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International Journal of Current Research
Vol. 4, Issue, 03, pp.126-131, March, 2012

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

THE EFFECT OF ROTATIONAL CONSTRAINT ON THERMO-BIOCONVECTION IN SUSPENSIONS OF GRAVITACTIC MICROORGANISMS-A NUMERICAL STUDY

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ARTICLE INFO

Article History:
Received 14th December, 2011
Received in revised form
09th January, 2011
Accepted 24th February, 2011
Published online 31st March, 2012

Key words:
Bioconvection,
Rotaion, Gravitactic,
Microorganisms,
Numerical study,
Permiability parameter.

ABSTRACT

This paper studies the cumulative effect of uniform rotation of the system and the heating/cooling from below on the stability of a suspension of motile gravitactic microorganisms in a shallow sparsely packed horizontal porous layer. The bioconvective system is described by the continuity, momentum, cell conservation, flux of micro-organisms and thermal energy equations. The basic state solution is determined and the perturbed equations are solved using a fast computational technique with the MATLAB tool. The Eigen value problem is solved and the profiles of the stream function, cell concentration, temperature along with the neutral stability curves are presented through graphs. The present results show an excellent agreement with the available results in the limiting cases.

INTRODUCTION

This paper studies the cumulative effect of uniform rotation of the system and the heating/cooling from below on the stability of a suspension of motile gravitactic microorganisms in a shallow sparsely packed horizontal porous layer. The model is based on a continuum model of a suspension of gravitactic microorganisms developed earlier by Pedley *et al.* (1988) supplemented by an energy equation and a buoyancy term in the momentum equation that results from an adverse temperature gradient in the porous layer. The boundaries are considered to be rigid, no-slip and zero-flux. Thermal boundary conditions are considered along with the above boundary conditions. The evolution of such a bioconvective system is described by the continuity, momentum, cell conservation, flux of micro-organisms and thermal energy equations. The basic state solution is determined and the perturbed equations are solved using a fast computational technique with the MATLAB tool. The Eigen value problem is solved and the profiles of the stream function, cell concentration, temperature along with the neutral stability curves are presented through graphs. The present problem is controlled by several non-dimensional parameters such as, bioconvection Rayleigh number, Permeability parameter, Thermal Rayleigh number, the Schimid number, the Lewis number and the Peclet number. For typical values of the parameters, the computation is done and the results are presented through graphs. It was found that the permeability parameter has a remarkable influence on the thermo-effects which may (i) either stabilize or destabilize the suspension and (ii) decrease or increase the wavelength of the bioconvective pattern. The phenomena of convective motion of fluid, well-known buoyancy driven phenomena, has attracted many researchers over the past few years. In this context, bioconvective motion due to the suspensions of microorganisms in a fluid/porous layer has been received a great deal of attention due to its very wide applications. The suspensions of microorganisms subject to spontaneous pattern formation were known as long ago as 1848 and the first quantitative study of the phenomena was published in 1911.Most experiments begin with an initially uniform suspension of the microorganism of interest, which is obtained by stirring. The fluid, which subsequently becomes quiescent, spontaneously develops the bioconvective instability as the microorganisms swim guided by their various taxes.

Bioconvection is a ubiquitous phenomenon of biological systems across spatial scales from cells (Okubo and Levin, 2002; Pedley and Kessler, 1992) to ecosystems (Mulched et al., 2001). The term bioconvection was recently developed in fluid mechanics, and refers to flows induced by the collective motion of a large number of motile microorganisms (Platt. 1961). This phenomenon can lead to pattern formation in aqueous media when motile microorganisms respond to certain stimuli (e.g., gravity, light, nutrients) by collectively swimming in particular directions (i.e. taxes). The basic mechanism underlying this phenomenon is similar to that of the well-known Benard thermal convection in the sense that

