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RESEARCH ARTICLE

NUMERICAL SOLUTION OF FUZZY MODIFIED NEWTON-RAPHSON METHOD FOR SOLVING NON-LINEAR EQUATIONS

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ABSTRACT

In this paper to find fuzzy modified Newton-Raphson method used for solving non linear equations of the form f(x)=0. The proposed numerical method has capability to solve fuzzy equations as well as algebraic ones. For this purpose with the motivation of avoiding the computation of the derivative of the function f(x), which is involved in fuzzy Newton-Raphson method. Finally, we provide a fuzzy linear interpolation method in solving a non linear equation f(x) =0.

INTRODUCTION

A fuzzy modified Newton-Raphson method is a Newton-Raphson method for solving non linear equations. The aim of this paper is to construct a new method using linear interpolation method to solve a non linear equation f(x) which is a modification of fuzzy Newton-Raphson formula. In this paper an efficient iterative method is build up to solve non linear equations. As it can be seen in our paper fuzzy modified Newton-Raphson method converges rapidly to exact solution. In section(2), we need for definition of triangular fuzzy number and membership function and also definitions of basic arithmetic operation on fuzzy triangular numbers which can be found in this section.In order to obtain a fuzzy modified Newton-Raphson method is explained in section (3). In section (4), the mentioned method has been applied in numerical example.

1. Preliminaries:

Definition 2.1:

A fuzzy number ã in a triangular fuzzy number denoted by $[a_1, a_2, a_3]$ where a_1, a_2, a_3 are real numbers and its membership function $\mu_{\tilde{a}}(x)$ is given below

$$\mu_{\tilde{a}}(x)=\begin{cases}(x-a_1)/(a_2-a_1) & for\ \ a_1 \le x \le a_2\\ (a_3-x)/(a_3-a_2) & for\ \ a_2 \le x \le a_3\\ 0 & otherwise\end{cases}$$

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Definition 2.2

Let ã= $[a_1, a_2, a_3]$ & $\tilde{b}=[b_1, b_2, b_3]$ be two triangular fuzzy numbers then

(i) $\tilde{a} \oplus \tilde{b} = [a_1+b_1, a_2+b_2, a_3+b_3]$

(ii) $\tilde{a} \Theta \tilde{b} = [a_1-b_3, a_2-b_2, a_3-b_1]$

(iii) $k\tilde{a} = [ka_1, ka_2, ka_3]$ for $k \ge 0$

(iv) $k\tilde{a} = [ka_3, ka_2, ka_1]$ for $k < 0$

(v) $\tilde{a} \otimes \tilde{b} = [c_1, c_2, c_3]$

Where c_1 = minimum $\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$

$c_2 = a_2b_2$

c_3 = maximum $\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}$

(vi) $\frac{1}{\tilde{b}} = \left[\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right]$, where b_1, b_2, b_3 are all non zero real numbers and

(vii) $\frac{\tilde{a}}{\tilde{b}} = \tilde{a} \otimes \frac{1}{\tilde{b}}$, where b_1, b_2, b_3 are all non zero real numbers.

Definition 2.3

The magnitude of the triangular fuzzy number $\tilde{\mu}=[a,b,c]$ is given by $\text{Mag}(\tilde{u}) = \frac{a+10b+c}{12}$.

2. Fuzzy Modified Newton Raphson Method:

The Fuzzy Newton Raphson method is a classical optimization technique for solving nonlinear equations. In this method, we start with an initial approximation x_0 and generate a sequence of approximations. The iterative procedure terminates when the relative error for two successive approximations becomes less than or equal to the prescribed tolerance.

Newton Raphson formula is:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n=0,1,\ldots \quad \_\_\_\_\_(1)$$

Newton's method requires that the derivative be calculated directly. In most practical problems, the function in question may be given by a long and complicated formula, and hence an analytical expression for the derivative may not be easily obtainable. It is clear from the formula for Newton's method that it will fail in cases where the derivative is zero. In these situations, it may be appropriate to approximate the derivative by using the slope of a line through two points on the function. In this case, the Secant method results. To avoid computing f'(x) may not always be available or may be costly to compute and to preserve the excellent convergence properties of the Newton-Raphson method, $f'(x_n)$ is replaced by $f[x_n,x_{n-1}]$ in equation (1)

