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# **RESEARCH ARTICLE**

# **STRONG LINE DOMINATION IN GRAPHS STRONG**

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### **ARTICLE INFO ABSTRACT ARTICLE INFO**

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For any graph  $G = (V, E)$ , the Line graph  $L(G)$  of a graph G is a graph whose set of vertices is the union of the set of edges of  $\vec{G}$  in which two vertices are adjacent if and only if the corresponding edges For any graph  $G = (V, E)$ , the Line graph  $L(G)$  of a graph G is a graph whose set of vertices is the union of the set of edges of G in which two vertices are adjacent if and only if the corresponding edges of G are adjacent in  $\langle V[L(G)] - D \rangle$  is strongly dominated by at least one vertex in D. Strong Line domination number  $\gamma_{SL}(G)$  of G is the minimum cardinality of strong Line dominating set of G. In this paper, we study  $\gamma_{SL}(G)$  of G is the minimum cardinality of strong Line dominating set of G. In this paper, we study graph theoretic properties of  $\gamma_{SL}(G)$  and many bounds were obtain in terms of elements of G and its relationship with other domination parameters were found.

**Subject Classification number:** AMS - 05C69, 05C70. **Subject** 

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# **INTRODUCTION INTRODUCTION**

In this paper, all the graphs consider here are simple and finite. In this paper, all the graphs consider here are simple and finite.<br>For any undefined terms or notation can be found in Harary (Harary, 1972). In general, we use  $\langle x \rangle$  to denote the subgraph induced by the set of vertices  $\Lambda$  and  $N(\nu)$  and denote open (closed) neighborhoods of a vertex  $\vec{v}$ . Let  $deg(\vec{v})$ is the degree of vertex **v** and as usual  $\mathcal{Q}(\mathcal{Q})(\mathcal{Q}(\mathcal{Q}))$  is the minimum (maximum) degree. A vertex of degree one is called an end vertex and its neighbor is called a support vertex. The degree of an edge  $e = uv$  of G is defined by  $deg(e) = deg(u) + deg(v)$  and  $\delta'(G)(\Delta'(G))$  is the minimum an end vertex and its neighbor is called a support vertex. The vertex degree of an edge  $e = uv$  of  $G$  is defined by adjace deg $(e) = deg(u) + deg(v)$  and  $\delta'(G)(\Delta'(G))$  is the minimum  $v_t(G)$  (maximum) degree among the edges of  $G$ . The is the minimum number of vertices (edges) in vertex (edge) cover of G. The notation  $\beta_0(G)(\beta_1(G))$  is the maximum cardinality of a vertex (edge) independent set in  $\mathcal{G}$ . A set  $S \subset V(G)$  is said to be a dominating set of G, if every vertex (edge) cover of **G**. The notation  $\beta_0(G)(\beta_1(G))$  is the commation maximum cardinality of a vertex (edge) independent set in **G**. a connecte A set  $S \subseteq V(G)$  is said to be a dominating set of **G**, if every restrained cardinality of vertices in such a set is called the domination number of  $\bullet$  and is denoted by  $V(G)$ . The concept of edge dominating sets were also studied by Mitchell and Hedetniemi in (Mitchell and Hedetniemi, 1977). (Harary, 1972). In general, we use  $\langle X \rangle$  to denote the subgraph induced by the set of vertices  $\hat{X}$  and  $N(v)$  and  $N(v)$  there denote open (closed) neighborhoods of a vertex  $\hat{v}$ . Let  $deg(v)$  comminimum is the degree cardinality of vertices in such a set is called<br>number of  $\boldsymbol{G}$  and is denoted by  $\boldsymbol{\gamma(G)}$ . The<br>dominating sets were also studied by Mitchell<br>in (Mitchell and Hedetniemi, 1977). Il the graphs consider here are simple and finite.<br>
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cover the called an edge on F. Equiviliant Equivalent in the set of vertices  $X$  and  $N(v)$  and  $N(v)$  there exists an edge

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An edge dominating set of  $\overline{G}$  if every edge in  $\overline{E} - F$  is adjacent to at least one edge in  $\overline{F}$ . Equivalently, a set  $\overline{F}$  edges in  $\overline{G}$  is called an edge dominating set of  $\mathbf{G}$  if for every edge  $e \in \mathbf{E} - \mathbf{F}$ , there exists an edge  $e_1 \in F$  such that  $\theta$  and  $\theta_1$  have a vertex in common. The edge domination number  $\gamma(G)$  of graph  $\tilde{G}$  is the minimum cardinality of an edge dominating set of  $\overline{G}$ . A dominating set  $\mathcal{S}$  is called the total dominating set, if for every vertex  $v \in V$ , there exists a vertex  $u \in S$ ,  $u \neq v$  such that  $u$  is adiacent to  $\overline{v}$ . The total domination number of  $\overline{G}$  is denoted by  $r_t(G)$  is the minimum cardinality of total dominating set of G. A dominating set  $S \subseteq V(G)$  is a connected dominating set, if the induced subgraph  $\leq s$  > has no isolated vertices. The connected domination number  $\gamma_{\epsilon}(G)$  of G is the minimum cardinality of a connected dominating set of  $\mathcal{G}$ . A dominating set  $\mathcal{S} \subseteq V(G)$  is restrained dominating set of  $\overline{G}$ , if every vertex not in  $\overline{S}$  is adjacent to a vertex in  $S$  and to a vertex in  $V(G) - S$ . The restrained domination number of a graph  $\overline{G}$  is denoted by  $\gamma_r(G)$  is the minimum cardinality of a restrained dominating set in  $\overline{G}$ . The concept of restrained domination in graphs was introduced by Domke *et al.* (1999). A dominating set  $\overline{D}$  of a graph  $G = (V, E)$  is an independent dominating set if the induced subgraph  $\leq D$  > has no edges. THE TRIME IS the same of disposition and the same of the same of

The independent domination number  $\mathfrak{t}(G)$  of a graph  $G$  is the minimum cardinality of an independent dominating set (Haynes *et al*., 1997; Robert B.Allan and Renu Laskar, 1978). The concept of a dominating set  $\overline{D}$  of a graph  $\overline{G}$  is a strong split dominating set if the induced subgraph  $(V - D)$  is totally disconnected with at least two vertices. The strong split domination number  $v_{ss}(G)$  of graph G is the minimum cardinality of a strong split dominating set of  $\overline{G}$ . A dominating set  $\overline{D}$  of a graph  $\overline{G}$  is a global dominating set if  $\overline{D}$  is also a dominating set of  $\overline{G}$ . The global domination number  $v_g(G)$  in the minimum cardinality of a global dominating set of  $\epsilon$ . This concept was introduced independently by Brigham and Dutton (Brigham and Dutton, 1990; Sampathkumar, 1989). The concept of Roman domination function (RDF) on a line graph  $L(G) = (V, E)$  is a function  $f: V' \rightarrow \{0, 1, 2\}$  satisfying the condition that every vertex  $\boldsymbol{u}$  for which  $f(u) = 0$  is adjacent to at least one vertex of  $\mathbf{v}$  for which  $f(\mathbf{v}) = 2$  in  $L(G)$ . The weight of a Roman dominating function is the value  $f(V) = \sum_{u \in V} f(u)$ . The minimum weight of a Roman dominating function on a line graph  $L(G)$  is called the Roman domination number of a graph  $L(G)$  and is denoted by  $\gamma_R(L(G))$  (see (9)). The concept of domination in graphs with its many were found in graph theory (Haynes *et al*., 1998; Haynes *et al*., 1999; Kulli *et al*., 1999; Panfarosh *et al.*, 2014). Analogously, a dominating set  $\overline{D}$  of a line  $L(G)$  is a cototal dominating set if the induced subgraph with  $\langle \nu(L(G)) - D \rangle$  has no isolated vertices. The cototal domination number  $v_{\text{ct}}(L(G))$  is the minimum cardinality of a cototal  $N(L) = V(L)$ dominating set of  $L(G)$ (Panfarosh *et al.*, 2014). The concept of Strong domination was introduced by Sampathkumar and Pushpa Latha in (1996) and well studied in (Muddebihal and Nawazoddin U. Patel, 2014; Muddebihal and Nawazoddin U. Patel, 2015; Muddebihal *et al*., 2015). Given two adjacent vertices  $\mu$  and  $\nu$  we say that  $\mu$  strongly dominates  $\nu$  if deg (u)  $\ge$  deg (v). A set  $D\subseteq V(G)$  is strong dominating set of G if very vertex in  $V - D$  is strongly dominated by at least one vertex in  $\overline{D}$ . The strong domination number  $\gamma_s(G)$  is the minimum cardinality of a strong dominating set of  $\mathbf{C}$ . A dominating set **D** of a graph  $L(G)$  is a strong Line dominating set if every vertex in  $\langle V[L(G)] - D \rangle$  is strongly dominated by at least one vertex in  $\overline{D}$ . Strong Line domination number  $v_{\text{SL}}(G)$  of  $\overline{G}$  is the minimum cardinality of strong Line dominating set of  $\mathcal G$ . In this paper, many bounds on  $v_{st}(G)$  were obtained in terms of elements of  $\mathbf{G}$  but not the elements of  $L(\mathbf{G})$ . Also its relation with other domination parameters were established. We need with deg  $(u_i) \ge 3.1 \le i \le k$ the following theorem for our further results.

