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RESEARCH ARTICLE

ON MAXIMAL PRODUCT OF TWO FUZZY GRAPHS

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ABSTRACT

In this paper, the maximal product of two fuzzy graphs is defined. It is proved that when two fuzzy graphs are effective then their maximal product is always effective. Also it is proved that the maximal product of two connected fuzzy graphs is connected. The degree of a vertex in the maximal product of two fuzzy graphs is obtained. It is illustrated that when two fuzzy graphs are regular then their maximal product need not be regular. But it is proved that the maximal product of two regular fuzzy graphs is regular with some restrictions.

1. INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975 (Rosenfeld, 1975). Later on, Bhattacharya (1987) gave some remarks on fuzzy graphs. Some operations on fuzzy graphs were introduced by Mordeson and Peng (2008). The conjunction of two fuzzy graphs was defined by Nagoor Gani and Radha (2008). We defined the direct sum (Radha and Arumugam, 2013), strong product (Radha and Arumugam, 2014) of two fuzzy graphs and studied their properties. In this paper, maximal product of two fuzzy graphs is defined. It is proved that when two fuzzy graphs are effective then their maximal product is always effective. Also it is proved that the maximal product of two connected fuzzy graphs is connected. The degree of a vertex in the maximal product of two fuzzy graphs is obtained. It is illustrated that when two fuzzy graphs are regular then their maximal product need not be regular. But it is proved that the maximal product of two regular fuzzy graphs is regular with some restrictions.

2. Preliminaries

First let us recall some preliminary definitions that can be found in (Bhattacharya, 1987; Mordeson and Peng, 2008; Nagoorgani and Radha, 2008; Radha and Arumugam, 2013; Radha and Arumugam, 2014).

A fuzzy graph G is a pair of functions (σ, μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ. The underlying crisp graph of $G:(\sigma,\mu)$ is denoted by $G^*(V,E)$ where $E \subseteq V \times V$.

Let $G:(\sigma, \mu)$ be a fuzzy graph. The underlying crisp graph of $G:(\sigma, \mu)$ is denoted by $G^*:(V, E)$ where $E \subseteq V\times V$. A fuzzy graph G is an effective fuzzy graph if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $(u,v)\in E$ and G is a complete fuzzy graph if $\mu(u,v) = \sigma(u) \wedge \sigma(v)$ for all $u,v\in V$. Therefore G is a complete fuzzy graph if and only if G is an effective fuzzy graph and G^* is complete.

The degree of a vertex u of a fuzzy graph G is defined as $d_G(u) = \sum_{u \neq v} \mu(uv) = \sum_{uv\in E} \mu(uv)$. If $d_G(v)=k$ for all $v\in V$, that is, if each vertex of G has the same degree k, then G is said to be a regular fuzzy graph of degree k or a k-regular fuzzy graph. The regular fuzzy graph G is called a full regular fuzzy graph if its underlying crisp graph G^* is a regular graph and called a complete regular fuzzy graph if its underlying crisp graph G^* is a complete graph.

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3. Maximal product

3.1Definition

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ denote two fuzzy graphs with underlying crisp graphs $G_1^*:(V_1,E_1)$ and $G_2^*:(V_2,E_2)$ respectively. Define $G:(\sigma, \mu)$, where $\sigma = \sigma_1 * \sigma_2$ and $\mu = \mu_1 * \mu_2$, with underlying crisp graph $G^*: (V, E)$ where $V = V_1 \times V_2$ and $E = \{(u_1, v_1)(u_2, v_2) / u_1= u_2, v_1 v_2 \in E_2$ or $v_1= v_2, u_1 u_2 \in E_1 \}$, by $\sigma(u_1, v_1) = \sigma_1(u_1) \vee \sigma_2(v_1)$, for all $(u_1, v_1) \in V_1 \times V_2$ and

$$\mu\left((u_1, v_1)(u_2, v_2)\right) = \begin{cases} \sigma_1(u_1) \vee \mu_2(v_1 v_2), \text{if } u_1 = u_2, v_1 v_2 \in E_2 \\ \mu_1(u_1 u_2) \vee \sigma_2(v_1), \text{if } v_1 = v_2, u_1 u_2 \in E_1 \end{cases}$$

Case(i)If $u_1= u_2$ and $v_1 v_2 \in E_2$ then, $\mu((u_1, v_1)(u_2, v_2)) = \sigma_1(u_1) \vee \mu_2(v_1 v_2) \le \sigma_1(u_1) \vee (\sigma_2(v_1) \wedge \sigma_2(v_2))$
$= (\sigma_1(u_1) \vee \sigma_2(v_1)) \wedge (\sigma_1(u_1) \vee \sigma_2(v_2)) = \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$.