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both are due to the force of buoyancy resulting from a density gradient which, in the case of bioconvection, occurs when a large number of microorganisms (which are slightly heavier than water) accumulates in a certain region of the fluid medium, while in the case of Benard convection, a density gradient is due to a temperature gradient (Childress et al., 1975; Pedley and Kessler, 1992).A thorough survey of literature pertaining to the subject reveals the following points: The phenomenon of bioconvection patterns in suspensions of swimming cells has been observed several decades ago. Ever since common algae, such *as Chlamydomonasnivalis, Euglena viridis, Crypthecodiniumcohnii and the ciliated protozoan Tetrahymenapyriformis* were isolated, plumes of aggregating cells have been noticed in the culturing flasks. Platt (1961) coined the term ''bioconvection'' to describe the phenomenon of pattern formation in shallow suspensions of motile micro-organisms at constant temperature, on a par with those found in convection experiments. However, this is by no means the first documented observation, which goes back to at least 1848 e.g., Wager (1911). Other experimental investigators have included Loeffer and Mefferd (1952) Nultsch and Hoff (1972) Plesset and Winet (1974) and, more recently, Kessler, (1984), (1985b) Bees (1996) and Bees and Hill (1998). Bioconvection is generally due to an overturning instability caused by micro-organisms swimming to the upper surface of a fluid which has a lower density than the micro-organisms. The first models of bioconvection were developed by Plesset and Winet (1974). They considered Rayleigh-Taylor instability in a continuously stratified, two-layer model and were able to investigate the preferred pattern wavelength as a function of the upper layer depth and the cell concentration. Levandowsky et al. (1975) investigated bioconvection patterns and proposed a more realistic model (Childress et al. (1975)) in which the micro-organisms could swim but were constrained to swim upwards only, due to their asymmetric density distribution. Their application of the Boussinesq approximation implies that the only way in which the cell concentration can affect the fluid flow is through vertical variations in the fluid density. One class of phenomena for which mathematical modeling is well advanced is spontaneous pattern formation, which has been observed in laboratory suspensions of swimming microorganisms from a variety of phyla, including algae (Wager, 1911; Kessler, 1985; Bees and Hill, 1997), protozoa (Platt, 1961; Childress et al., 1975) and bacteria (Kessler et al., 1994). The mechanism of pattern formation is a convective one, driven by the up-swimming of cells that are denser than the medium in which they swim, and is called bioconvection (Platt, 1961), the mathematical modeling of which has been discussed by Pedley and Kessler (1992a,b).Some of the recent works include Srimani and Sudhakar(1992), Srimani and Padmasini(2001), Srimani and Anuradha (2007), Srimani and Roopa(2011), Srimani and Sujatha(2011).

MATERIALS AND METHODS

In this section the continuum model in boundary conditions and asymptotic analysis are discussed.

Mathematical Formulation

In this section, in order to study the instability of motile suspension in a horizontal thermally stratified porous layer (sparsely packed), the mathematical formulation is discussed. The randomly swimming microorganisms are assumed to be gravitactic in behavior and on the average swimming upwardly with a constant velocity. Here, we consider a 2D-thermally stratified porous layer of infinite horizontal extent containing a large number of gravitactic microorganisms swimming in the porous layer with an upward velocity V_c. The major assumptions are(i)all the physical properties of the fluid are assumed to be constant (ii) the porous layer is isotropic and homogeneous (iii) the density of the medium is everywhere constant except in the buoyancy force term (Boussinesq approximations). The pattern formation within the suspension is described by the Navier-Stokes equation with Boussinesq approximation for the fluid flow, the diffusion convection equation for the concentration of the motile microorganisms together with the thermal energy equation for the temperature. For the forthcoming section, the stability of the equilibrium diffusive-conductive state is discussed through the linear stability analysis for a wide range of the swimming velocity and permeability of the medium the governing equations are:

Continuity equation

$$\nabla \cdot \vec{u}' = 0 \qquad \ldots(1)$$

Momentum equation

$$\rho_w \frac{\partial \vec{u}}{\partial t} + \rho_w \left(\vec{u}' \cdot \nabla\right)\vec{u}' + \rho_w 2\Omega \vec{k} \times \vec{u}' = -\nabla p' + \mu \nabla^2 \vec{u}' + \vartheta \Delta \rho n' \vec{g} - \rho_w \beta T \vec{g} \qquad \ldots(2)$$

