Available online at http://www.journalajst.com

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

International Journal of Current Research
Vol. 5, Issue, 06, pp.1582-1584, June, 2013

ISSN: 0975-833X

RESEARCH ARTICLE

CONSTRUCTION OF FREE DISTRIBUTIVE SEMIGROUPS THROUGH COMMUTATIVE SEMI GROUP

*Rushadije Ramani

Department of Mathematics, State University of Tetova, Avenue "Bllagoja Toska" Bllok 82-A1/6, Tetova 1200, Maqedoni

ARTICLE INFO

Article History:
Received 07th March, 2013
Received in revised form
30th April, 2013
Accepted 16th May, 2013
Published online 15th June, 2013

Key words:
Distributive semi group,
Commutative semi group,
Homomorphism.

ABSTRACT

In The paper [1] represents the construction of free medial semi group through commutative semi group. In this project the structure of distributive semi group [2], [3], [4] and the commutative semi group are been used for constructing one distributive semi group through commutative semi group. The first part explains the distributive semi groups and commutative semi groups, represents the conditions that the commutative semi group should complete whereof to construct distributive semi group. The second part contains the contruction of the free distributive semi group using indications of the first part. Pointing that in this project I will try the same technique to accompany the different semi groups.

INTRODUCTION

A semi group S is called left(right) distributive semi group if $xyz = xyxz$ $(yzx = yxzx)$ where x, y, z belong to S. A semi group S is distributive semi group if it is both left and right distributive semi group. S is a commutative semi group if it satisfies $xz = zx$, for any $x, z \in S$.

Definition: $x \in S$ is an idempotent if $x^2 = x$.Also we define the set of idempotents in S to be

$$Id(S) = \left\{ x \in S \mid x^2 = x \right\}$$

Now, $Id(S)$ may be empty, but $Id(S)$ may also be S.

Let A be a finite alphabet, $F[A]$ denote the set of nonempty words $a_1 a_2 ... a_n$ over A.

The binary operation "$\cdot$" an $F[A]$ is defined

(*) …….. $$\left(a_1 a_2 ... a_n\right)\left(b_1 b_2 ... b_n\right) = a_1 a_2 ... a_n b_1 b_2 ... b_n$$

The set $F[A]$ with a binary multiplicative operation defined by (*) is called free semi group over A.

If $A = \left\{a_1, a_2, ..., a_n\right\}$ is finite and if the congruence ρ on $F[A]$ is generated by a finite set

$R = \left\{\left(w_1, z_1\right), ..., \left(w_r, z_r\right)\right\}, \left(w_i, z_i\right) \in F[A] \times F[A]$ then we said $F[A]/\rho$ is constructed with a finite number of generators, so

$$F[A]/\rho = < a_1, a_2, ..., a_n > \mid w_1 = z_1, ..., w_r = z_r >$$

$F[A]/\rho$ have the system of free generators $a_1, a_2, ..., a_n$ and the relations …….. $w_1 = z_1, ..., w_r = z_r$.

Let $(S, +)$ be a commutative semi group and φ, ψ its idempotent permutable endomorphisms:

***Corresponding author:** Rushadije Ramani
Department of Mathematics, State University of Tetova, Avenue "Bllagoja Toska" Bllok 82-A1/6, Tetova 1200, Maqedoni

(1) … $\varphi^2=\psi,\psi^2=\psi,\varphi\psi=\psi\varphi$ the $\varphi\psi(x)=\varphi\psi(y)$ for any $x,y\in S$.

Proposition1: Let $(S,+)$ be a commutative semi group and φ,ψ its endomorphisms satisfying (1). By the definition

(2)... $xy=\varphi(x)+\psi(y)$

Then, $(S,.)$ is a distributive semi group.

Proof: Let $x,y,z\in S$,

then

$$xyz=\varphi(x)+\psi(yz)=\varphi(x)+\psi(\varphi(y)+\psi(z))=$$
$$=\varphi(x)+\psi(\varphi(y)+\psi(z)=\varphi(x)+\psi(\varphi/y))+\psi\varphi(x)+\psi(z)=$$
$$=\varphi(x,y)+\psi(x,z)=\underline{xyxz}$$
$$yzx=\varphi(yz)+\psi(x)=\varphi(\varphi(y)+\psi(z)+\psi(x)=$$
$$=\varphi(y)+\varphi(\psi(z))+\psi(x)=$$
$$=\varphi(y)+\varphi\psi(x)+\varphi\psi(z)+\psi(x)=$$
$$=\varphi(yx)+\psi(zx)=$$
$$=\underline{\underline{yxzx}}$$

We proof that $(S,.)$ is a distributive semi group. #

Construction

Now we will construct free distributive semi groups with system of free generators $\{x_i\},i\in I.$

Let $(F,+)$ be free commutative semi group of idempotent with system of free generators

$$\{a_i\}\cup\{b_i\}\cup\{c_i\}\cup\{d\},\ \{a_i\}\cup\{b_i\}\cup\{c_i\}\cup\{d\},\ i\in I.$$

