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**RESEARCH ARTICLE**

**MINIMIZATION OF MULTIPLICATIVE GRAPHS**

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**ABSTRACT**

In this paper, the new concept minimization of multiplicative labelings has been introduced and a formula for minimization of multiplicative labeling has been introduced. A function  $f$  is called a minimization of multiplicative labeling of a graph  $G$  with  $q$  edges, if  $f$  is a bijective function from the vertices of  $G$  to the set  $\{1,2,3,\dots,p\}$  such that when each edge  $UV$  is assigned the label  $f(UV)=f(u)*f(v)-\min\{f(u),f(v)\}$ , then the resulting edge labels are distinct numbers. Moreover, some families of graphs which are coming under minimization of multiplicative labeling are being investigated.

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**INTRODUCTION**

The graphs that are taken in this paper are finite and undirected. The symbols  $V(G)$  and  $E(G)$  denotes the vertex set and edge set of a graph  $G$ . The cardinality of the vertex set is called the order of  $G$  denoted by  $p$ . The cardinality of the edge set is called the size of  $G$  denoted by  $q$  edges is called  $(p,q)$  graph. A graph labeling is an assignment of integers to the vertices or edges. A dynamic survey on graph labeling is regularly updated by Gallian (Gallian, 2000) and it is published by electronic journal of combinatority. Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past three decades. In this paper, we introduced the new concept minimization of multiplicative labeling and established a formula. Some basic definitions are taken from Harary (Harary, 2001).

**Definition 1.1**

Let  $G=(V(G), E(G))$  be a graph  $G$ . A graph  $G$  is said to be minimization of multiplicative labeling if there exist a bijective function from the vertices of  $G$  to the set  $\{1,2,3,\dots,p\}$  such that when each edge  $uv$  is assigned the label  $f(uv)=f(u)*f(v)-\min\{f(u), f(v)\}$ , then the resulting edge labels are distinct numbers.

**Definition 1.2**

A minimization of multiplicative graph with weight as perfect square numbers are called perfect minimization of multiplicative graph

**Main Results**

**Theorem 2.1**

The path  $p_n$  is perfect minimization of multiplicative graphs.

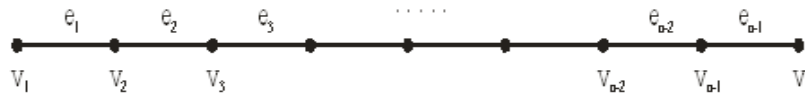
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**Proof**

Let the graph  $G$  be path  $p_n$ .

Let  $V(G)=n$  and  $E(G)=n-1$ .

Let us set an arbitrary labeling as follows:



The mapping  $f:V(G) \rightarrow \{1,2,3,\dots,p\}$  is defined by  $f(uv)=f(u)*f(v)-\min\{f(u),f(v)\}$ .

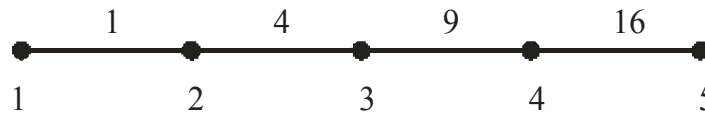
Define  $f(v_i)=i$  for  $1 \leq i \leq n$

$f(e_i)=i^2$  for  $1 \leq i \leq n-1$

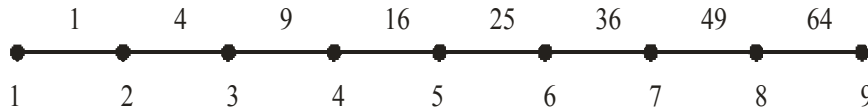
The edges of the path receives distinct values with perfect squares.

Hence the graph  $p_n$  is perfect minimized multiplicative graphs.

**Example 2.1**



**Example 2.2**



**Theorem 2.2**

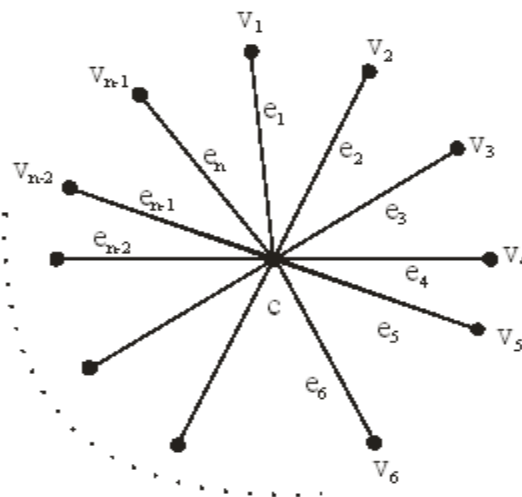
The star graph  $k_{1,n}$  are minimization of multiplicative graphs.