**Theorem A(4):** for any  $(p, q)$  graph  $G$ ,  $r(G) = \frac{p}{2}$ .

## **Main results**

**Theorem 1:** For any non trivial  $(p,q)$  tree with  $p \ge 3$  and m end vertices, then  $y_{5L}(G) \leq m$ . Equality holds if  $T = P_n, 4 \le n \le 7$ 

**Proof:** Let  $A = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(T)$  be the set of all end vertices in 7 with  $|A| = m$ . Suppose  $D \subset V - A$  be the set of all non end vertices then each block incident with the vertices of  $\overline{D}$  gives a complete subgraph in  $L(T)$ .

If  $\deg(u) \geq 2$ ,  $u \in V(L(T))$ , then  $D' = \{u_1, u_2, u_3, \dots, u_m\} \subseteq V[L(T)]$ such that  $\deg(u_m) \geq \deg(u_k)$   $\forall u_k \in V(L(T)) - D'$  and  $\forall u_m \in D$ Suppose  $D'' = \{u_1, u_2, u_3, ..., u_i\}$ ,  $1 \le i \le m$  with  $D' \subset$  $V[L(T)] - D$  and  $\deg(u_i) = \deg(u_i)$   $\forall u_i \in V[L(T)] - D$  Then  $[D' \cup D'']$  forms a minimal Strong dominating set of  $L(T)$ . Therefore,  $|D' \cup D''| \le m$  which gives  $\gamma_{SL}(G) \le m$ . For equality if  $P_n$ ,  $4 \le n \le 7$  holds, then for each  $P_4$ ,  $P_5$ ,  $P_6$  and  $P_7$  have  $m = 2$  Since by Theorem A,  $v_{5L}(P_n) = 2 = m + 4 \le n \le 7$ Then deg  $(u_i) \ge \deg(u_k)$   $\forall u_i \in D'$  and  $\forall u_k \in V(L(G)) - D'$ Hence the equality.

**Theorem 2:** For any connected  $(p,q)$  graph  $q$ ,  $\gamma_{SL}(G) + \gamma(G) \leq P - 1$ 

**Proof:** Let  $R = \{v_1, v_2, v_3, \dots, v_m\} \subseteq V(G)$  be the set of vertices with  $\deg(v_i) \geq 2, \forall v_i \in R, 1 \leq j \leq m$  Further let there exists a set  $R_1 \subset R$  of vertices with  $diam(u, v) \geq 3$ ,  $\forall u, v \in R_1$ which covers all the vertices in  $\overline{G}$ . Clearly  $\overline{R}_1$  forms a dominating set of  $\overline{G}$ . Otherwise if  $\frac{diam(u,v)}{ }$  < 3 then there exists at least one vertex  $x \notin R_1$  such that  $R' = R_1 \cup \{x\}$  form a minimal  $\gamma$  - set of G. Now by definition of  $L(G)$ , let  $H = \{u_1, u_2, u_3, \dots, u_n\} \subseteq V[L(G)]$  be the set of vertices such that  $\{u_i\} = \{e_i\} \in E(G)$ ,  $1 \le i \le n$  where  $\{e_i\}$  are incident with the vertices of  $\overline{R}$ . Further let  $D \subseteq H$  be the set of vertices  $deg(w) \ge 3$  for every  $w \in D$  such that  $N[D] = V(L(G))$  and if  $\forall v_i \in V[L(G)] - D$  Then  $\{D'\} \cup \{v_i\}$ Strong line dominating set. Clearly  $|\{D\} \cup \{v_i\}| \cup |R'| = |V(G)| - 1$  and hence  $\gamma_{SL}(G) + \gamma(G) \le P - 1$ 

**Theorem 3:** For any connected  $(p, q)$  graph  $G$ ,  $\gamma_{SL}(G) \leq p - \gamma_t(G)$ 

**Proof:** Let  $H = \{v_1, v_2, v_3, \dots, v_m\}$  be the minimum set of vertices which covers all the vertices in  $\epsilon$ . Suppose  $deg(v_j) \ge 1, \forall v_j \in H_1, 1 \le j \le m$  in the subgraph  $\le H_1 >$  then  $H_1$  forms a  $\gamma_t$  (G) – set of G. Otherwise if deg (v<sub>j</sub>) < 1 then attach the vertices  $w_i \in N(v_i)$  to make  $\deg \ge 1$  such that  $\langle H_1 \cup \{w_i\} \rangle$  does not contains any isolated vertex. Clearly  $H_1 \cup \{w_i\}$  forms a minimal total dominating set of  $\,$ . Now in (G), let  $A \subset V(L(G))$  be the set of vertices corresponding to the edges which are incident to the vertices of  $\overrightarrow{H}$  in  $\overrightarrow{G}$ . Let there exists a subset  $D = \{u_1, u_2, u_3, \dots, u_k\}$  of vertices and  $N[u_i] = V(L(G))$  Further  $|\deg(u) - \deg(w)| \leq 2 \forall u \in D$  and  $w \in V[L(G)] - D$  has at least one vertex in  $\overline{D}$ . Clearly  $\overline{D}$  forms a minimal Strong dominating set in  $L(G)$ . Therefore it follows that  $|D| \leq |V(G)| - |H \cup \{w_i\}|$  and hence  $\gamma_{SL}(G) \leq P - \gamma_t(G)$ For any connected  $(p,q)$  graph  $G$ ,<br>  $g(x) = \{v_1, v_2, v_3, \dots, v_m\}$  be the minimum set<br>
thich covers all the vertices in  $G$ . Suppose<br>  $v_i \in H_1 \cup 1 \leq j \leq m$  in the subgraph  $\in H_1 \geq h$  then<br>  $g(x) = s e t$  of  $G$  obterwise if  $\deg(g$ *A* 1 be the minimum set<br>
vertices in  $G$ . Suppose<br>  $P$  subgraph  $\leq H_1 \geq$  then<br>
rwise if deg  $(v_j) \leq 1$  then<br>
rake deg  $\geq 1$  such that<br>
isolated vertex. Clearly<br>
nating set of  $G$ . Now in<br>
vertices corresponding to<br> *Y<sub>H</sub>* (*G*)  $\leq p - \gamma_r$  (*G*).<br> **Proof:** Let  $H = \{v_1, v_2, v_2, \dots, \dots, v_m\}$  be the minimum set<br>
of vertices which covers all the vertices in  $G$ . Suppose<br>  $\deg(v_i) \geq 1, \forall v_i \in H_1, 1 \leq j \leq m$  in the subgraph  $\lt H_1 >$  then<br> *H<sub>*</sub> **Proof:** Let  $H = \{v_1, v_2, v_3, \dots, v_m\}$  be the minimum set<br>
of vertices which covers all the vertices in  $\overline{G}$ . Suppose<br>  $\text{deg}(v_j) \geq 1, \forall v_j \in H_1, 1 \leq j \leq m$  in the subgraph  $\lt H_1 \geq$  then<br>  $H_1$  forms a  $\gamma_r(G) - s\epsilon t$  o

**Theorem 4:** For any connected  $(p,q)$  graph  $G$ ,  $\gamma_{SL}(G) + \gamma_c(G) + 2 \leq \alpha_o(G) + \beta_o(G) + \gamma(G)$ 

