilde{c}^{2}_{121}) = 7 - 4\theta$	$\pi_{\tilde{c}^{2}_{121}}(7) = \pi_{\tilde{c}^{2}_{121}}(3) = 0$
$\sup(\tilde{c}^{2}_{221}) = 7 + 4\theta$	$\pi_{\tilde{c}^{2}_{221}}(7) = \pi_{\tilde{c}^{2}_{221}}(11) = 0$
$\sup(\tilde{c}^{2}_{131}) = 2 + 4\theta$	$\pi_{\tilde{c}^{2}_{131}}(2) = \pi_{\tilde{c}^{2}_{131}}(6) = 0$
$\sup(\tilde{c}^{2}_{231}) = 6 + 4\theta$	$\pi_{\tilde{c}^{2}_{231}}(6) = \pi_{\tilde{c}^{2}_{231}}(10) = 0$
$\sup(\tilde{c}^{2}_{112}) = 6 - 4\theta$	$\pi_{\tilde{c}^{2}_{112}}(6) = \pi_{\tilde{c}^{2}_{112}}(2) = 0$
$\sup(\tilde{c}^{2}_{212}) = 14 + 4\theta$	$\pi_{\tilde{c}^{2}_{212}}(14) = \pi_{\tilde{c}^{2}_{212}}(18) = 0$
$\sup(\tilde{c}^{2}_{122}) = 10 - 4\theta$	$\pi_{\tilde{c}^{2}_{122}}(10) = \pi_{\tilde{c}^{2}_{122}}(6) = 0$
$\sup(\tilde{c}^{2}_{222}) = 5 + 4\theta$	$\pi_{\tilde{c}^{2}_{222}}(5) = \pi_{\tilde{c}^{2}_{222}}(9) = 0$
$\sup(\tilde{c}^{2}_{132}) = 7 - 4\theta$	$\pi_{\tilde{c}^{2}_{132}}(7) = \pi_{\tilde{c}^{2}_{132}}(3) = 0$
$\sup(\tilde{c}^{2}_{232}) = 7 + 4\theta$	$\pi_{\tilde{c}^{2}_{232}}(7) = \pi_{\tilde{c}^{2}_{232}}(11) = 0$

Table3. The supports of $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{e}_k$ and the corresponding possibly distributions

Support ($\tilde{a}_i$)	$\pi_{\tilde{a}_i}(\cdot)$
$\sup(\tilde{a}_1) = 20 - 4\theta$	$\pi_{\tilde{a}_1}(20) = \pi_{\tilde{a}_1}(16) = 0$
$\sup(\tilde{a}_2) = 16 + 4\theta$	$\pi_{\tilde{a}_2}(16) = \pi_{\tilde{a}_2}(20) = 0$
Support ($\tilde{b}_j$)	$\pi_{\tilde{b}_j}(\cdot)$
$\sup(\tilde{b}_1) = 16 - 4\theta$	$\pi_{\tilde{b}_1}(16) = \pi_{\tilde{b}_1}(12) = 0$
$\sup(\tilde{b}_2) = 7 + 4\theta$	$\pi_{\tilde{b}_2}(7) = \pi_{\tilde{b}_2}(11) = 0$
$\sup(\tilde{b}_3) = 11 + 4\theta$	$\pi_{\tilde{b}_3}(11) = \pi_{\tilde{b}_3}(15) = 0$
Support ($\tilde{e}_k$)	$\pi_{\tilde{e}_k}(\cdot)$
$\sup(\tilde{e}_1) = 22 + 4\theta$	$\pi_{\tilde{e}_1}(22) = \pi_{\tilde{e}_1}(26) = 0$
$\sup(\tilde{e}_2) = 14 - 4\theta$	$\pi_{\tilde{e}_2}(14) = \pi_{\tilde{e}_2}(10) = 0$