**Proof**

Let  $v_1, v_2, v_3, \dots, v_n$  be the pendent vertices of the star graph  $k_{1,n}$  and  $v_0$  be the centre vertex.

Let  $V(G)=n+1$  and  $E(G)=n$

Let us set an arbitrary labeling as follows:



Define  $f:V(G) \rightarrow \{1,2,3,\dots,p\}$

$f(v_0)=1$

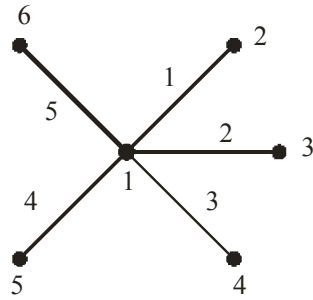
$f(v_i)=i+1$  for  $1 \leq i \leq n$

$f(e_i)=i$  for  $1 \leq i \leq n$

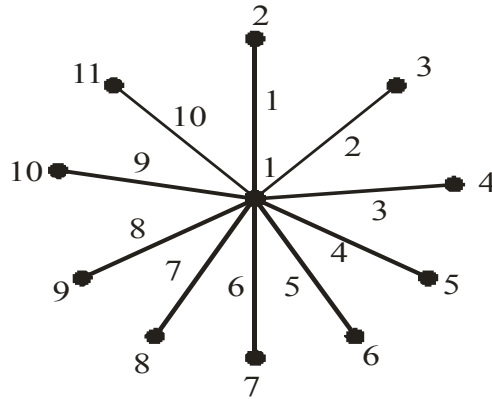
The edges of the star graph receives distinct values.

Therefore, star graph  $k_{1,n}$  are minimized multiplicative graph.

**Example 2.3**



**Example 2.4**



**Theorem 2.3**

S graph is minimization of multiplicative graph for every positive integer n.

**Proof**

S graph consists of n vertices and n-1 edges.

Let  $V(G)$  denotes the set of all vertices

ie.,  $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$

Let  $E(G)$  denotes the set of all edges

ie.,  $E(G) = \{e_1, e_2, e_3, \dots, e_{n-1}\}$

Define  $f: V(G) \rightarrow \{1, 2, 3, \dots, p\}$  as follows

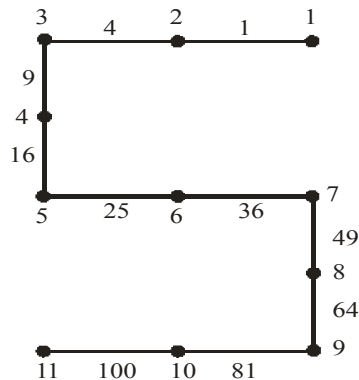
$f(v_i) = i$  for  $1 \leq i \leq n$

$f(e_i) = i^2$  for  $1 \leq i \leq n-1$

The edges receive weight as a perfect square numbers.

Hence S graph is a perfect minimized multiplicative graph.

**Example 2.5**



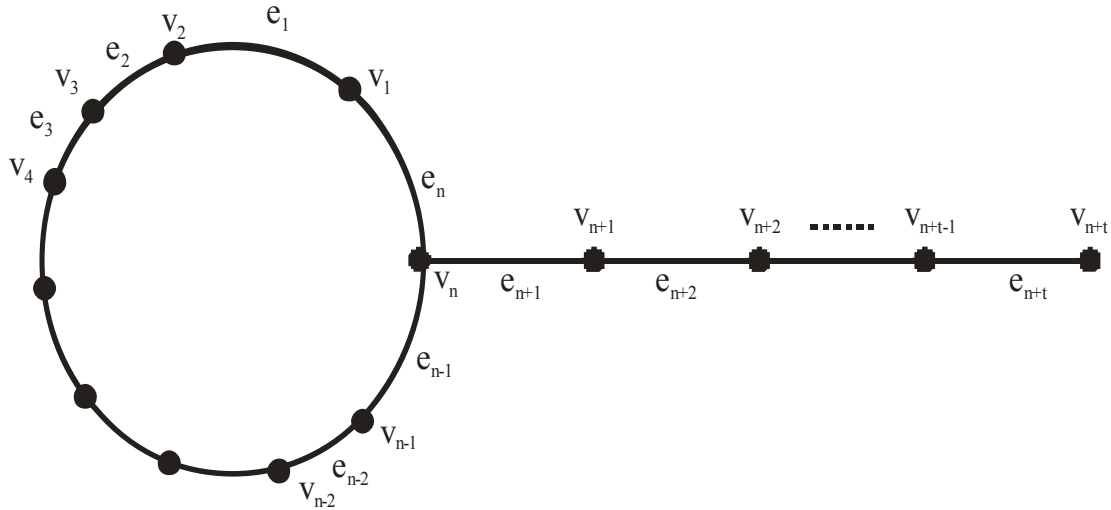
**Theorem 2.4**

The graph  $C_3@P_t$  is a minimization of multiplicative graphs.