**Proof:** Let  $A = \{v_1, v_2, v_3, ..., ..., v_m\} \subseteq V(G)$  be the set of vertices with  $\deg(v_j) \geq 2, \forall v_j \in A, 1 \leq j \leq m$  which are at distance at least two covers all the edges in  $\overline{G}$ . Clearly  $|A| = \alpha_0(G)$ . Further if for any vertex  $x \in A$ ,  $N(x) \in V(G)-A$ . Then A itself is an independent vertex set. Otherwise  $A_1 \cup A_2$  where  $A_1 \subseteq A$  and  $A_2 \subseteq V(G)-A$  forms a maximum independent set of G with

Now let  $S = A \cup A$  where  $A \subseteq A$  and  $A \subseteq V(G) - A$  be the minimal S set of vertices which covers all the vertices in  $G$ . Clearly S forms a minimal  $X$  - set of G. Suppose the subgraph  $\langle S \rangle$  has  $A = \{v_1, v_2, ..., v_i\} \subseteq V(L(T))$  be the set of vertic only one component. Then  $S$  itself is a connected dominating  $S$  itself set of G. Otherwise if the subgraph  $\langle S \rangle$  has more than one  $\frac{G}{\langle \text{Jac}(x), \text{Jac}(y) \rangle}$ component, then attach the minimum number of vertices  $\{w_j\} \in V(G) - S$  where  $\deg(w_j) \geq 2$ , which are between the vertices of S such that  $S_1 = S \cup \{w_j\}$  forms exactly one component in  $S_2$ the subgraph  $\langle S_1 \rangle$ . Clearly  $S_1$  forms a minimal  $X_{\epsilon}$ - set of G. Let  $D = {u_1, u_2, u_3, ..., u_k} \subseteq C$  where C is the set of vertices corresponding to the edges which are incident with the vertices of S in G. The minimal set of vertices with  $N[D] = V(L(G))$  **Theorem** 7: A and  $\forall u_k \in D$  has degree greater or equal to those vertices  $u_j \in V(L(G)) - D$ . Clearly *D* forms a Strong line dominating set in  $L(G)$  Therefore  $|D|\cup|S_1|+2\leq |A|\cup|A_1\cup A_2|\cup|S|$  and hence  $X_{st}(G) + X_{s}(G) + 2 \leq \Gamma_0(G) + S_0(G) + X(G)$ International Journal of Current Research, Vol. 08, Issue, 10, pp.39782-39787, October, 2016<br>  $S = A \cup A'$  where  $A \subseteq A$  and  $A' \subseteq V(G) - A$  be the minimal<br>  $S = A \cup A'$  where  $A' \subseteq A$  and  $A' \subseteq V(G) - A$  be the minimal<br>  $S = A \cup A'$  where **19784**<br> *Mermational Journal of Current Research, Vol. 08, Issue, 10, pp.39782-39787, October, 2016***<br>
<b>Now let**  $S = A \cup A$  where  $A \subseteq A$  and  $A \subseteq V(G) - A$ be the minimal Suppose the set  $V_2$  dominates  $V_6$ . Then  $S$ <br>
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where*  $A \subseteq A$  *and*  $A' \subseteq V(G) - A$ *be the minimal<br>
Suppose the set*  $V_2$  *dominates*  $V_1$ *. Then*  $S = V_1 \cup V_2$  *form<br>
incide covers all t* **D**<br> **Durational Journal of Current Research, Vol. 08, Issue, 10, pp.39782-39787, October, 2016<br>
Now let**  $S = \lambda \cup A$  **where**  $\Lambda \subset \Lambda$  **and**  $A' = V(G) - A$  **be the minimal express in**  $G$ **. Clearly**  $S$ **<br>
For the second all the vertices Example 10 International Journal of Current Research, Vol. 08, Issue, 10, pp.39782-39787, October, 2016<br>
V let**  $S = A \cup A$  **where**  $A \subset A$  **and**  $A' \subseteq V(G) - A$ **be the minimal Suppose the set**  $V_2$  **dominates**  $V_1$ **. Then<br>
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Now let  $S = A \vee A$  where  $A \subseteq A$  and  $A' \subseteq V(G) - A$  be the minimal Suppose the set  $V$ , dominates  $V$ . Then S<br>
but intern **EXERCISE THE CONDUCT THE CONDUCTER CONSULTER (EXECUT) THE CONDUCTER (EXECUT) AND CONDUCTER (EXECUT) AND CONDUCTER (EXECUT) AND CONDUCTER (EXECUT) AND CONDUCTER (EXECUT) THE CONDUCTER (EXECUT) AND CONDUCTER (EXECUT) AND C EXAMPLE AT A SURFADE AT A SURFADE AT A SURFADE IN THE CALCULATION (S) FOR A CONSIDENT (S)**  $\mathcal{E} = \{c, \mathcal{E} \}$  **and**  $\mathcal{E} = \{c, \mathcal{E} \}$  **and SPARE THE CONDUCT ACT CALCE ANTERNATION** (SEE ALT FOR A SURVEY) **FOR A CONDUCT ANTERNATION** (SEE ALT FOR CONDUCT) **FOR A CONDUCT CONDUCT CONDUCT CONDUCT CONDUCT CONDUCT CONDUCT CONDUCT CONDUCT CONDUCT** *Letermational Journal is Current Research, Vol. 06, Leave, 10, <i>np.37932-39787, October, 2016*<br>  $\sqrt{A}$  where  $A' \in V(G) - A_{\text{DS}}$  the number of  $G$ , Clearly  $S$  minimal Sompose the set  $V$  dominates  $V$ . Then  $\bar{B} = V_e / V_e$ *Marmational Journal Bcearch, Vol. 68, Izme, 10, npm3 (2017)237-2078). Codobe, 2016<br>*  $\lambda$ *-4 where*  $\lambda$ *-2 and*  $\lambda$ *<sup>-2</sup>*  $\mathcal{C}(\mathcal{V})$ *-4 be the minimal Suppose the set*  $V_i$  *dominates*  $V_i$ *. Then*  $S = V_i \cup V_i$  *forms and*  $N_i$ *-set* Now let  $5 = A \cup A$  where  $4 \subset 4$  and  $A \subset V(G) = 3$  **cominalisation**  $\mathbf{R} = \mathbf{R} \times \mathbf{R}$  *i* (a)  $\mathbf{R} = \mathbf{R} \times \mathbf{R} \times \mathbf{R}$  (a)  $\mathbf{R} = \mathbf{R} \times \mathbf{R} \times \mathbf{R}$  (a)  $\mathbf{R} = \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf$ 