Case(ii)If $u_1 u_2 \in E_1$ and $v_1= v_2$ then, $\mu((u_1, v_1)(u_2, v_2)) = \mu_1(u_1 u_2) \vee \sigma_2(v_1) \le (\sigma_1(u_1) \wedge \sigma_1(u_2)) \vee \sigma_2(v_1)$
$= (\sigma_1(u_1) \vee \sigma_2(v_1)) \wedge (\sigma_1(u_2) \vee \sigma_2(v_1)) = \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$.

Hence $\mu((u_1, v_1)(u_2, v_2)) \le \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$. Therefore, $G:(\sigma, \mu)$ is a fuzzy graph. This is called the maximal product of the fuzzy graphs G_1 and G_2 and denoted by G_1*G_2.

3.2 Example

The following Figure 1 illustrates the maximal product G_1*G_2 of the two fuzzy graphs G_1 and G_2.

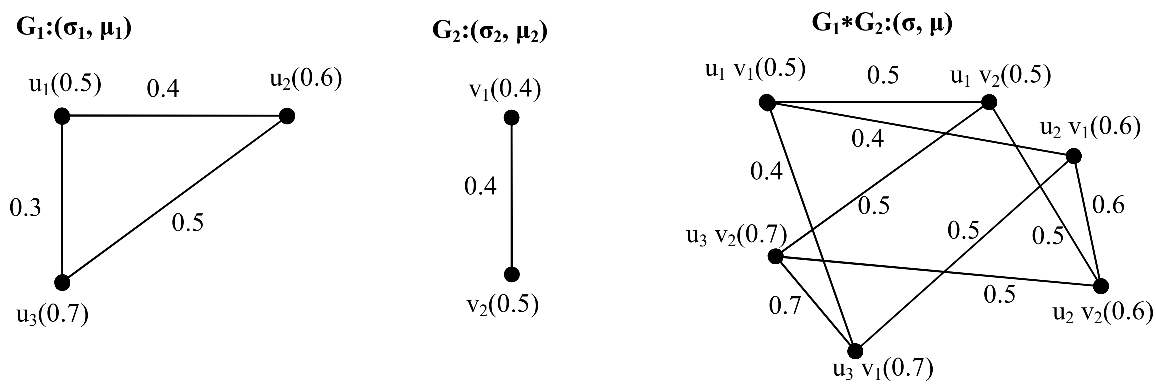

Fig. 1.

3.3 Theorem

The maximal product of two effective fuzzy graphs is an effective fuzzy graph.

Proof:

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be effective fuzzy graphs.

Then $\mu_1(u_1u_2) = \sigma_1(u_1) \wedge \sigma_1(u_2)$ for any $u_1u_2 \in E_1$ and $\mu_2(v_1v_2) = \sigma_2(v_1) \wedge \sigma_2(v_2)$ for any $v_1v_2 \in E_2$.Then proceeding as in the definition,
Case(i)If $u_1= u_2$ and $v_1 v_2 \in E_2$ then, $\mu((u_1, v_1)(u_2, v_2)) = \sigma_1(u_1) \vee \mu_2(v_1 v_2) = \sigma_1(u_1) \vee (\sigma_2(v_1) \wedge \sigma_2(v_2))$
$= (\sigma_1(u_1) \vee \sigma_2(v_1)) \wedge (\sigma_1(u_1) \vee \sigma_2(v_2)) = \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$.
Case(ii)If $u_1 u_2 \in E_1$ and $v_1= v_2$ then, $\mu((u_1, v_1)(u_2, v_2)) = \mu_1(u_1 u_2) \vee \sigma_2(v_1) = (\sigma_1(u_1) \wedge \sigma_1(u_2)) \vee \sigma_2(v_1)$
$= (\sigma_1(u_1) \vee \sigma_2(v_1)) \wedge (\sigma_1(u_2) \vee \sigma_2(v_1)) = \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$.
Thus $\mu((u_1, v_1)(u_2, v_2)) = \sigma(u_1, v_1) \wedge \sigma(u_2, v_2)$ for all edges in the maximal product.
Hence G_1*G_2 is an effective fuzzy graph.