Therefore equation (1) becomes $x_{n+1} = x_n -$

$$\frac{f(x_n)}{f[x_n, x_{n-1}]} \quad \_\_\_\_\_\_\_\_(2)$$

Where $f[x_n, x_{n-1}] = [f(x_n)-f(x_{n-1})]/(x_n, x_{n-1})$

This result in equation (2) is secant method. The Newton-Raphson iteration requires two functions evaluations, one of f(x) and another of f'(x), per iteration. Starting with two initial approximations x_0 and x_1, we compute x_2 by secant method. Then we apply Modified Newton-Raphson method to calculate x_3,x_4,x_5 … using interpolation table. We replace $f'(x_n)$ in Newton-Raphson Formula equation(1) by $g_{n,k}(x)$. For this, we write $g_{n,k}(x)$ in Newtonian form as

$$g_{n,k}(x) = f(x_n) + \sum_{i=1}^{k} f[x_n, x_{n-1,\ldots,} x_{n-i}] \prod_{j=0}^{i-1} (x - x_{n-j}) \quad \_\_\_\_\_(3)$$

Where $f[x_n, x_{n-1,\ldots,} x_{n-i}]$ are divided difference of f(x).

We can define the divided differences as $f[x_n]=f(x_n)$ and

$$f[a,b] = \frac{f[a]-f[b]}{a-b}, a \neq b$$

By equation (3), $g_{n,k}(x)$ is compute by ordering the x_i as $x_n, x_{n-1,\ldots,} x_{n-i}$ for i=1,2,…k.

Using this ordering we can compute $g_{n,k}(x)$ in (3), and let $x=x_n$,
we obtain,

$$g_{n,k}(x) = f(x_n) + \sum_{i=2}^{k} f[x_n, x_{n-1,\ldots,} x_{n-i}] \prod_{j=1}^{i-1} (x - x_{n-j}) \quad \_\_\_\_\_(4)$$

The divided difference table is

x	f	I diff.	II diff.			nth diff.
x_0	f_0	∇f_1	$\nabla^2 f_1$			
x_1	f_1					
x_2	f_2	∇f_2				
						$\nabla^n f_n$
			$\nabla^2 f_n$			
x_n	f_n					
		∇f_n				

Where the first difference is:

$$\nabla f_1 = \frac{f_1 - f_0}{x_1 - x_0}, \nabla f_2 = \frac{f_2 - f_1}{x_2 - x_1}, \ldots, \nabla f_n = \frac{f_n - f_{n-1}}{x_n - x_{n-1}}$$

The second difference is:

$$\nabla^2 f_2 = \frac{\nabla f_2 - \nabla f_1}{x_2 - x_0}, \nabla^2 f_3 = \frac{\nabla f_3 - \nabla f_2}{x_3 - x_1}, \ldots, \nabla^2 f_n = \frac{\nabla f_n - \nabla f_{n-1}}{x_n - x_{n-2}}$$

For k=2 equation (4) becomes

$g_{n,k}(x) = f[x_n, x_{n-1}] + f[x_n, x_{n-1}, x_{n-2}](x_n, x_{n-1})$ ________(5)

We write $f[x_n, x_{n-1}] = \nabla f_n$, $f[x_n, x_{n-1}, x_{n-2}] = \nabla^2 f_n$

Therefore equation (5) becomes $g_{n,k}(x) = \nabla f_n + \nabla^2 f_n (x_n, x_{n-1})$

Modified Newton Raphson formula for k=2 is

$$x_{n+1} = x_n - \frac{f(x_n)}{\nabla f_n + \nabla^2 f_n (x_n, x_{n-1})} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_(6)$$

Similarly for k=3 equation (4) becomes

$g_{n,k}(x) = f[x_n, x_{n-1}] + f[x_n, x_{n-1}, x_{n-2}](x_n, x_{n-1}) + f[x_n, x_{n-1}, x_{n-2}, x_{n-3}](x_n, x_{n-1})(x_n, x_{n-2})$ ___(7)

We write $f[x_n, x_{n-1}, x_{n-2}, x_{n-3}] = \nabla^3 f_n$

Then equation (7) becomes

$g_{n,k}(x) = \nabla f_n + \nabla^2 f_n (x_n, x_{n-1}) + \nabla^3 f_n (x_n, x_{n-1})(x_n, x_{n-2})$

Modified Newton Raphson formula for k=3 is $x_{n+1} = x_n -$

$$\frac{f(x_n)}{\nabla f_n + \nabla^2 f_n (x_n, x_{n-1}) + \nabla^3 f_n (x_n, x_{n-1})(x_n, x_{n-2})} \quad \_\_\_\_\_\_\_\_(8)$$

In this way we put the values in the above equations of the backward diagonal from the table. To compute next iteration, we put the values of the previous iteration in the table. In this way to compute the next iteration we need only the backward diagonal of table.