**Theorem 5:** For any connected  $(p,q)$ , graph  $G$ , .  $x_{_{SL}}(G) + x_{_r}(G) \le r_{_1}(G) + s_{_1}(G) + u'(G)$ 

**Proof:** Let  $A = \{e_1, e_2, ..., e_n\} \subseteq E(G)$  be the maximal set of edges with  $N(e_i) \cap N(e_j) = e$  for every  $e_i, e_j \in A$ ,  $1 \le i \le n$ .  $1 \le j \le n$  and  $e \in E(G) - A$ . Clearly A forms a maximal independent edge set in G. Suppose  $B = \{v_1, v_2, ..., v_n\}$  be the set of vertices which are incident with the edges of  $\overline{A}$  and if  $|B|=P$ . Then A itself is an edge covering number. Otherwise consider the minimum number of edges  ${e_n} \subseteq E(G) - A$  such that  $A_1 = A \cup {e_m}$  forms a minimal edge covering set of G. Let  $C = \{v_1, v_2, ..., v_k\} \subseteq V(G)$  be the set of all end vertices. Then  $S = C \cup C'$  where  $C \subseteq V(G)-C$  be the set of Si vertices covering all the vertices with  $diam(u, v) \ge 3 \quad \forall u \in C$ ,  $v \in C$  or for every vertex  $w \in V$   $(G)-S$  there exists at least one vertex  $z \in V(G) - S$  where  $z \cap w = \emptyset$  and  $y \in S$ . Clearly S forms a minimal  $x_i$ - set of G. Suppose  $C = \emptyset$ . Then S itself D is a minimal Strong line dominat forms a minimal  $X_r$  –set of G. Let  $D = \{u_1, u_2, ..., u_k\} \subseteq V(L(G))$ be the minimum set of vertices with  $N[u_j] = V(L(G))$  for every  $u_j \in D$ ,  $1 \le j \le k$ . If  $\forall v_i \in V(L(G))$  has degree at most 2 and  $v_i \in V[L(G)] - D$  then  $\{D\} \cup \{v_i\}$  forms a strong line dominating set. Hence  $|{D} \cup {v_i}| = \gamma_{s}$  (G) Since for any graph G there exists at least one edge *e* with  $\left|\frac{\text{deg}(e)}{e}\right| = u(G)$ . Thus which are *B* component, then attach the minimum number of vertices in  $\int_{\mathbb{R}}^{\infty} B(s|x) - ds| \leq \int_{\mathbb{R}}^{\infty} \int_{\mathbb{R}}^{\infty} \int_{\mathbb{R}}^{\infty} \int_{\mathbb{R}}^{\infty} \int_{\mathbb{R}}^{\infty} \int_{\mathbb{R}}^{\infty} \int_{\mathbb{R}}^{\infty} \int_{\mathbb{R}}^{\infty} \int_{\mathbb{R}}^{\infty} \int_{\mathbb{R}}$  *v C w V G S y*<sub>1</sub><sup>2</sup> C (*x*) is a line store of vertices  $P^{(1)}(x|z) = \cos(2\pi i)$ . Clearly,  $\pi_E(T) \leq \pi_g(T) - \Delta(T) + 1$ <br> *X* So the store of vertices with  $N[P] = V(E(G))$  **Theorem 7:** A Shong line dominating set the metric of vertices with  $N[P]$ *L*(0) Therefore Dollars ( $P_0$ +1  $\mu$ ,  $Q_1$  is the coincident of  $\mathbf{R}_0$  **l**  $\mathbf{R}_1$  **u**  $\mathbf{R}_2$  **l**  $\mathbf{R}_3$  **u**  $\mathbf{R}_4$  **l**  $\mathbf{R}_5$  **u**  $\mathbf{R}_6$  **l**  $\mathbf{R}_7$  **l**  $\mathbf{R}_8$  **l**  $\mathbf{R}_9$  **l**  $\mathbf{R}_1$  **l** F<sub>1</sub>(*G*) + x<sub>1</sub>(*G*) + x<sub>1</sub>(*G*) + x<sub>1</sub>(*G*) + x<sub>1</sub>(*G*) + x<sub>1</sub>(*G*) + x<sub>1</sub>(*G*) + *x*<sub>(*G*)+ x<sub>1</sub>(*G*) + *x*<sub>(*G*)+ x<sub>1</sub>(*G*) + *x*<sub>(*B*)+ *x*<sup>(*n*</sup>)</sub> + *i* (*G*) + *x*<sub>(*D*)+ *N*<sub>(*C*) + *n*<sup>(*n*</sup>)+ *i* (*G*)+ *n*(*N*(</sub></sub></sub></sub> *A* = { $e_1,e_2,...,e_n$ } =  $E(G)$  be the maximal set of  $\begin{cases} \frac{1}{2}E(G) \\ 0.01 \text{ N} \end{cases}$   $\begin{cases} \frac{1}{2}E(G) \\ 0.01 \text{ N} \end{cases}$  consider the main and number of equipment of equipment of equipment of equipment  $f(x) = (x_1 - y_1)$ . From the meteors  $x_1 - y_2 = (x_1 - y_1)$ ,  $y_1 - y_2 = (x_1 - y_2)$ ,  $y_2 - y_1 = (x_1 - y_1)$ ,  $y_2 - y_2 = (x_1 - y_2)$ ,  $y_1 - y_2 = (x_1 - y_1)$ , *V* Surface in  $S = C \cup C$  where  $\overline{x} = V$  (*V*)  $\overline{Y} = \frac{V}{V}$  (*C*)  $\overline{Y} = \frac{V}{V}$  (*C*)  $\overline{Y} = \frac{V}{$ 

There fore  $x_{sL}(G) + x_r(G) \leq L_1(G) + S_1(G) + \omega(G)$ . The following theorem relates the Strong line domination number and Roman domination number of 7.

**Theorem 6:** For any non trivial tree  $\overline{T}$  with  $P \ge 3$ , then  $\gamma_{SL}(T) \leq \gamma_R(T) - \Delta(T) + 1$ 

**Proof:** Let  $f: V(T) \to \{0,1,2\}$  and partition the vertex set  $V(T)$ into  $(V_0, V_1, V_2)$  induced by  $\overline{f}$  with  $|V_i| = n_i$  for  $i = 0, 1, 2$ .