**Proof**

Let  $\{v_1, v_2, v_3, v_{n+1}, v_{n+2}, \dots, v_{n+t}\}$  be the vertices of  $C_3@P_t$  and  $\{e_1, e_2, e_3, e_{n+1}, e_{n+2}, e_{n+3}, \dots, e_{n+t}\}$  be the edges of  $C_3@P_t$   
 Let  $V(G)=n+t$  and  $E(G)=n+t$

Let us set an arbitrary labeling as follows:



Define  $f:V(G) \rightarrow \{1,2,3,\dots,p\}$  as follows

$$f(v_i)=i ; 1 \leq i \leq n+t$$

$$f(e_i)=i^2 ; 1 \leq i \leq n-1$$

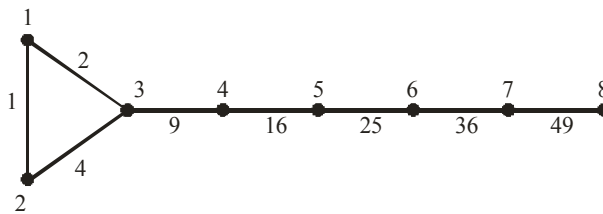
$$f(e_n)=n-1$$

$$f(e_i)=(n-1+i)^2 ; n+1 \leq i \leq n+t$$

The edges receive weight as a distinct value.

Hence Kite graph is a minimization of multiplicative graphs.

**Example: 2.6**



**Theorem: 2.5**

The flower pot graph is a minimization of multiplicative graphs.

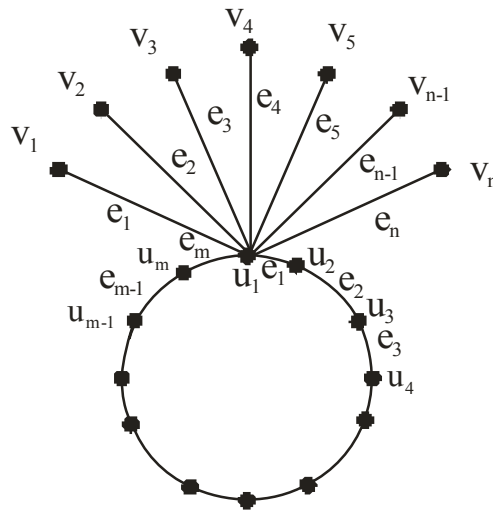
**Proof**

Let G be a graph of flower pot.

Let  $\{v_1, v_2, v_3, \dots, v_n, u_1, u_2, u_3, \dots, u_n\}$  be the vertices of the flower pot.

Let  $V(G)=n+3$  and  $E(G)=n+3$

Let us set an arbitrary labeling as follows



Define  $f: V(G) \rightarrow \{1, 2, 3, 4, \dots\}$  as follows

$$f(v_i) = i + 1 \quad ; \quad 1 \leq i \leq n$$

$$f(u_1) = 1$$

$$f(u_i) = n + i \quad ; \quad 2 \leq i \leq m$$

$$f(e_i) = i \quad ; \quad 1 \leq i \leq n$$

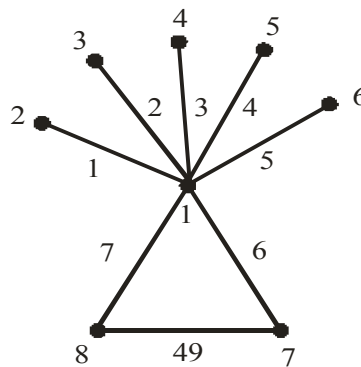
$$f(e_i^?) = u_i * u_{i+1} - \min\{u_i, u_{i+1}\} \quad ; \quad 1 \leq i \leq m - 1$$

$$f(e_m^?) = u_m * u_1 - \min\{u_1, u_m\}$$

The edges receive weight as a distinct numbers.

Hence the flower pot graph is a minimization of multiplicative graphs

**Example: 2.7**



**Theorem 2.6**

Windmill graph is a minimization of multiplicative graphs.

**Proof**

Let  $G$  be a graph of Windmill  $WM_n$ .