3.4 Example

Consider the following Figure 2. The two fuzzy graphs G_1 and G_2 are effective fuzzy graphs and their maximal product G_1*G_2 is also an effective fuzzy graph.

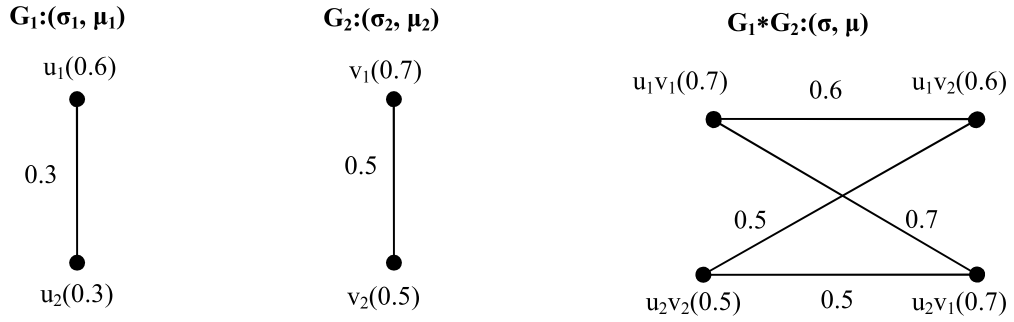

Fig.2.

But for the maximal product G_1*G_2 to be an effective fuzzy graph, G_1 and G_2 need not be effective fuzzy graphs. The following Figure 2(a) illustrates this

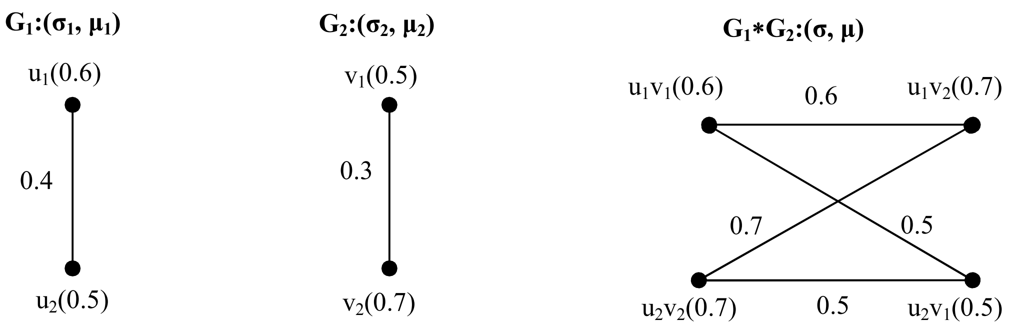

Fig.2(a).

3.5 Theorem

The maximal product of two connected fuzzy graphs is always a connected fuzzy graph.

Proof

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two connected fuzzy graphs with underlying crisp graphs $G_1^*:(V_1,E_1)$ and $G_2^*:(V_2,E_2)$ respectively. Let $V_1 = \{u_1, u_2, \ldots\ldots\ldots, u_m\}$ and $V_2 = \{v_1, v_2, \ldots\ldots\ldots, v_n\}$.
Then $\mu_1^\infty(u_iu_j) > 0$ for all $u_i, u_j \in V_1$ and $\mu_2^\infty(v_iv_j) > 0$ for all $v_i, v_j \in V_2$.

The maximal product of $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ can be taken as $G:(\sigma, \mu)$. Now consider the 'm' sub graphs of G with the vertex sets $\{u_iv_1, u_iv_2, \ldots\ldots\ldots, u_iv_n\}$ for $i=1,2,\ldots\ldots,m$. Each of these sub graphs of G is connected since the u_i's are the same and since G_2 is connected, each v_i is adjacent to at least one of the vertices in V_2. Also since G_1 is connected, each u_i is adjacent to at least one of the vertices in V_1.