Suppose the set  $V_2$  dominates  $V_0$ . Then  $S = V_1 \cup V_2$  forms a minimal Roman dominating set of  $T$ . Further let  $A = \{v_1, v_2, ..., v_i\} \subseteq V(L(T))$  be the set of vertices with  $\deg(v_j) \geq 3$ . Suppose there exists a vertex set  $D \subseteq A$  with  $N[D] = V(L(T))$  and if  $|\deg(x) - \deg(y)| \leq 2$ ,  $\forall x \in D$ ,  $0, y \in V(L(T)) - D$ . Then p forms a Strong line dominating set in  $L(T)$ . Otherwise there exists at least one vertex  $\{w\} \subseteq A$  where  $\{w\} \notin D$  such that  $D \cup \{w\}$  forms a minimal  $x_{st}$  - set in  $L(T)$ . Since for any tree G there exists at  $\int_{S_1}^{S_2}$ . Clearly  $S_1$  forms a minimal  $X_{c-}$  set of G. Let least one vertex  $V \in V(T)$  of maximum degree  $\Delta(T)$ , then  $|D \cup \{w\}| \leq |S| - |\text{deg}(v)| + 1$ , Clearly,  $\gamma_{SL}(T) \leq \gamma_R(T) - \Delta(T) + 1$ *A*  $\subseteq V(G) - A$  be the minimal suppose the set  $V_2$  dominates  $V_6$ . Then  $S = V_1 \cup V_2$  forms a vertices in  $G$ . Clearly  $S$  minimal Roman dominating set of  $T$ . Further let pose the subgraph  $\langle S \rangle$  has  $A = \{v_1, v_2, ..., v_i\} \$ *Current Research, Vol. 08, Issue, 10, pp.39782-39787, October, 2016<br>
<sup>1</sup> Le the minimal Suppose the set*  $V_2$  *dominates*  $V_1$ *. Then*  $S = V_1 \cup V_2$  *forms a<br>*  $G$ *. Clearly*  $S$  *minimal Roman dominating set of*  $T$ *. Further let<br> p q*, *G* 2-39787, October, 2016<br> *V*<sub>2</sub> dominates  $v_0$ . Then  $S = V_1 \cup V_2$  forms a<br> *L*(*T*)) be the set of vertices with  $\frac{\deg(v_j) \ge 3}{\deg(v_j) \ge 3}$ .<br> *S*ts a vertex set  $D \subseteq A$  with  $N[D] = V(L(T))$  and<br>  $\le 2$ ,  $\forall x \in D$ ,  $0 \quad y \in V(L(T)) - D$ . *A*, *Issue, 10, pp.39782-39787, October, 2016*<br>
Suppose the set  $V_2$  dominates  $V_6$ . Then  $S = V_1 \cup V_2$  forms a<br>
minimal Roman dominating set of *T*. Further let<br>  $A = \{v_1, v_2, ..., v_i\} \subseteq V(L(T))$  be the set of vertices with  $\$ 16<br> *V<sub>0</sub>* . Then  $S = V_1 \cup V_2$  forms a<br>
set of *T*. Further let<br>
of vertices with  $\deg(v_j) \ge 3$ .<br>  $D \subseteq A$  with  $N[D] = V(L(T))$  and<br>  $\in V(L(T)) - D$ . Then *p* forms<br> *T*). Otherwise there exists at<br>  $\in D$  such that  $D \cup \{w\}$  forms a<br> Issue, 10, pp.39782-39787, October, 2016<br>
ppose the set  $V_2$  dominates  $v_6$ . Then  $S = V_1 \cup V_2$  forms a<br>
inimal Roman dominating set of T. Further let<br>  $\{v_1, v_2, ..., v_i\} \subseteq V(L(T))$  be the set of vertices with  $\deg(v_j) \ge 3$ .<br>
pp **2016**<br> *V<sub>0</sub>* . Then  $S = V_1 \cup V_2$  forms a<br>
set of *T*. Further let<br>
of vertices with  $\deg(v_j) \ge 3$ .<br>  $D \subseteq A$  with  $N[D] = V(L(T))$  and<br>  $y \in V(L(T)) - D$ . Then *D* forms<br>  $L(T)$ . Otherwise there exists at<br>  $L(f) \notin D$  such that  $D \cup \{w\}$  *w*<sub>2</sub> dominates  $v_0$ . Then  $S = V_1 \cup V_2$  forms a<br> *w*<sub>2</sub> dominating set of *T*. Further let  $(L(T))$  be the set of vertices with  $\deg(v_j) \ge 3$ .<br>
ists a vertex set  $D \subseteq A$  with  $N[D] = V(L(T))$  and  $\le 2$ ,  $\forall x \in D$ ,  $0$   $y \in V(L(T)) - D$ . *SLAPA SPACE 1 S PLAPAPE 1 <b>C C <i>CLAPE 1 <b>C CLAP CLAP* 39787, October, 2016<br>
<sup>7</sup><sub>2</sub> dominates  $v_6$ . Then  $S = V_1 \cup V_2$  forms a<br>
dominating set of *T*. Further let<br>  $(T)$  be the set of vertices with  $\deg(v_j) \ge 3$ .<br>
ts a vertex set  $D \subseteq A$  with  $N[D] = V(L(T))$  and<br>  $2, \forall x \in D, 0, y \in V(L(T)) -$ *B*, *Issue, 10, pp.39782-39787, October, 2016*<br>
Suppose the set  $V_z$  dominates  $V_s$ . Then  $S = V_1 \cup V_z$  forms a<br>
inimial Roman dominating set of *T*. Further let<br>  $A = \{v_1, v_2, ..., v_r\} \subseteq V(L(T))$  be the set of vertices with  $\frac{\deg$ *Z*<sub>1</sub>  $\cup$  *V*<sub>2</sub> forms a<br>
Further let<br>  $\deg(v_j) \ge 3$ <br>  $D] = V(L(T))$  and<br>
Then *D* forms<br>
there exists at<br>  $\lim_{y \to 0} \frac{\deg(v_j)}{\deg(v_j)}$  forms a<br>
there exists at<br>  $\deg(u_j)$ <br>  $\deg(v_j)$ , then<br>  $\deg(v_j) = \deg(v_j)$ <br>  $\deg(v_j)$ , one of the<br>  $\deg(v_j) \ge$ *x* =  $V_1 \cup V_2$  forms a<br> *x* with deg( $v_j$ ) ≥ 3<br> *x* with deg( $v_j$ ) ≥ 3<br> *x*  $[D] = v(L(T))$  and<br> *n*  $D \cup \{w\}$  forms wise there exists at<br> *x*  $D \cup \{w\}$  forms a<br> *x* e  $\frac{\Delta(T)}{T}$ , then<br> *x* = *D*, one of the<br> *x* = *D*, o *ber, 2016*<br> *y* Exerces V. Then  $S = V_1 \cup V_2$  forms a<br>
ing set of T. Further let<br>  $\csc{g}$  and  $\sin{g}$   $\sin{g}$   $\sin{g}$   $\sin{g}$   $\sin{g}$ <br>  $\csc{g}$  and  $\sin{g}$   $\sin{g}$ <br>  $\sin{g}$   $\sin{g}$   $\sin{g}$ <br>  $\sin{g}$   $\cos{g}$   $\sin{h}$ <br>  $\sin{g$ *I0, pp.39782-39787, October, 2016*<br>
2 *If*  $p$ , *P.39782-39787, October, 2016*<br>
2 If Roman dominating set of *T*. Further let<br>  $w_1, w_2 \in V(L(T))$  be the set of vertices with  $\frac{\deg(v_1) \geq 3}{\deg(v_1) \geq 3}$ ,<br>  $v_1$  there exists *B*, *Issue, 10, pp.39782-39787, October, 2016<br>
Suppose the set*  $\frac{V_2}{V_2}$  *dominates*  $\frac{V_1}{V_2}$ *. Then*  $S=V_1\cup V_2$  *forms a<br>
ninimal Roman dominating set of*  $T$ *. Further let<br>*  $A = \{r_1, v_2, ..., v_i\} \in V(L(T))$  *be the set of v* **DS,** Issue, 10, pp.39782-39787, October, 2016<br>
Suppose the set  $V_z$  dominates  $V_x$ . Then  $S = V_y \cup V$ , forms a<br>
minimal Roman dominating set of  $T$ . Further let<br>  $A = \{v_1, v_2, ..., v_n\} = V(L(T))$  be the set of vertices with  $\frac{d\exp(V$ 

**Theorem 7:** A Strong line dominating set  $D \subseteq V(L(G))$  is minimal if and only if for each vertex  $x \in D$ , one of the following condition holds.

- a) There exists a vertex  $y \in V(L(G)) D$  such that  $N(y) \cap D = \{x\}$
- b)  $\bar{x}$  is an isolated vertex in  $\langle D \rangle$ .
- c)  $\langle (V(L(G))-D)\cup \{x\} \rangle$  is connected.