Now on $(F,+)$ we define the homomorphisms φ and ψ :

(3) …….. $$\varphi=\begin{pmatrix} a_i & b_i & c_i & d\\ b_i & b_i & d & d\end{pmatrix} \quad \text{and} \quad \psi=\begin{pmatrix} a_i & b_i & c_i & d\\ c_i & d & c_i & d\end{pmatrix}$$

Proposition 2: Homomorphisms φ and ψ of free idempotent commutative semi group $(F,+)$ defined by (3) satisfies (1) and $(F,.)$ where $"\cdot"$ is defined by (2) is distributive semi group.

Proof: Homomorphisms φ and ψ satisfies (1) because this conditions satisfied on system of free generators. By the proposition1 $(F,\cdot)$ is a distributive semi group.

Proposition 3:_ Let $\{a_i\}$, $i\in I$ be a system of free generators. Then $(F[a_i,i\in I],\cdot)$, where $"\cdot"$ is defined by (2) is a free distributive semi group with a system of free generators $\{a_i\}$, $i\in I$.

The elements from $F[a_i,i\in I]$, have the form

(4a)... $$w_1=a_{i_1}^{n_1}(\prod_{k=2}^{r-1}a_{ik})a_{i_r}^{n_r},$$

where

$a_{i_\nu}\neq a_{i_\mu}$ for $\nu\neq\mu$ and $n_1,n_r\in\{1,2\}$ or

$$(4b)\ldots \quad w_1 = a_{i_1}(\prod_{k=2}^{r} a_{i_k})a_{i_1}, \text{ where } a_{i_\nu} \neq a_{i_\mu} \text{ for } \nu \neq \mu.$$

Let:

$$(5a) \qquad w_2 = a_{j_1}^{m_1}(\prod_{1=2}^{s-1} a_{j_1})a_{j_s}^{m_s} \quad \text{or}$$

$$(5b) \qquad w_2 = a_{j_1}(\prod_{1=2}^{s} a_{j_1})a_{j_1}$$

From $w_1 = w_2 \Rightarrow r = s$ and $a_{i_1} = a_{j_1}, n_1 = m_1$ such that the equations (a) or (b) take the form:

$$(6) \qquad \prod_{k=2}^{k-1} a_{i_k} \quad \text{and} \quad \prod_{1=2}^{s-1} a_{j_1}.$$

From $w_1 \neq a_{i_1}$ and $w_2 \neq a_{j_1}$ we have

$$w_1 = \varphi(a_{i_1}) + d + \psi(a_{i_n}) = b_{j_1} + d + c_{i_r} \quad \text{and}$$
$$w_2 = \varphi(a_{j_1}) + d + \psi(a_{i_s}) = b_{j_1} + d + c_{j_s}$$

From $w_1 = w_2 \Rightarrow i_1 = j_1$ and $i_r = j_s$, so from $(6) \Rightarrow a_{i_k} = a_{j_k}$, and $r = s$, $n_1 = m_1$, $n_r = m_s$.

It is obvious that for the construction of free distributive semi groups $(F[a_i, i \in I], \cdot)$ it is not taken minimal commutative idempotent semi group $(F, +)$.

Let $(F,+)$ be free commutative idempotent semi group with a system of free generators

$\{a\} \cup \{b\} \cup \{c\} \cup \{d\}$.

Let φ and ψ be homomorphisms of $(F,+)$ defined as follows:

$$\varphi = \begin{pmatrix} a & b & c & d \\ b & b & d & d \end{pmatrix} \quad \text{and} \quad \psi = \begin{pmatrix} a & b & c & d \\ c & d & c & d \end{pmatrix}$$

Homomorphisms φ and ψ satisfy (1).

Proposition4: $(F$ [a,c]$, \cdot)$ is a free distributive semi group with a system of free generators $\{a\} \cup \{c\}$ where "$\cdot$" is defined by (2).

Proof: All elements of $F[a, c]$ have the form $a^m, c^n, a^m c^n, c^n a^m, a^m c^n a$ and $c^n a^m c$ because by (2) this elements are different in $(F,+)$ so they are different in $(F[a,b], \cdot)$ #

REFERENCES

[1] R. Strecker., *Konstruktion freier medialer Hlbgrupen aus komutativen Halbgrupen*, Heft 2 265-270 , 1982.
[2] A. Zejnullahu, *Free left distributive semi groups*, Acta Universitatis Carolinae-Mathematica et Physica, Vol. 30, No.1, 29-32 , 1989.
[3] S. Markovski, *On distributive semi groups*, God. Zbor. Matem. Fak. 30, 15-27, 1979 [4].
[4] T. Kepka, *Varieties of left distributive semi groups*, Acta. Univ. Car. Math. Phyz.25/1, 3-18, 1984.

****