Therefore there exists at least one edge between any pair of the above 'm' sub graphs. Thus we have $\mu^\infty((u_i,v_j)\ (u_k,v_l)) > 0$ for all $(u_i,v_j)\ (u_k,v_l) \in E$. Hence G is a connected fuzzy graph.

3.6 Remark

The maximal product of two complete fuzzy graphs is not a complete fuzzy graph because we do not include the case $u_1u_2 \in E_1$ and $v_1v_2 \in E_2$ in the definition of the maximal product. Since every complete fuzzy graph is effective, from Theorem 3.3, we have the maximal product of two complete fuzzy graphs is an effective fuzzy graph. Consider the following Figure 3 where G_1 and G_2 are complete fuzzy graphs and their maximal product G_1*G_2 is an effective fuzzy graph.

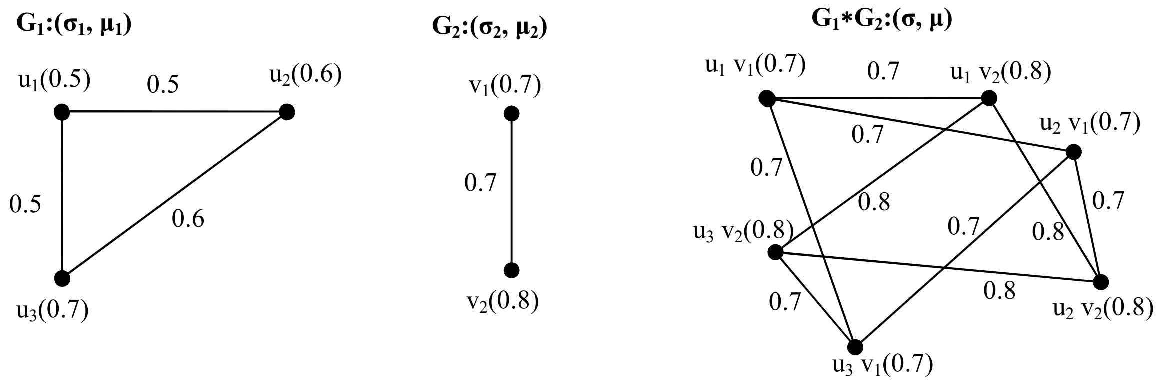

Fig.3.

4. Degree of a vertex in the maximal product

The degree of any vertex in the maximal product G_1*G_2 of the fuzzy graph $G_1:(\sigma_1,\mu_1)$ with $G_2:(\sigma_2,\mu_2)$ is given by,

$$d_{G_1*G_2}(u_i, v_j) = \sum_{u_iu_k \in E_1, v_j = v_\ell} \mu_1(u_iu_k) \vee \sigma_2(v_j) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \sigma_1(u_i) \vee \mu_2(v_jv_\ell) \ .$$

4.1 Notation

The relation $\sigma_1 \le \sigma_2$ means that $\sigma_1(u) \le \sigma_2(v)$ for every $u \in V_1$ and for every $v \in V_2$ where σ_i is a fuzzy subset of V_i, i =1,2.

4.2 Theorem

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \le \mu_2$ then the degree of any vertex in the maximal product of the two fuzzy graphs is given by,

$$d_{G_1*G_2}(u_i, v_j) = d_{G_1^*}(u_i)\sigma_2(v_j) + d_{G_2}(v_j).$$

Proof:

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \le \mu_2$ then $\sigma_2 \ge \mu_1$.

Then the degree of any vertex in the maximal product is given by,

$$\begin{aligned} d_{G_1*G_2}(u_i, v_j) &= \sum_{u_iu_k \in E_1, v_j = v_\ell} \mu_1(u_iu_k) \vee \sigma_2(v_j) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \sigma_1(u_i) \vee \mu_2(v_jv_\ell) \\ &= \sum_{u_iu_k \in E_1, v_j = v_\ell} \sigma_2(v_j) + \sum_{u_i = u_k, v_jv_\ell \in E_2} \mu_2(v_jv_\ell) \\ &= d_{G_1^*}(u_i)\sigma_2(v_j) + d_{G_2}(v_j). \end{aligned}$$

4.3 Theorem

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \le \mu_2$ and σ_2 is a constant function of value 'c', then the degree of any vertex in the maximal product of the two fuzzy graphs is given by,

$$d_{G_1*G_2}(u_i, v_j) = d_{G_1^*}(u_i)\,c + d_{G_2}(v_j).$$

Proof:

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\sigma_1 \le \mu_2$ and σ_2 is a constant function of value 'c'. Also, $\sigma_1 \le \mu_2$ implies that $\sigma_2 \ge \mu_1$.