**Proof:** Suppose D is a minimal Strong line dominating set of  $G$  and there exists a vertex  $x \in D$  such that  $\chi$  does not hold any of the above conditions. Then for some vertex  $V$  the set  $D_1 = D - \{v\}$  forms a Strong line dominating set of G by the conditions (a) and (b). Also by (c),  $\langle V(L(G)) - D \rangle$  is disconnected. This implies that  $\overline{D}$  is a Strong line dominating set of  $G$ , a contradiction. Conversely, suppose for every vertex A and if  $x \in D$  one of the above statements hold. Further if  $D$  is not minimal. Then there exists a vertex  $x \in D$  such that  $D-\{x\}$  is a Strong line dominating set of G and there exists a vertex  $y \in D-\{x\}$  such that  $\mathcal{Y}$  dominates  $\mathcal{X}$ . That is  $y \in N(x)$ . Therefore  $\bar{x}$  does not satisfy (a) and (b). Hence it must satisfy (c). Then there exists a vertex  $y \in V(L(G)) - D$  and  $N(y) \cap D = \{x\}$ . Since  $D-\{x\}$  is a Strong line dominating set of G, then there exists a vertex  $z \in D-\{x\}$  and  $z \in N(y)$ . Therefore  $w \in N(y) \cap D$  where  $w \neq x$ , a contradiction to the fact that  $N(y) \cap D = \{x\}$  and  $\langle V[(L(G)) - D] \cup \{x\} \rangle$  is connected. Clearly  $D$  is a minimal Strong line dominating set of  $G$ . *i of Crement Research, Vol. 08, Issne, 10, pn,39782-39787, October, 2016*<br>
<sup>-A</sup> be the minimal Suppose the set  $V_x$  dominates  $V_x$ . Then  $S = V_x \cup N_x$  forms a<br>
in *G*, Clearly *S* minimal Roman dominating set of *T*. Furthe **Example 11:** the vertices in G. Clearly S minimal Roman dominating set of T.<br>
of G. Suppose the subgraph <sup>(8)</sup> has  $A = \{v_1, \ldots, v_n\} \in V(L(T))$  be the set of vertices with<br>
the subgraph (8) has more than one than one than th Ex or C. Once<br>Point  $\sum_{k=1}^{\infty} (kx)^{2} \times 2^{k} \times 2^{k$ *<sup>G</sup> C v v v V G* 1 2 , ,..., *<sup>k</sup> S* $C(v_i)$  forms exactly one component in last one vertex  $\{v = 0$  where  $\{v = 0\}$  where  $\{v = 0\}$  and  $\{v = 0\}$  a Least one vertices<br>  $\text{C}^{\text{C}}$  component in lamimal  $x_k = \pi^{\text{C}}$  in  $\frac{|\psi(x_0)|}{\sqrt{2}}$  forms a state of  $G$ . Let  $\text{C}^{\text{C}}$  let  $\mathbb{R}^{\text{C}}$  component in minimal  $x_k = \pi^{\text{C}}$  in  $\mathbb{R}^{\text{C}}$ . Therefore we be th **and** only controllar  $M_0 = 5\pi^2$  in  $L(1)$ . Since for any tree feats at the set of vertices  $|D^2 + |z| = |z| - |z| \cos(\gamma + \alpha)$  or maximum degree  $\Delta(T)$ , then the set of vertices  $|D^2 + |z| \le |z| + |z| \cos(\gamma + \alpha)$ . Clearly,  $x_R(T) \le x_R(T)$ **EVALUATION THE SET AS THE CONSERVATION CONSERVATION (Fig. 1)**  $\int \frac{1}{2} f(x) f(x) dx = 0$  $\int \frac{1}{2} f(x) f(x) dx = 0$  **(b)**  $\int \frac{1}{2} f(x) f(x) dx = 0$  (b)  $\int \frac{1}{2} f(x) f(x) dx = 0$  (c)  $\int \frac{1}{2} f(x) f(x) dx = 0$  (c)  $\int \frac{1}{2} f(x) f(x) dx = 0$  and  $\int \frac{$ **Equipe to the exists a vertex**  $\sqrt{v_1}(L(x)) - D(x)$  **and there exists a vertex**  $\sqrt{v_2}$  **and**  $\sqrt{v_1}$  **and**  $\sqrt{v_2}$  **and hence**  $\frac{1}{2}$  **and hence**  $\frac{1}{2}$  **is a minimal Strong line dominating set of**  $\sqrt{v_1}(L(x)) - D(x)$ **. By a min Example 10**<br>
The maximal set of and there exists a vertex  $x \in D$  such that  $X$  does not be<br>  $e_i, e_j \in A$ ,  $1 \le i \le n$ ,  $2 - |z|$  forms a Strong line dominating set of  $G$  by<br>
forms a maximal set of  $Q_i$  and this implies that *V L G D <sup>G</sup> x* =  $\langle v_i, v_i, \ldots, v_i \rangle \subseteq V(L(T))$  be the set of vertices with  $\frac{\deg(v_i) \ge 3}{\chi(b_i) \le 1}$ <br>suppose there exists a vertex set  $D \subseteq 4$  with  $N[D]-V(L(T))$  and<br>suppose there exists a vertex set  $D \subseteq V \setminus V(L(T)) - D$ . Then *n* forms<br>strong lin *x* and  $N[D] = V(L(T))$  and  $V(L(T)) - D$ . Then *b* forms of  $(L(T)) - D$ . Then *b* forms a such that  $D \cup \{w\}$  forms a any tree *G* there exists at such that  $D \cup \{w\}$  forms a any tree *G* there exists at mum degree  $\Delta(T)$ , then suppose there exists a werts  $D \in \mathcal{H}$  with  $n^{y}P_{2} - x$  with  $n^{y}P_{1} - x(x + y)$  and<br> *y*  $|p(x)| \to 2$ ,  $\sqrt{x} \in D$ ,  $0 \neq v(L(T)) - D$ . Then  $\nu$  forms<br>
Strong line dominating set in  $\frac{L(T)}{T}$  observations there exists at<br>
east *y*(*v*). Otherwise there exists at  $\partial^k e^D$  such that  $D \cup \{w\}$  forms a<br>for any tree *G* there exists at<br>aximum degree  $\Delta(T)$ , then<br>aximum degree  $\Delta(T)$ , then<br>ominating set  $D \subseteq V(L(G))$  is<br>h vertex  $x \in D$ , one of the<br> $L$ *D*  $\sin \theta$   $\cos \theta$  *z*(*v*). Since for any tree **<sup>***z***</sup>** there exists at  $V(T)$  of maximum degree  $\Delta(T)$ , then Clearly,  $y_{\Omega}(T) \leq y_{\Omega}(T) - \Delta(T) + 1$ <br>
Ong line dominating set  $D \subseteq V(L(G))$  is if for each vertex  $x \in D$ , one of the ololds.<br>
vertex  $y$ Least one vertex  $e^{-x} \times e^{x} \$ *N* Fried 11 (*N* Fried 1 (*N* Fried 1 (*N* Fried 1 (*N* Friedment 7: A Strong line dominating set  $D \subset V(L(G))$  is **Chooren** 7: A Strong line dominating set  $D \subset V(L(G))$  ( $\langle V(L(G)) - D \rangle \cup \langle V(L(G)) - D \rangle$  ( $\langle V(L(G)) - D \rangle \cup \langle V(L(G)) - D \rangle$ ) ( $\langle$ rollowing condution holds.<br>
a) There exists a vertex  $x \in V(L(G)) - D$  such that  $N(y) \cap D = \{x\}$ <br>
b) *X* is an isolated vertex in  $\langle D \rangle$ .<br>
c)  $\langle |V(L(G)) - D \rangle \cup \{x\} \rangle$  is connected.<br> **Proof:** Suppose *D* is a minimal Strong line d *A*  $D$  is a minimal Strong line dominating set of exists a vertex  $x \in D$  such that  $X$  does not hold bove conditions. Then for some vertex  $V$  the set rms a Strong line dominating set of  $G$  by the set rms a Strong line inimal Strong line dominating set of<br>
ex  $x \in D$  such that X does not hold<br>
ns. Then for some vertex V the set<br> *J* line dominating set of G by the<br> *Also* by (c),  $\langle V(L(G)) - D \rangle$  is<br> *S* that D is a Strong line dominating<br> *J* and  $\infty$  does not not<br>
some vertex  $V$  the set<br>
ting set of  $G$  by the<br>  $(C)$ ,  $\langle V(L(G)) - D \rangle$  is<br>
Strong line dominating<br>
uppose for every vertex<br> *d*. Further if  $D$  is not<br> *D* such that  $D^{-\{x\}}$  is a<br>  $D$  such that and the tect sake stellar the verte sake the minimal of violation<br>
any of the above conditions. Then for some vertex *V* the set<br>  $D_n = D_n^{-1}v^1$  forms a Strong line dominating set of *G* by the<br>
conditions (a) and (b). Als

**Theorem 8:** For any connected  $(p,q)$  graph  $G$ ,  $x_{st}(G) + x_c(G) \leq diam(G) + x(G)-1$ . Equality holds with  $P \geq 3$ . *G*

**Proof:** Let  $A \subseteq V(G)$  be the minimal set of vertices. Further, G there there exists an edge set  $J \subseteq J$  where  $J'$  is the set of edges  $(e) = u(G)$  Thus which are incident with the vertices of A constituting the longest path in G such that  $|J| =$ alam(G). Let S  $\bigcap_{i=1}^{n} V_1, V_2, \dots, V_k \bigsubseteq A$  be the minimal set of vertices which covers all the vertices in  $G$ . Clearly  $S$  forms a minimal dominating set of G. Suppose the subgraph  $\lt s$  is connected. Then  $\overline{S}$  itself is a  $X_c - set$ . Otherwise there exists at least one vertex  $x \in V(G) - S'$  and  $S' = S \cup \{x\}$  forms a minimal connected dominating set of  $G$ . Now in  $L(G)$ , let  $\mathcal{L} = \{u_1, u_2, ..., u_n\} \subseteq V(L(G))$  be the set of  $\{u_j\} = \{e_j\} \in E(G)$ ,  $1 \le j \le n$  where  $\{e_j\}$  are incident with the vertices of S. iction. Conversely, suppose for every vertex<br>above statements hold. Further if  $D$  is not<br>exist a vertex  $x \in D$  such that  $D - \{x\}$  is a<br>ating set of  $G$  and there exists a vertex<br>hat  $Y$  dominates  $X$ . That is  $y \in N(x)$ ,<br> Expressed to the vertex is hold. Further if *D* is not  $x \in D$  such that  $D^{-{x}}$  is a id there exists a vertex nates *X*. That is  $y \in N(x)$ , and there exists a vertex nates *X*. That is  $y \in N(x)$ , and (b). Hence it must sati *x*∈*D* such that  $L^{(n)}$  is a<br>
and there exists a vertex<br>
ates *X*. That is  $y \in N(x)$ .<br>  $d(b)$ . Hence it must satisfy<br>  $(L(G)) - D$  and  $N(y) \cap D = \{x\}$ .<br>
aating set of *G*, then there<br>
tradiction to the fact that<br>  $\{x\} >$  is co iminal. Then there exists a vertex  $x \in P$  such that  $P^{-1/2}$  is a<br>throng line dominating set of  $G$  and there exists a vertex<br> $y \in D-[x]$  such that  $Y$  dominates  $X$ . That is  $y \in N(x)$ .<br>Therefore  $X$  does not satisfy (a) and  $y \in D^2(x)$  such that y dominates X. That is  $y \in N(x)$ .<br>
Therefore X does not satisfy (a) and (b). Hence it must satisfy<br>
Since  $D - [x]$  is a Strong line dominating set of  $G$ , then there<br>  $e^{-\beta}$  is a Strong line dominating