Let  $|V(G)| = 2n + 1$  and  $|E(G)| = 3n$ .

Let us set an arbitrary labeling of graphs.

The mapping  $f: V(G) \rightarrow \{1, 2, 3, \dots, p\}$  is defined by

$$f(v_0) = 1$$

$$f(v_i) = i + 1 \quad ; \quad 1 \leq i \leq n$$

$$f(v_i^?) = n + i + 1 \quad ; \quad 1 \leq i \leq n$$

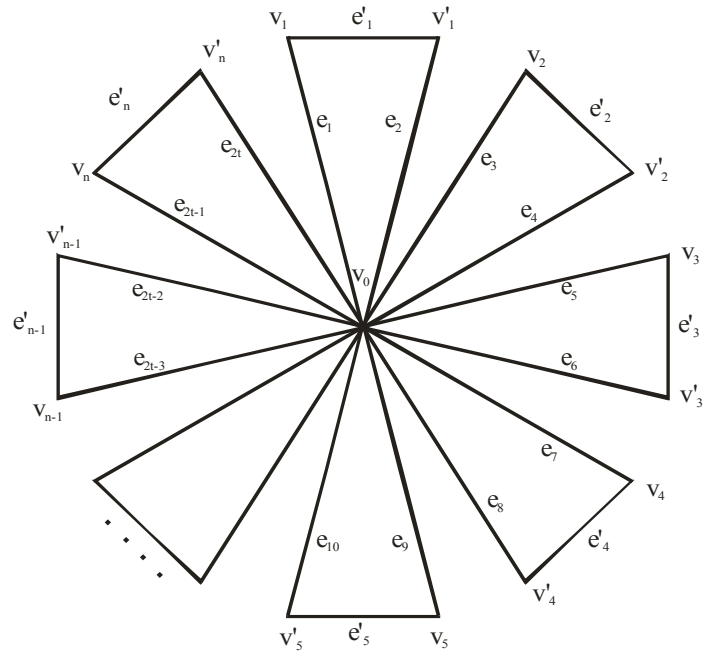
$$f(e_i) = i \quad \text{if } i \text{ is odd}$$

$$f(e_i) = n + i \quad \text{if } i \text{ is even}$$

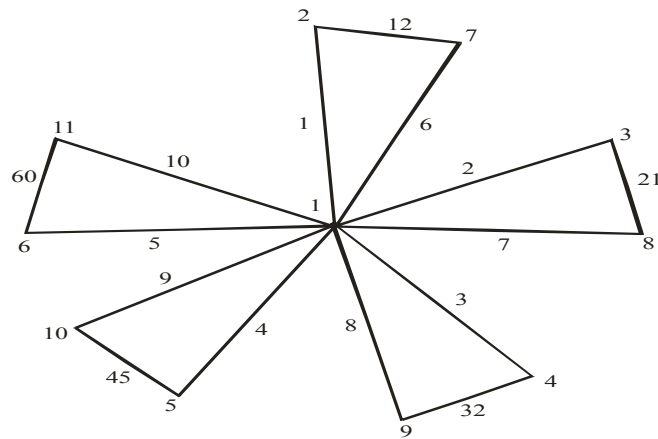
$$f(e_i^?) = v_i * v_i^? - \min\{v_i, v_i^?\}$$

The edges receive weight as a distinct numbers.

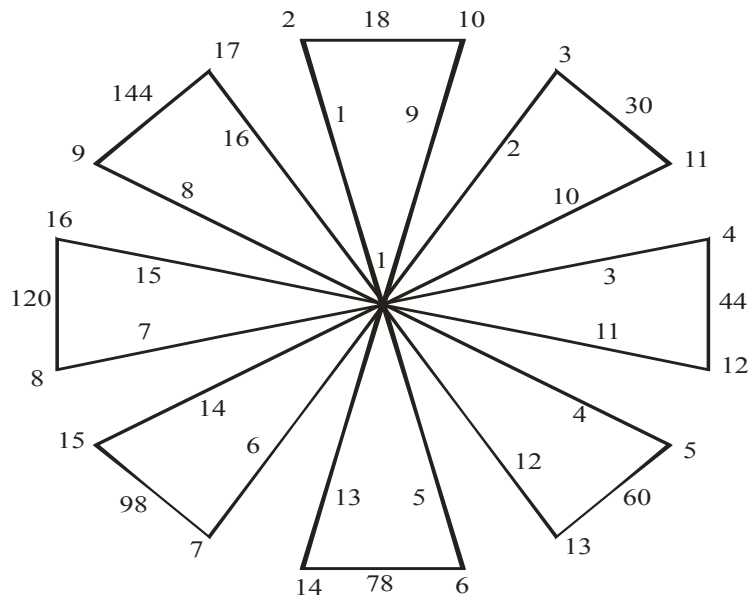
Hence Windmill graph  $WM_n$  admits differences of cubic and squared difference graphs.