Cell conservation equation

$$\frac{\partial n'}{\partial t'} = -\nabla \cdot \vec{J}' \qquad \ldots(3)$$

with flux the of microorganisms .

$$\vec{J}' = \left(\vec{u}' + V_c \vec{k}\right) n' - D_c \nabla n' \qquad \ldots(4)$$

Thermal energy equation

$$\frac{\partial \mathrm{T}'}{\partial t'} + \nabla \cdot \left(\vec{u}' \mathrm{T}'\right) = \alpha_t \nabla^2 \mathrm{T}' \qquad \ldots(5)$$

The boundary conditions at the impermeable boundaries are the condition of rigid no-slip and zero-flux:

$$u' = 0 \quad and \quad \vec{J}' \cdot \vec{n} = 0 \quad at \quad y' = 0, H \qquad \ldots(6)$$

And the thermal boundary conditions are

$$T' = T_0 + \Delta T \quad at \quad y' = 0 \qquad \ldots(7)$$
$$T' = T_0 \quad at \quad y' = H$$

Dimensionless Equations

The governing equations along with the boundary conditions aredimensionalised by using the following scales:

$$\vec{u} = \frac{D_c}{H} u, \quad n' = \frac{\bar{n} H V_c n}{D_c}, \quad T' = \Delta T \cdot T \quad , \nabla = \frac{I}{H} \nabla \quad , t = \frac{H^2}{D_c} t,$$
$$P' = \frac{\nu D_c \rho_w}{H^2} P, \quad \vec{g} = g \hat{k}, \qquad \ldots(8)$$

Dimensionless equationsare :

$$Sc^{-1}\left\{\frac{\partial u}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}\right\}+\frac{2H^2\Omega}{\nu}\vec{k}\times\vec{u}= -\nabla p+\nabla^2\vec{u}+Ran\hat{k}-Ra_T LeT\hat{k} \quad \ldots(9)$$

$$\frac{\partial n}{\partial t}+u\frac{\partial n}{\partial x}+(\nu+Pe)\frac{\partial n}{\partial y}=\nabla^2 n \quad \ldots(10)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+\nu\frac{\partial T}{\partial y}=Le\nabla^2 T \quad \ldots(11)$$

$$Le=\frac{\alpha_t}{D_c}=Lewis\ number$$

$$Pl=\frac{H^2}{k_p}=Permeability\ parameter$$

$$R_a=\frac{g\vartheta\Delta\rho\bar{n}PeH^3}{\rho\nu D_c}=Thermal\ Rayleighnumber$$

$$R_{aT}=\frac{g\beta\Delta TH^3}{\nu\alpha_t}=Thermal\ Raleigh\ number$$

$$Sc=\frac{\nu}{D_c}=Schimid\ number,\quad Pe=\frac{HV_c}{D_c}=Peclet\ number$$

Boundary conditions (in terms of dimensionless variables):

$$\vec{u}=0,\quad \vec{J}\cdot\vec{n}=0,\quad T=1\quad at\quad y=0 \quad \ldots(12)$$
$$\vec{u}=0,\quad \vec{J}\cdot\vec{n}=0,\quad T=0\quad at\quad y=1$$

Equation (9) can be reduced to

$$Sc^{-1}\left\{\frac{\partial u}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}\right\}+\Im\vec{k}\times\vec{u}= -\nabla p+\nabla^2\vec{u}+Ran\hat{k}-Ra_T LeT\hat{k} \quad \ldots(13)$$

Where $\Im=\frac{2H^2\Omega}{\nu}$ = Taylor number …(14)

Equation (13) is resolved into the following equations to eliminate the pressure

$$Sc^{-1}\left\{\frac{\partial \vec{u}}{\partial t}+(\vec{u}\cdot\nabla)u\right\}-\Im w= -\frac{\partial p}{\partial x}+\nabla^2 u \quad \ldots(15)$$

$$Sc^{-1}\left\{\frac{\partial w}{\partial t}+(\vec{u}\cdot\nabla)w\right\}+\Gamma u=-\frac{\partial p}{\partial z}+\nabla^2 w \quad \ldots(16)$$

$$Sc^{-1}\left\{\frac{\partial v}{\partial t}+(\vec{u}\cdot\nabla)v\right\}=-\frac{\partial p}{\partial y}+\nabla^2 v+Ran+Ra_T LeT \quad \ldots(17)$$