Further let  $D \subseteq F$  be the set of vertices with  $N[D] =$ and  $\forall u_k \in \langle V(L(G)) - D \rangle$  deg  $(u_k) \leq$  deg  $(u_j)$  where  $\forall u_j \in D$ Then  $\overline{D}$  forms a Strong line dominating set of  $G$ . Otherwise there exists at least one vertex  $\{u\} \in V(L(G)) - D$  such that deg  $(u)$  > deg  $(u_j)$ ,  $\forall u_j \in D$ . Clearly  $D \cup \{u\}$  forms a minimal  $X_{SL} - set$  of G. Thus  $|D \cup \{u\}| \cup |S| \le |J| \cup |S| - 1$ . Hence vertices. Suppose there exist . *Muddebihal and Nawazoddin U. Patel, Strong line domination*<br>  $D \subseteq F$  be the set of vertices with  $N[D] = V(L(G))$  **Proof:** Suppose *A*<br>  $V(L(G)) - D$ , deg  $(u_k) \leq$  deg  $(u_j)$  where  $\forall u_j \in D$  set of vertices whic<br>
ms a Strong line dom

**Theorem 9:** For any non trivial tree  $\overline{T}$  with  $P \ge 3$  vertices and  $C$  number of cut vertices, then  $\gamma_{SL}(T) \leq C$ .

**Proof:** Let  $F = \{v_1, v_2, ..., v_k\} \subseteq V(T)$  be the set of all cut  $\begin{array}{ccc} \text{Otherwise} & \text{if} & \text{if} & \text{if} \\ \text{if} & \text{if} & \text{if} & \text{if} \end{array}$ vertices in T with  $|F| = C$ . Further, let A be the set of edges which are incident with the vertices of  $\overline{F}$ . Now by the definition of line graph, suppose  $\{u_1, u_2, ..., u_j\} \subseteq A'$  be the set of vertices which covers all the Otherwise there exists at vertices in  $L(T)$ , deg  $(u_k) \ge \text{deg } (u_n)$  where  $\forall u_k \in D$  and . Clearly D forms a minimal Strong line dominating set of  $L(T)$ , which gives  $|D| \le |F|$ . Hence form  $X_{SL}(T) \leq C$ *Muddebihal and Nawazoddin U. Patel, Stro*<br>  $\downarrow$  e set of vertices with  $N[D] = V(L(G))$  **Proc**<br>  $\downarrow$  deg  $(u_k) \leq \deg(u_j)$  where  $\nabla u_j \in D$  set  $c$ <br>  $\downarrow$  line dominating set of  $G$ . Otherwise<br>  $\downarrow u_j \in D$ . It includes the set of Further let  $D \subset F$  be the set of vertices with  $N[D] = V(L(G))$  **Proof:** Sund  $\nabla u_k \in (V(L(G)) - D)$ , deg  $(u_k) \le \deg(u_i)$  where  $\nabla u_i \in D$ , est of vertical and  $\log(u_k) \le \deg(u_i)$  where  $\nabla u_k \in D$ , est of vertical and  $\log(u_k) \le \deg(u_k)$ , where *V V*,  $G$  *S diam*(*G*) + *x*(*G*) -1<br> **Pen 9:** For any non trivial tree *T* with  $P \ge 3$  vertices  $V$  *Vi*,  $\in B_1$ ,  $V_j \in B$ <br>
number of cut vertices, then  $\gamma_{2k}(T) \le C$ <br>  $\therefore$  Let  $F = \{\gamma_1, \gamma_2, ..., \gamma_r\} \subseteq V(T)$  be the

**Theorem 10:** For any connected  $(p,q)$  graph  $G$ ,  $x_*(G) \leq \left[\frac{p}{2}\right]$ .

**Proof:** Let  $D = \{v_1, v_2, ..., v_m\} \subseteq V(L(G))$  be the minimal Strong  $|V(L(G))| - 1$ . line dominating set of G. Suppose  $|V(L(G)) - D| = 0$ . Then the result follows immediately. Further if  $|V(L(G)) - D| \ge 2$ then  $V(L(G)) - D$  contains at least two vertices such that  $2n < p$  Hence  $X_{SL}(G) = n < \lceil p/2 \rceil$ 

**Theorem 11:** For any non trivial tree  $T$  and  $T \neq K_{1,n}$   $n \geq 2$ , then  $X_{SL}(T) \leq q - \Delta(T)$ 

**Proof:** Let  $D = \{v_1, v_2, ..., v_n\} \subseteq V(L(T))$  be the set of all vertices. Suppose there exists a set of vertices line  $S = {u_1, u_2, ..., u_m} \subseteq V(L(T)) - B$  such that  $dist(u_j, v_k) \ge 2$ ,  $v_k \in B$ ,  $1 \le j \le m$ ,  $1 \le k \le n$ . Then  $S = B$ forms a Strong line dominating set of  $T$ . Otherwise if  $B \subset V(L(T))$ , then select the set of vertices  $S = B$  such that  $N[S] = v(L(T))$  and the subgraph is disconnected. Clearly in any case  $S$  forms a minimal Strong line dominating set of  $T$ . Since for any tree T there exists at least one edge  $\mathcal{L} \subset \mathcal{L}(T)$ with  $\deg(e) = \Delta(T)$ , We obtain  $|S| \le |E(T)| - \Delta(T)$ . Therefore  $X_{SL}(T) \leq q - \Delta(T)$ is the set of edges which are incident with the vertices of  $F'$ .<br>  $V(A(G)Y - D)$ , then definition of time graph suppose control is equivalently the measure of  $G'$ .<br>  $D = \{n_1, n_2, \ldots n_r\} \subseteq A$  for the graph suppose  $\text{Cone}$  an *Now* by the definition of line graph, suppose Charly *D* forms a Strong line denotes  $\omega_2$  **b**  $\omega_2$  *k*  $\omega_3$  *k*  $\omega_4$  *b*  $\omega_5$  *k*  $\omega_6$  *b*  $\omega_7$  *k*  $\omega_8$  **c** *k*  $\omega_7$  *k*  $\omega_8$  *k*  $\omega_7$  *c*  $\omega_8$  *c B*  $B = \frac{W(L(V) - D)}{N}$  Chanky B from a minimal Strong line<br> **Control of**  $L(P)$ , which gives  $\left[ P_1 e^{i\frac{1}{2} E_1} P_2$  **From a minimal**  $X_{\infty} - \text{set } G$  **From (***B, A***)**<br> **Control of** *D* **Example 2.1 (***B, which* **gives \left[ P\_2 \ Theorem 10:** For any connected  $\binom{p,q}{p}$  **c**  $\binom{p,q}{q}$  **Proof:** Let  $D = \{v_1, v_2, \ldots, v_n\} \subset V(L(G))$  be the minimal Strong  $\begin{aligned}\nV(L(G)) &= 1 \\
V(L(G)) &= 1\n\end{aligned}$ <br> **SCLUMB THE CONSTRANGE (CONSTRANGE)** The minimal Strong  $\begin{aligned}\nV(L(G)) &= 1 \\
V(L(G)) &= 0\n\end{aligned}$  **Proof:** To prove this resul

**Theorem 12:** For any acyclic  $(p, q)$  graph  $G$ ,  $\gamma_{SL}(G) \leq i(G)$ . Where  $i(G)$  is an independent domination number G.