Then the degree of any vertex in the maximal product is given by,

$$\begin{aligned} d_{G_1*G_2}(u_i,v_j) &= \sum_{u_iu_k\in E_1, v_j=v_\ell} \mu_1(u_iu_k)\vee\sigma_2(v_j) + \sum_{u_i=u_k, v_jv_\ell\in E_2} \sigma_1(u_i)\vee\mu_2(v_jv_\ell) \\ &= \sum_{u_iu_k\in E_1, v_j=v_\ell} \sigma_2(v_j) + \sum_{u_i=u_k, v_jv_\ell\in E_2} \mu_2(v_jv_\ell) \\ &= d_{G_1^*}(u_i)c + d_{G_2}(v_j). \end{aligned}$$

4.4 Theorem

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_2 \le \mu_1$ then the degree of any vertex in the maximal product of the two fuzzy graphs is given by,

$$d_{G_1*G_2}(u_i,v_j) = d_{G_1}(u_i) + d_{G_2^*}(v_j)\sigma_1(u_i).$$

Proof

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_2 \le \mu_1$ then $\sigma_1 \ge \mu_2$. Then the degree of any vertex in the maximal product is given by,

$$\begin{aligned} d_{G_1*G_2}(u_i,v_j) &= \sum_{u_iu_k\in E_1, v_j=v_\ell} \mu_1(u_iu_k)\vee\sigma_2(v_j) + \sum_{u_i=u_k, v_jv_\ell\in E_2} \sigma_1(u_i)\vee\mu_2(v_jv_\ell) \\ &= \sum_{u_iu_k\in E_1, v_j=v_\ell} \mu_1(u_iu_k) + \sum_{u_i=u_k, v_jv_\ell\in E_2} \sigma_1(u_i) \\ &= d_{G_1}(u_i) + d_{G_2^*}(v_j)\sigma_1(u_i). \end{aligned}$$

4.5 Theorem

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_2 \le \mu_1$ and σ_1 is a constant function of value 'c', then the degree of any vertex in the maximal product of the two fuzzy graphs is given by,

$$d_{G_1*G_2}(u_i,v_j) = d_{G_1}(u_i) + d_{G_2^*}(v_j)c.$$

Proof

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\sigma_2 \le \mu_1$ and σ_1 is a constant function of value 'c'. Also, $\sigma_2 \le \mu_1$ implies that $\sigma_1 \ge \mu_2$. Then the degree of any vertex in the maximal product is given by,

$$\begin{aligned} d_{G_1*G_2}(u_i,v_j) &= \sum_{u_iu_k\in E_1, v_j=v_\ell} \mu_1(u_iu_k)\vee\sigma_2(v_j) + \sum_{u_i=u_k, v_jv_\ell\in E_2} \sigma_1(u_i)\vee\mu_2(v_jv_\ell) \\ &= \sum_{u_iu_k\in E_1, v_j=v_\ell} \mu_1(u_iu_k) + \sum_{u_i=u_k, v_jv_\ell\in E_2} \sigma_1(u_i) \\ &= d_{G_1}(u_i) + d_{G_2^*}(v_j)c. \end{aligned}$$

4.6 Theorem

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two fuzzy graphs such that $\sigma_1 \ge \mu_2$ and $\sigma_2 \ge \mu_1$ then the degree of any vertex in the maximal product of the two fuzzy graphs is given by,

$$d_{G_1*G_2}(u_i,v_j) = d_{G_1^*}(u_i)\sigma_2(v_j) + d_{G_2^*}(v_j)\sigma_1(u_i).$$