Differentiating (15) with respect to z and (16) w. r .t to x and taking the difference, the following equation results:

$$Sc^{1}\left[\frac{\partial}{\partial t}(-u_z+w_x)\right]-\nabla^2(-u_z+w_x)+\Im(w_x+u_z)=0$$

i.e., $\left(Sc^{-1}\left(\frac{\partial}{\partial t}\right)-\nabla^2\right)\varsigma+\Im(-v_y)=0$

I.e., $\left(Sc^{-1}\left(\frac{\partial}{\partial t}\right)-\nabla^2\right)\varsigma=\Im v_y$ …(18)

$where\quad V=w_x-u_z\quad ;Vorticity$

Now $\frac{\partial}{\partial y}\left(\frac{\partial(15)}{\partial x}+\frac{\partial(16)}{\partial z}\right)-\nabla_1^2(17)\Rightarrow$

$$\left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)\nabla^2 v+\Im\frac{d\varsigma}{dy}=\nabla_1^2(Ran-Ra_T LeT) \quad \ldots(19)$$

Operating $\left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)$ on (19) :

$$\left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)^2\nabla^2 v+\Im\frac{d}{dy}\left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)\varsigma= \left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)\nabla_1^2(Ran-Ra_T LeT) \quad \ldots(20)$$

Substituting (17) in (20) :

$$\left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)^2\nabla^2 v+\Im^2 v_{yy}= \left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)\nabla_1^2(Ran-Ra_T LeT) \quad \ldots(21)$$

Substituting for $v=-\psi_x$ in the above equation:

$$\left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)^2\nabla^2(-\psi_x)+\Im^2(-\psi_x)_{yy}= \left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)\nabla_1^2(Ran-Ra_T LeT) \quad \ldots(22)$$

Finally the governing equations are:

$$\left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)^2\nabla^2(-\psi_x)+\Im^2(-\psi_x)_{yy}= \left(Sc^{-1}\frac{\partial}{\partial t}-\nabla^2\right)\nabla_1^2(Ran-Ra_T LeT) \quad \ldots(23)$$

$$\frac{\partial n}{\partial t}+u\frac{\partial n}{\partial x}+(\nu+Pe)\frac{\partial n}{\partial y}=\nabla^2 n \quad \ldots(24)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=Le\nabla^2 T \quad \ldots(25)$$

Thus, we have a sixth-order differential equation in ψ in the rotating case.

Basic state solution

In the basic state the fluid is motionless and accordingly a possible equilibrium state can be determined:

$$Pe\frac{¶ n}{¶ y}=\frac{¶^2 n}{¶ y^2} \quad \ldots(26)$$

$$\frac{¶^2 T}{¶ y^2}=0 \qquad \partial \quad \ldots(27)$$

Now solving (26) and (27) ,we obtain

$$n_b=\frac{e^{Pey}-1}{e^{Pe}-1} \quad \ldots(28)$$

$$T_b = 1 - y \quad \ldots(29)$$

Linear stability analysis

It is a well-known fact that, the linear stability analysis with regard to a convection problem, facilitates the prediction of critical conditions and provides the base for the nonlinear analysis. To study the linear stability, a small perturbation to the basic state is considered. A small perturbation is given to the basic state to study the linear stability analysis:

$$\vec{u} = \in u,$$
$$n = n_b + \in n_1$$
$$T = T_b + \in T_1 \quad \ldots(30)$$

where n_b and T_b are given by (3) and (4) Substituting for $\vec{u}$, n, T the in governing equations (9), (10), and (13) and collecting only $\in$ terms we get the following equations

$$\omega_1 = -\nabla^2\psi_1 = \frac{\partial v_1}{\partial x} - \frac{\partial u_1}{\partial y} \quad \ldots(31)$$

$$\left(Sc^{-1}\frac{\partial}{\partial t} - \nabla^2\right)^2 \nabla^2(-\psi_1)_x + \Im^2(-\psi_1)_{xyy} = \left(Sc^{-1}\frac{\partial}{\partial t} - \nabla^2\right)\nabla_1^2(Ran_1 - Ra_T LeT_1) \quad \ldots(32)$$

$$\frac{\partial n_1}{\partial t} + v_1\frac{\partial n_b}{\partial y} + Pe\frac{\partial n_1}{\partial y} = \nabla^2 n_1 \ldots(33)$$

$$\frac{\partial T_1}{\partial t} + v_1\frac{\partial T_b}{\partial y} = Le\nabla^2 T_1 \quad \ldots(34)$$