At $\alpha = 0$, the corresponding 0-PMOSTP problem is

$$\min z_1(\theta) = (9+4\theta)x_{111} + (14-4\theta)x_{211} + (18+4\theta)x_{121} + (10-4\theta)x_{112}$$
$$+(7-4\theta)x_{212} + (11+4\theta)x_{122} + (18-4\theta)x_{222} + (5+4\theta)x_{212}$$
$$+(11+4\theta)x_{232}$$

subject to

$$x \in G = \{x \in R^{12} : x_{111} + x_{121} + x_{131} + x_{112} + x_{122} + x_{132} = 20 - 4\theta,$$
$$x_{211} + x_{221} + x_{231} + x_{212} + x_{222} + x_{232} = 16 + 4\theta,$$
$$x_{111} + x_{211} + x_{112} + x_{212} = 16 - 4\theta,$$
$$x_{121} + x_{221} + x_{122} + x_{222} = 7 + 4\theta,$$
$$x_{131} + x_{231} + x_{132} + x_{232} = 11 + 4\theta,$$
$$x_{111} + x_{211} + x_{121} + x_{221} + x_{131} + x_{231} = 22 + 4\theta,$$
$$x_{112} + x_{212} + x_{122} + x_{222} + x_{132} + x_{232} = 14 - 4\theta,$$
$$x_{ijk} \geq 0,\ i = 1, 2;\ j = 1, 2, 3;\ k = 1, 2;\ 0 \leq \theta \leq 1\}$$
$$\min z_2(\theta) = (14 - 4\theta)x_{111} + (5 + 4\theta)x_{211} + (7 - 4\theta)x_{121} + (7 + 4\theta)x_{221}$$
$$+(2 + 4\theta)x_{131} + (6 + 4\theta)x_{231} + (6 - 4\theta)x_{112} + (14 + 4\theta)x_{212}$$
$$+(10 - 4\theta)x_{122} + (5 + 4\theta)x_{222} + (7 - 4\theta)x_{132} + (7 + 4\theta)x_{232}$$

subject to

$x \in G$.

For $w^* = (w_1^*, w_2^*) = (0.4, 0.6)$ and $\theta = 0.5$.

(STP (w))$'$ $\min\ (11.6x_{111} + 9x_{211} + 14x_{121} + 8.2x_{221} + 11.6x_{131} + 5.6x_{112}$
$+11.6x_{212} + 10x_{122} + 10.6x_{222} + 5.8x_{132} + 9.8x_{232})$

subject to

$$x_{111} + x_{121} + x_{131} + x_{112} + x_{122} + x_{132} = 18,$$
$$x_{211} + x_{221} + x_{231} + x_{212} + x_{222} + x_{232} = 18,$$
$$x_{111} + x_{211} + x_{112} + x_{212} = 14,$$
$$x_{121} + x_{221} + x_{122} + x_{222} = 9,$$
$$x_{131} + x_{231} + x_{132} + x_{232} = 13,$$
$$x_{111} + x_{211} + x_{121} + x_{221} + x_{131} + x_{231} = 24,$$
$$x_{112} + x_{212} + x_{122} + x_{222} + x_{132} + x_{232} = 12,$$
$$x_{ijk} \geq 0,\ i = 1, 2;\ j = 1, 2, 3;\ k = 1, 2.$$

The solution is

$(x_{111}, x_{211}, x_{121}, x_{221}, x_{131}, x_{231}, x_{112}, x_{212}, x_{122}, x_{222}, x_{132}, x_{232}) =$
$(0, 2, 0, 9, 6, 7, 12, 0, 0, 0, 0, 0,)$,
$z^{\min} = 237$,

and

$$S(x^*) = \{w \in R^2 : 2w_1 + w_2 \geq 0,\ w_1 + 5w_2 \geq 0,\ w_1, w_2 \geq 0,\ w_1 + w_2 = 1\}.$$

Conclusion

The main objective of this paper is to present a solution procedure for multiobjective solid transportation problem with possibilistic variables (Poss MOSTP). A multi-objective solid transportation problem with possibilistic objective functions coefficients, possibilistic supplies, possibilistic demands and possibilistic conveyances has been introduced. The relation between the α-possibly for α-PMOSTP problem has been given. A parametric analysis to characterize the set of all α-parametric efficient solution for α-PMOSTP problem has been given. An algorithm to determine the stability set of the first kind corresponding to one of the parametric efficient solution of α-PMOSTP problem has been presented. A numerical example has been included in the sake of the paper for illustration. However, WINQSB computer package has been used to obtain the results.

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