Proof

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be two fuzzy graphs such that $\sigma_1 \ge \mu_2$ and $\sigma_2 \ge \mu_1$. Then the degree of any vertex in the maximal product is given by,

$$d_{G_1*G_2}(u_i,v_j) = \sum_{u_iu_k\in E_1, v_j=v_\ell} \mu_1(u_iu_k)\vee\sigma_2(v_j) + \sum_{u_i=u_k, v_jv_\ell\in E_2} \sigma_1(u_i)\vee\mu_2(v_jv_\ell)$$
$$= \sum_{u_iu_k\in E_1, v_j=v_\ell} \sigma_2(v_j) + \sum_{u_i=u_k, v_jv_\ell\in E_2} \sigma_1(u_i)$$
$$= d_{G_1^*}(u_i)\sigma_2(v_j) + d_{G_2^*}(v_j)\sigma_1(u_i).$$

4.7 Example

Consider the maximal product of the two fuzzy graphs $G_1:(\sigma_1, \mu_1)$ and $G_2:(\sigma_2, \mu_2)$ such that $\sigma_1 \le \mu_2$ given in Figure 3 of Remark 3.6. Then the degree of any vertex in the maximal product of the two fuzzy graphs is given by, $d_{G_1*G_2}(u_i,v_j) = d_{G_1^*}(u_i)\sigma_2(v_j) + d_{G_2}(v_j)$.

Now,

$d_{G_1*G_2}(u_1,v_1) = 0.7+0.7+0.7 = 2.1$ and $d_{G_1^*}(u_1)\sigma_2(v_1)+d_{G_2}(v_1) = 2(0.7)+0.7 = 2.1$

$d_{G_1*G_2}(u_3,v_2) = 0.7+0.8+0.8 = 2.3$ and $d_{G_1^*}(u_3)\sigma_2(v_2)+d_{G_2}(v_2) = 2(0.8)+0.7 = 2.3$

5. Regular property of maximal product

If G_1: (σ_1, μ_1) and G_2: (σ_2, μ_2) are two regular fuzzy graphs then their maximal product G_1*G_2 need not be a regular fuzzy graph. It is illustrated with the following example.

5.1 Example

Consider the following two fuzzy graphs $G_1:(\sigma_1, \mu_1)$ and $G_2:(\sigma_2, \mu_2)$ which are regular but their maximal product G_1*G_2 is not a regular fuzzy graph.

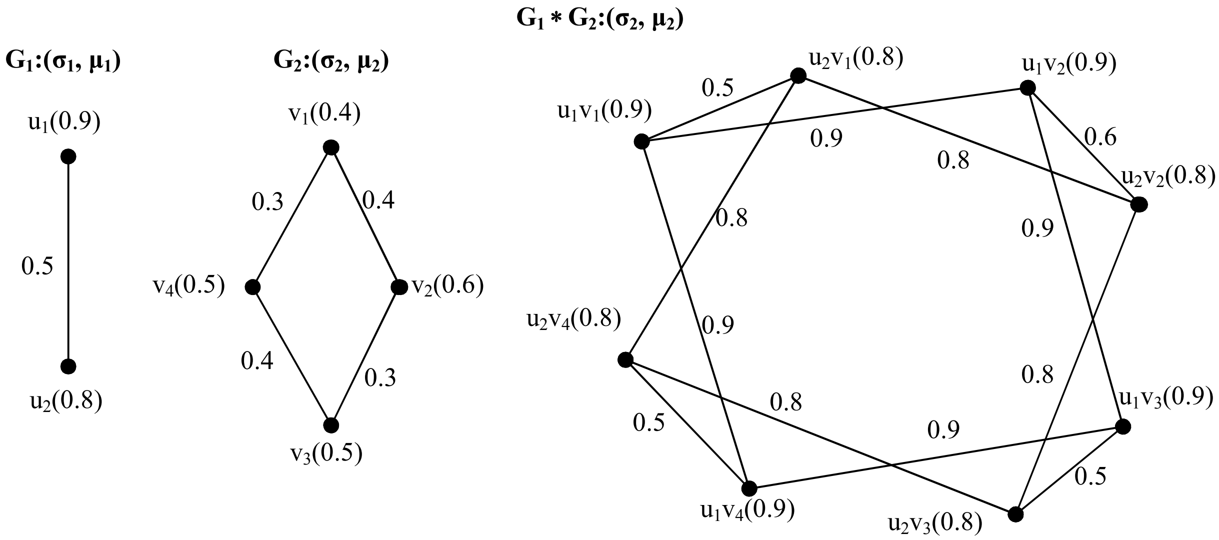

Fig.4.