The quantities ω_1, n_1 and T_1 can be resolved into normal modes by considering the solution of the form $e^{\sigma t}e^{i\alpha x}f(y)$. Now substituting these solutions in the governing equations (3) to (5), the equations reduce to.

$$(Sc^{-1})^2\sigma^2\left(\frac{d_2}{dy_2} - \alpha^2\right)\psi_1 + \left(\frac{d^2}{dy^2} - \alpha^2\right)^3\psi_1$$
$$-2\sigma Sc^{-1}\left(\frac{d_2}{dy_2} - \alpha^2\right)^2\psi_1 - \Im^2\left(\frac{d^2\psi_1}{dy^2}\right) =$$
$$-\sigma Sc^{-1}(Ran_1 - Ra_T LeT_1)(i\alpha)^2$$
$$+\left(\frac{d^2}{dy^2} - \alpha^2\right)(Ran - Ra_T LeT)(i\alpha)^2 \quad \ldots(35)$$

$$\sigma n_1 - i\alpha\frac{\partial n_b}{\partial y}\psi_1 + Pe\frac{dn_1}{dy} = \left(\frac{d^2}{dy^2} - \alpha^2\right)n_1 \quad \ldots(36)$$

$$\sigma T_1 - i\alpha\frac{\partial T_b}{\partial y}\psi_1 = Le\left(\frac{d^2}{dy^2} - \alpha^2\right)T_1 \quad \ldots(37)$$

The boundary conditions are

$$\psi_1 = 0, \quad \frac{d\psi_1}{dy} = 0, \quad at \quad y = 0, 1 \quad \ldots(38)$$

$$n_1 Pe = \frac{dn_1}{dy}, \quad T_1 = 0 \quad at \quad y = 0,1 \quad \ldots(39)$$

The equations (6), (7) and (8) in the boundary conditions (8) and (9) are solved numerically. For this purpose, CODES are designed and implemented through MATLAB. The computed results are presented through graphs and the accuracy achieved is remarkable.

RESULTS AND DISCUSSION

In this section, the results of the linear stability analysis of a suspension of gravitactic microorganisms in a horizontal sparsely packed porous layer heated or cooled from below and subject to a uniform rotation about the vertical axis are presented. Predictions are made with regard to the markedly different behaviour of rotating and non-rotating systems. The boundaries are assumed to be rigid and hence no-slip and zero-flux conditions are applied along with the thermal boundary conditions. A computer code is designed and implemented by using MATLAB. The coupled differential equations are solved numerically by using the boundary conditions. The computed results are presented through graphs in figures 1 to 9. In all our computations, the typical values of the Schmid number Sc = 1.0 and Lewis number Le = 1.0 are considered. The results are computed for small as well as large rotation rates. The parameters being (i) the bioconvection Rayleigh number Ra (ii) the thermal Rayleigh number Ra_T (iii) the Taylor number $\Im$ (iv) the Schmidt number Sc (v) the Lewis number Le and (vi) the peclet number Pe. From the figures, the following observations are made:

(In the figures, $\Im$ = t; the Taylor number)

I. Figures 1 to 3 present the graphs of stream function (ψ vs. y) for the combinations of the parameters (Pe, t, Ra_T) = (0.1, 5, 5000), (0.1, 10, 5000), (0.1, 0.15, 5000) respectively. The figures clearly indicate the drastically/markedly different behaviour of rotating and non-rotating bioconvective porous systems with the suspension of gravitacticmicroorganisms.Even for small rotation rates considered here, the stream function profiles exhibit highly nonlinearities throughout the region under consideration. The random behaviour of the curves suggests that there might be an oscillatory type of bioconvective mechanism under the constraint of uniform rotation as in the case of ordinary convection.