3. Numerical Example:

Consider the following nonlinear equation:

$[0.5, 1, 1.5][x_1, x_2, x_3]^2 - [4,5,6][x_1, x_2, x_3] + [1,2,3] = 0$

Let , $f[x_1, x_2, x_3] = [0.5,1,1.5][x_1, x_2, x_3]^2 - [4,5,6][x_1, x_2, x_3] + [1,2,3]$

Now , $f[3,4,5] = [0.5,1,1.5][3,4,5]^2 - [4,5,6][3,4,5] + [1,2,3]$

= [-24.5,-2, 28.5] i.e., negative

f[4,5,6]= [0.5,1,1.5][4,5,6] ^{2} -[4,5,6][4,5,6]+[1,2,3]

= [-27, 2, 41] i.e., positive

Therefore the root lies between [3, 4, 5] & [4, 5, 6].

Let x_0 = [4, 5, 6]

x_1 = [3, 4, 5]

Therefore f_0 = [-27, 2, 41]

f_1 = [-24.5,-2, 28.5]

First approximation:

The Secant Formula is, $x_{n+1} = x_n - \dfrac{f(x_n)}{f[x_n, x_{n-1}]}$ ______ (9)

Where $f[x_n, x_{n-1}] = [f(x_n)-f(x_{n-1})]/(x_n, x_{n-1})$

Compute x_2 from above equation (8)

x_2=[0.4865,4.5,5.4414], f_2=[-28.5570,-0.25,7.4672]

Second approximation

For k=2 our modified Newton Raphson formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{\nabla f_n + \nabla^2 f_n(x_n, x_{n-1})} \qquad (10)$$

Table 2

x	f	I Diff.	II Diff.
[4,5,6]	[-27, 2, 41]		
		[-65.5,4,55.5]	
[3,4,5]	[-24.5,-2, 28.5]		[-55.6082,1,63.0511]
		[-24.657,3.5,25.818]	
[2.4865,4.5,5.4414]	[-28.5570,-0.25,7.4672]		

Putting the values of the backward diagonal from the above table in equation (9)

x_3= [2.2776,4.5625,5.6460]

Third approximation

For k=3 our modified Newton Raphson formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{\nabla f_n + \nabla^2 f_n(x_n, x_{n-1}) + \nabla^3 f_n(x_n, x_{n-1})(x_n, x_{n-2})} \qquad (11)$$

Now, extending the Divided Difference Table

Table 3

x	f	I Diff.	II Diff.	III Diff.
[4,5,6]	[-27, 2, 41]			
		[-65.5,4,55.5]		
[3,4,5]	[-24.5,-2, 28.5]		[-55.6082,1,63.0511]	
		[-24.657,3.5,25.818]		[-49.226,0.0005, 44.555]
[2.4865,4.5,5.4414]	[-28.5570,-0.25,7.4672]		[-17.9857,0.9998, 17.7219]	
		[-22.28,4.0624,22.285]		
[2.2776,4.5625,5.6460]	[-30.2822,0.009, 41.7056]			

Putting the values of the backward diagonal from the above table in equation (11)

x_4= [2.1959,4.5616,5.7276]

Fourth approximation

For k=4 our modified Newton Raphson formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{\nabla f_n + \nabla^2 f_n(x_n,x_{n-1}) + \nabla^3 f_n(x_n,x_{n-1})(x_n,x_{n-2}) + \nabla^4 f_n(x_n,x_{n-1})(x_n,x_{n-2})(x_n,x_{n-3})}$$

Now, extending the Divided Difference Table

Table 4

x	f	I Diff.	II Diff.	III Diff.	IV Diff.
[4,5,6]	[-27, 2, 41]				
		[-65.5,4,55.5]			
[3,4,5]	[-24.5,-2, 28.5]		[-55.6082,1,63.0511]		
		[-24.657,3.5,25.818]		[-49.226,0.0005,44.555]	
[2.4865,4.5,5.4414]	[-28.5570,-0.25, 7.4672]		[-17.9857,0.9998, 17.7219]		[-32.4033,0.8508, 35.1675]
		[-22.28,4.0624,22.285]		[-11.4294,-0.3725, 11.5228]	
[2.2776,4.5625,5.6460]	[-30.2822,0.009, 41.7056]		[-13.4529,0.7906, 13.4438]		
		[-21.3637,4.1111,21.3643]			
[2.1959,4.5616,5.7276]	[-30.9546,0.0002, 43.4246]				

Putting the values of the backward diagonal from the above table in equation (12)

x_5= [2.1625,4.5616,5.7590]

Conclusion

This modified method seems to be very easy to employ with reliable results.

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