But with few restrictions it can be proved that the maximal product of two regular fuzzy graphs is regular. The following theorems explain the conditions for the maximal product of two regular fuzzy graphs to be regular.

5.2 Theorem

If $G_1:(\sigma_1,\mu_1)$ is a partially regular fuzzy graph and $G_2:(\sigma_2,\mu_2)$ is a fuzzy graph such that $\sigma_1 \le \mu_2$ and σ_2 is a constant function of value 'c', then their maximal product is regular if and only if G_2 is regular.

Proof

Let $G_1:(\sigma_1,\mu_1)$ be a partially regular fuzzy graph such that G_1^* is r_1-regular and $G_2:(\sigma_2,\mu_2)$ be any fuzzy graph with $\sigma_1 \le \mu_2$ and σ_2 is a constant function of value 'c'. Now assume that $G_2:(\sigma_2,\mu_2)$ is a k-regular fuzzy graph. Then,

$$d_{G_1*G_2}(u_i,v_j)=d_{G_1^*}(u_i)\sigma_2(v_j)+d_{G_2}(v_j)=r_1c+k.$$

This is a constant for all vertices in $V_1\times V_2$. Hence G_1*G_2 is a regular fuzzy graph.
Conversely assume that G_1*G_2 is a regular fuzzy graph. Then, for any two vertices (u_1,v_1) and (u_2,v_2) in $V_1\times V_2$,

$$\begin{aligned}
& d_{G_1*G_2}(u_1,v_1)=d_{G_1*G_2}(u_2,v_2)\\
\Rightarrow\quad & d_{G_1^*}(u_1)\sigma_2(v_1)+d_{G_2}(v_1)=d_{G_1^*}(u_2)\sigma_2(v_2)+d_{G_2}(v_2)\\
\Rightarrow\quad & r_1c+d_{G_2}(v_1)=r_1c+d_{G_2}(v_2)\\
\Rightarrow\quad & d_{G_2}(v_1)=d_{G_2}(v_2)
\end{aligned}$$

This is true for all vertices in G_2. Hence G_2 is a regular fuzzy graph.

5.3 Theorem

If $G_1:(\sigma_1,\mu_1)$ is a fuzzy graph and $G_2:(\sigma_2,\mu_2)$ is a partially regular fuzzy graph such that $\sigma_2\le\mu_1$ and σ_1 is a constant function of value 'c', then their maximal product is regular if and only if G_1 is regular.

Proof: Proof of this theorem is similar to that of the theorem 5.2.

5.4 Theorem

If $G_1:(\sigma_1,\mu_1)$ is a partially regular fuzzy graph and $G_2:(\sigma_2,\mu_2)$ is a fuzzy graph such that $\sigma_1\le\mu_2$ and σ_2 is a constant function of value 'c', then their maximal product is regular if and only if G_2 is regular.

5.5 Theorem

If $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are two partially regular fuzzy graphs such that $\sigma_1\ge\mu_2$, $\sigma_2\ge\mu_1$ and σ_2 is a constant function of value 'c', then their maximal product is regular if and only if σ_1 is a constant function.

Proof

Let $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ be partially regular fuzzy graphs such that $\sigma_1\ge\mu_2$, $\sigma_2\ge\mu_1$ and σ_2 is a constant function of value 'c' with G_i^* is r_i-regular, i=1,2. Now assume that σ_1 is a constant function of value 'k'. Then,

$$d_{G_1*G_2}(u_i,v_j)=d_{G_1^*}(u_i)\sigma_2(v_j)+d_{G_2^*}(v_j)\sigma_1(u_i)=r_1c+r_2k$$

This is a constant for all vertices in $V_1\times V_2$. Hence G_1*G_2 is a regular fuzzy graph.
Conversely, assume that G_1*G_2 is a regular fuzzy graph. Then, for any two vertices (u_1,v_1) and (u_2,v_2) in $V_1\times V_2$,

$$\begin{aligned}
& d_{G_1*G_2}(u_1,v_1)=d_{G_1*G_2}(u_2,v_2)\\
\Rightarrow\quad & d_{G_1^*}(u_1)\sigma_2(v_1)+d_{G_2^*}(v_1)\sigma_1(u_1)=d_{G_1^*}(u_2)\sigma_2(v_2)+d_{G_2^*}(v_2)\sigma_1(u_2)\\
\Rightarrow\quad & r_1c+r_2\sigma_1(u_1)=r_1c+r_2\sigma_1(u_2)\\
\Rightarrow\quad & \sigma_1(u_1)=\sigma_1(u_2)
\end{aligned}$$