II. The variation of critical wave number α_c with the thermal Rayleigh number Ra_T for different rotation rates (= 5, 10, 15) and Pe = 0.1, 1, 10 are presented through graphs in figures 4 to 6. In all the three cases, the qualitative as well as the quantitative nature of the curves is almost the same and the effect of rotation is significant for certain values of Ra_T. Further, in all the cases, α_c increases with Ra_T continuously.

III. The variation of critical bio-Rayleigh number with the thermal Rayleigh number (Rac vs. Ra_T) for t = $\Im$ = 5, 10, 15 and Pe = 0.1 are presented through graphs in figures 7 to 9. In all the cases, the curves exhibit the same peculiar behaviour. The interesting observations are (a) only for t $= \Im = 15$ the curve exhibits a maximum for $Ra_T = 1.5 \times 10^4$ and local minima for certain values. But, for t $= \Im =$ 10 and 5, the curves show exactly an opposite behaviour. This clearly indicates the sensitivity of BPC system for rotation. A very interesting and remarkable feature of the present study is that the cumulative effect of thermal stratification and rotation on bioconvection in suspensions of gravitactic microorganism is highly of stochastic type and challenges for an in-depth research which might explore certain hidden phenomenological aspects. This markedly different behaviour of rotating and non-rotating bioconvective systems with the suspension of gravitactic microorganisms is quite interesting from the experimental as well as theoretical view points. No work in this direction is available. The results of the present investigation are in excellent agreement with the earlier works in the limiting cases as discussed earlier.