Example: 2.8



Example: 2.9



**Theorem 2.7**

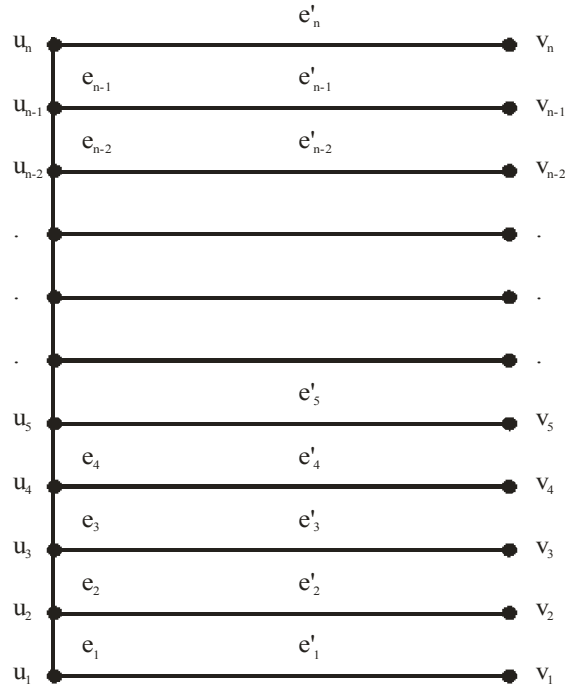
A comb graph  $(P_n \odot K_1)$  is a minimization of multiplicative graphs.

**Proof**

Let  $G$  be a graph of comb  $(P_n \odot K_1)$

A comb graph consists of  $2n$  vertices and  $2n - 1$  edges.

Let us set an arbitrary labeling as follows:



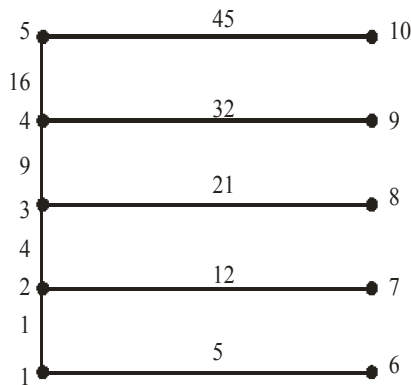
The mapping  $f: V(G) \rightarrow \{1, 2, 3, \dots, p\}$  is defined by

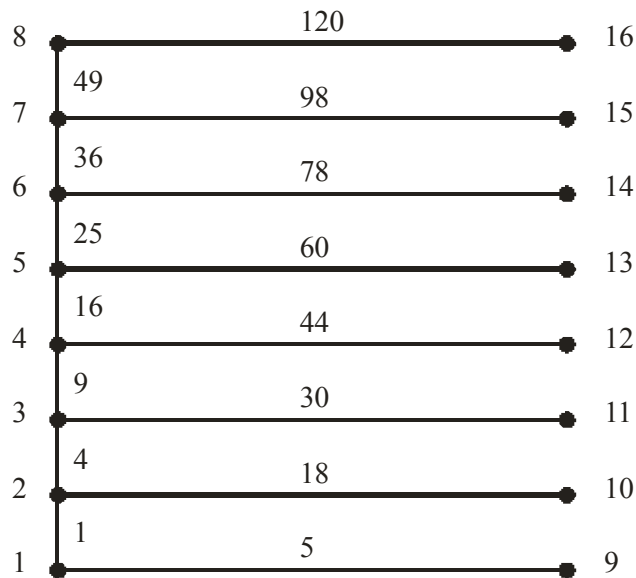
$$\begin{aligned}
 f(u_i) &= i & ; 1 \leq i \leq n \\
 f(v_i) &= n + i & ; 1 \leq i \leq n \\
 f(e_i) &= i & ; 1 \leq i \leq n-1 \\
 f(e'_i) &= u_i * v_i - \min\{u_i, v_i\} & ; 1 \leq i \leq n
 \end{aligned}$$

The edges receive weight as a distinct numbers.

Hence the comb graph is a minimization of multiplicative graphs.

**Example: 2.10**



**Example: 2.11****Conclusion**

We established a formula for minimization of multiplicative graphs. In this paper, we investigated some families of graphs such as path, star, S, flower pot, wind mill, comb and kite graph which satisfies the formula. Finally, we conclude that, the above mentioned graphs are minimization of multiplicative graphs.

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