This is true for all vertices in G_1. Hence σ_1 is a constant function

5.6 Remark

The maximal product of two full regular fuzzy graphs need not be full regular. In the Example 5.1, the fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ are full regular and their maximal product G_1*G_2 is partially regular and not a full regular fuzzy graph. Also the maximal product of two complete regular fuzzy graphs is partially regular and not complete regular. This is illustrated through the following example. Consider the two complete regular fuzzy graphs $G_1:(\sigma_1,\mu_1)$ and $G_2:(\sigma_2,\mu_2)$ and their maximal product in Figure 5.

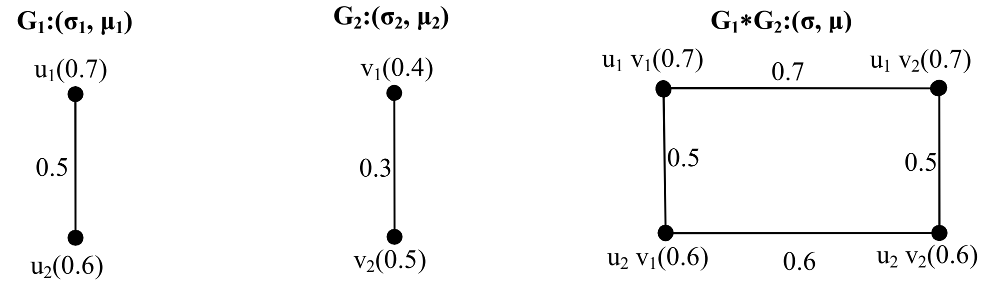

Fig.5.

6. Conclusion

In this paper, maximal product of two fuzzy graphs is defined. It is proved that when two fuzzy graphs are effective then their maximal product is always effective. Also it is proved that the maximal product of two connected fuzzy graphs is connected. The degree of a vertex in the maximal product of two fuzzy graphs is obtained. It is illustrated that when two fuzzy graphs are regular then their maximal product need not be regular. But it is proved that the maximal product of two regular fuzzy graphs is regular with some restrictions. In addition to the existing ones, this operation will be helpful to study large fuzzy graph as a combination of small fuzzy graphs and to derive its properties from those of the small ones.

REFERENCES

Bhattacharya, P. 1987. Some Remarks on Fuzzy Graphs, Pattern Recognition Letter 6, 297-302.

Mordeson J.N. and Peng C.S. 1994. Operations on fuzzy graphs, Information Sciences 79, 159-170.

Nagoorgani, A. and Radha, K. 2008. Conjunction of Two Fuzzy Graphs, International Review of Fuzzy Mathematics, Vol. 3, 95-105.

Nagoorgani, A. and Radha, K. 2008. Regular Property of Fuzzy Graphs, Bulletin of Pure and Applied Sciences, Vol.27E(No.2) 411-419.

Radha, K. and Arumugam, S. 2013. On Direct Sum of Two Fuzzy Graphs, *International Journal of Scientific and Research Publications,* Volume 3, Issue 5, May 2013, ISSN 2250-3153.

Radha, K. and Arumugam, S. 2014. On Strong Product of Two Fuzzy Graphs, *International Journal of Scientific and Research Publications*, Volume 4, Issue 10, October 2014, ISSN 2250-3153.

Radha, K. and Arumugam, S. 2014. Path Matrices of Fuzzy Graphs, Proceedings of the International Conference on Mathematical Methods and Computation, *Jamal Academic Research Journal*, Special Issue, February 2014, ISSN 0973-0303.

Radha, K. and Arumugam, S. 2014.Some Regular Properties of the Strong Product of Two Fuzzy Graphs, *Global Journal for Research Analysis*, Volume 3, Issue 11, November 2014, ISSN 2277-8160.

Rosenfeld, A. 1975. Fuzzy graphs, In: Zadeh, L.A., Fu, K.S., Shimura, M., Eds., Fuzzy Sets and their Applications, Academic Press, New York, ISBN 9780127752600, pp.77–95. 1975.

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