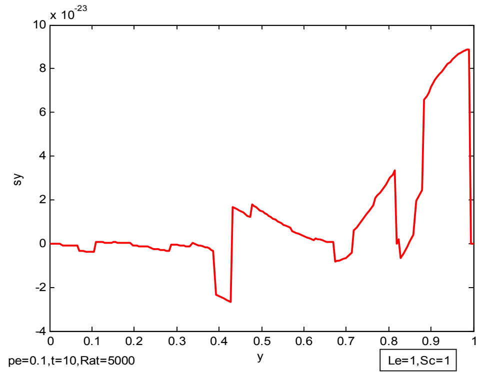

Figure 1: ψ vs. Y

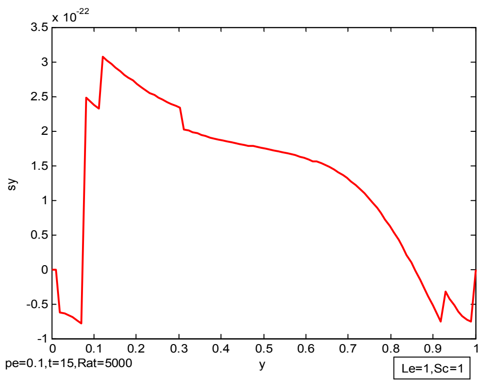

Figure 2: ψ vs. Y

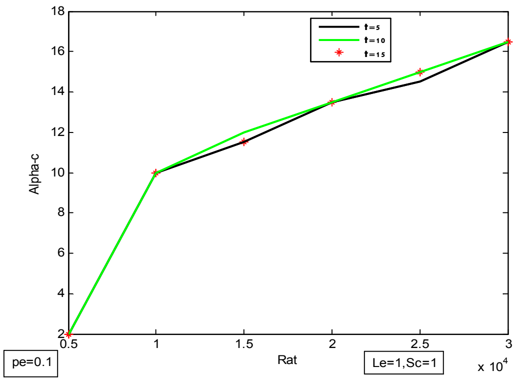

Figure 3 :ψvs.Y

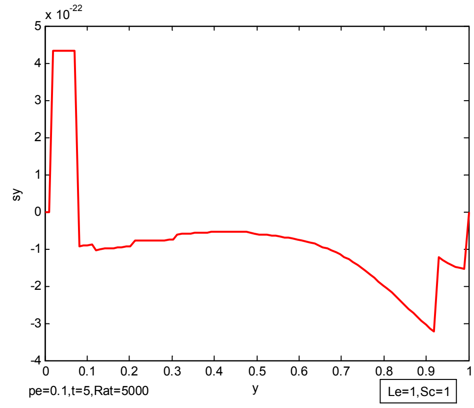

Figure 4 :α_c vs. Ra_T

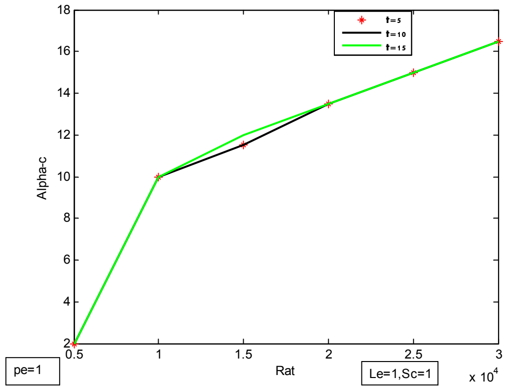

Figure 5 :α_c vs. Ra_T

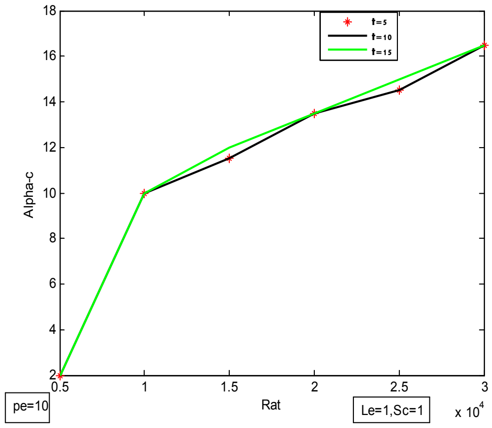

Figure 6 :α_c vs. Ra_T

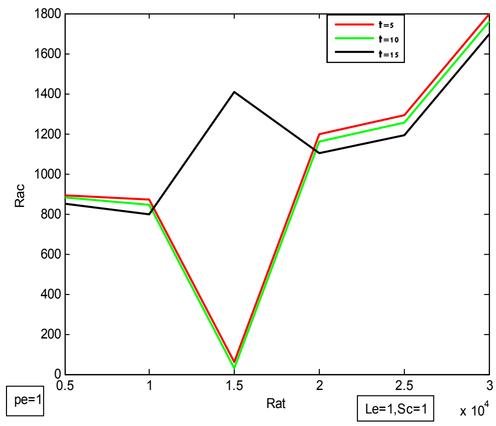

Figure 7 : Ra_C vs. Ra_T

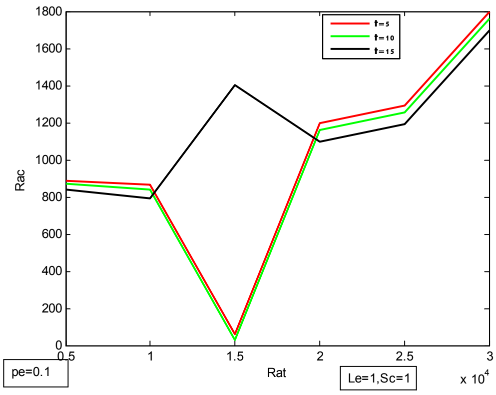

Figure 8 : Ra_C vs. Ra_T

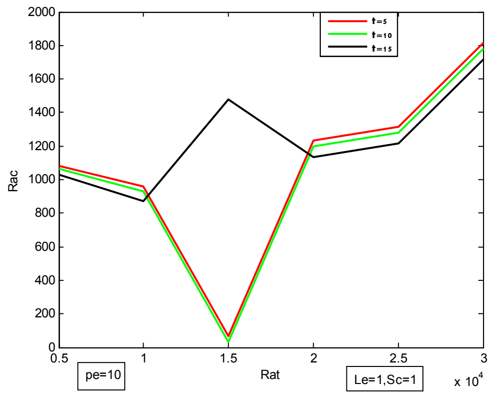

Figure 9 : Ra_C vs. Ra_T

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