**Proof:** Suppose  $A = \{v_1, v_2, v_3, ..., ..., v_n\} \subseteq V(G)$  be the set of vertices which covers all the vertices in  $\mathcal G$ . Further, if  $\forall v_i \in A$ ,  $deg v_i = 0$ , then A itself is an independent dominating set of G. Otherwise  $S = A' \cup I$ , where  $A \subseteq A$  and  $I \subset V(G) - A$  forms a minimal independent dominating set of **6**. Now let  $B = \{v_1, v_2, ..., v_m\} \subseteq V(L(G))$  be the set of all vertices. Suppose there exists a set of vertices  $B_1 = {u_1, u_2, ..., u_n} \subseteq V(L(G)) - B$  and deg  $(u_i) \ge$  deg  $(v_j)$ ,  $\forall u_i \in B_1$ ,  $v_j \in B$ ,  $1 \le i \le n$ ,  $1 \le j \le m$ . Then  $D = B \cup B_1$  forms a Strong line dominating set of  $G$ . Otherwise if  $B \subset V(L(G))$ , then select the set of vertices such that  $N[D] = V(L(G))$  and  $\forall u_k \in$  $F = \frac{V(L(\circ)) - D}{L}$ , then deg  $(u_k) \le \deg(u_j)$  where  $\nabla u_j \in D$ . Clearly  $\overline{D}$  forms a Strong line dominating set of  $\overline{G}$ . Otherwise there exists at least one vertex  $\{u\} \in V(L(G)) - D$ such that deg (u) > deg (u<sub>j</sub>)  $\forall u_j \in D$  Clearly  $D \cup \{u\}$ forms a minimal  $x_{st}$  – set of G. Hence  $|D \cup \{u\}| \leq |V(G)|$  and clearly  $\gamma_{5L}(G) \leq i(G)$ . *M Nawazoddin U. Patel, Strong line domination in graphs*<br>  $N[D] = V(L(G))$  **Proof:** Suppose  $A = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be the<br>
where  $\forall u_j \in D$  set of vertices which covers all the vertices in G. Further, if<br>  $\forall v_i \in A$ , *Muddebihal and Nawazoddin U. Patel, Strong line domination in graphs*<br>  $D \subseteq F$  be the set of vertices with  $N[D] = V(L(G))$ <br>  $V(L(G)) - D)$ , deg  $(u_k) \le \deg(u_l)$  where  $\forall u_j \in D$ <br>
set of vertices which covers all the vertices in G. Fund<br> *unddebihal and Nawazoddin U. Patel, Strong line domination in graphs*<br>
vices with  $N[D] = V(L(G))$  **Proof:** Suppose  $A = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be the<br>  $\leq \deg(u_j)$  where  $\forall u_j \in D$  set of vertices which covers all the vertice *D* debihal and Nawazoddin U. Patel, Strong line domination in graphs<br>  $\mathcal{B} = \{v_1, v_2, v_3, \ldots, v_m\}$ <br>  $\mathcal{B} = \{v_1, v_2, v_3, \ldots, v_m\}$ <br>  $\mathcal{B} = \{v_1, v_2, v_3, \ldots, v_m\}$ <br>  $\mathcal{B} = \{v_i\}$ <br>  $\mathcal{B} = \{v_i\}$ <br>  $\mathcal{B} = \{v_i\}$ <br> Muddebihal and Nawazoddin U. Patel, Strong line domination in graphs<br>
vertices with  $N[D] = V(L(G))$  **Proof:** Suppose  $A = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(G)$  be the<br>  $u_k$ )  $\leq \deg(u_j)$  where  $\nabla u_j \in D$  set of vertices which covers all th 39785<br>
Hurther let  $D \subseteq F$  be the set of vertices with  $N[D] = V(L(G))$  **Proof:** Suppose  $A = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(G)$  be the<br>
flurther let  $D \subseteq F$  be the set of vertices with  $N[D] = V(L(G))$  **Proof:** Suppose  $A = \{v_1, v_2, v_3, \ldots, v_n$ **Muddebihal and Noveroddin U. Patd, Shrong line domination in graphs**<br>  $D \subset F$  be the set of vertices with  $N[D] = V(L(G))$  **Proof:** Suppose  $A = \{v_1, v_2, v_3, ..., w_n\} \subseteq V(G)$  be the<br>  $V(L(G)) - D$ , deg  $(u_k) \leq \deg(u_j)$  where  $\forall u_j \in D$  set of *Muddebited and Nawazoddin U. Patel, Strong line domination in graphs*<br> *For extract some*  $N[D] = V(L(G))$  **Proof:** Suppose  $A = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(G)$  be<br> *For extract some*  $\forall v_i \in A$ *, deg* $\{v_i = 0\}$ *, then*  $A$  *itself is an hal and Nawazoddin U. Patel, Strong line domination in graphs*<br>  $\text{trh}(M[D] = V(L(G))$  **Proof:** Suppose  $A = \{v_1, v_2, v_3, \ldots, v_n\}$ <br>  $\text{trh}(G)$  where  $\nabla u_j \in D$  set of vertices which covers all the<br>  $\forall v_i \in A$  deg $v_i = 0$ , then A **EXAMPLE 12** The set of vertices with  $N[D] = V(L(G))$ . **Proof:** Suppose  $A = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(L(G))$ <br>
Then  $P$  forms a Strong line dominanting set of vertices with  $N[D] = V(L(G))$ . **Proof:** Suppose  $A = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(L$ *nd Nawazoddin U. Patel, Strong line domination in graphs*<br> *N*[*D*]= $V(L(G))$  **Proof:** Suppose  $A = \{v_1, v_2, v_3, \dots, \dots, v_n\}$ <br> *N*[*D*]= $V(L(G))$  **Proof:** Suppose  $A = \{v_1, v_2, v_3, \dots, \dots, v_n\}$ <br>  $V_{\text{up}} \in \mathcal{B}$ . **C** *Otherwise p*)  $\leq$  deg (*u<sub>i</sub>*) where  $\forall u_i \in D$  we for vertices which covers all the vertices in  $G$ . Forming set of  $G$ , otherwise  $\frac{1}{4}$  deg  $u_i = 0$ , then  $\hat{A}$  is lest f is sent in  $\{u_i \in V(G) - D$  such that  $\sum_{i=1}^{n} V(x_i(G))$  $V(L(G))$  Proof: Suppose  $A = \{v_1, v_2, v_3, \dots, v_{n_B}\} \subseteq V(G)$  be the<br>  $tu_j \in D$  set of vertices which covers all the vertices in G. Further, if<br>
therwise<br>
denoting set of G. Otherwise  $S = A' \cup I$ , where  $A'_{C} \subseteq A$  and<br>
then that<br>  $I$ **EVALUAT ALL CONTAGES** (*V*) is equilable that the video  $\sigma$  is a substrained of  $G$ . Otherwise  $\sigma^2 = 0$ , then  $\sigma^2$  is a substrained at least one vertex  $\langle v \rangle$  with  $\epsilon^2 \rho^2$  and  $\sigma^2$  (*D*). Then  $\sigma^2$  is a subst line diminuiting set of  $\overline{G}$ . Cherwise  $\overline{G} = \overline{A} \cup I$ , where  $\overline{A} \subset \overline{A}$  and<br>  $\overline{E} = \overline{C}$ . Cluerty  $D^{-1}$  is once that  $\overline{C} = \overline{A} \cup I$ , where  $\overline{A} \subset \overline{A}$  and<br>  $\overline{E} = \overline{C}$ . Cluerty  $D^{-1}$  is on *V* Such that  $I \subseteq V(b) - A$  forms a minimal integralant dominating set of<br> *V* Let  $S$  C Low let  $B = \{v_1, v_2, ..., v_m\} \subseteq V(L(G))$  be the set of all<br>  $v_2v_1|S = 1$ . Hence critics. Suppose there exists a set of critics<br>  $B_i = \{u_1, u_2$ **From 9:** For a convertion that by the Example of the set of  $V(E(G)) = B$  and do  $\frac{1}{2}$  and do  $\frac{1}{2}$  and do  $\frac{1}{2}$  and do  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  ( $E(B_1, B_2, B_3) = 2$ ) ( $E(B_1, B_3) = 2$  and  $\frac{1}{2}$  and F number of cut vertices, then  $y_{2n}(T) \leq C$ <br> **S**  $D - B \cup B$ , forms a Strong line dominating set<br> **S** Ex  $F = \{v_1, v_2, ..., v_n\} \leq V(T)$  be the set of all cut Otherwise if  $B \subset V(L(G))$ , then select the set of<br>  $S = B$ ,<br>
set of edges w *F* = [ $v_1, v_2, v_3$  in the set of all cut values in the  $N$  ( $I_1$ ( $I_2$ ) and  $I_2$  ( $I_3$ ) and  $V_4$  ( $I_4$ )  $I_5$  ( $I_6$ )  $I_7$  ( $I_8$ ) ( $I_9$ ) (*B v v v V L G* 1 2 , ,..., *<sup>m</sup> Brong line domination in graphs*<br> **Proof:** Suppose  $A = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be the<br> *E* of vertices which covers all the vertices in G. Further, if<br>  $\forall v_i \in A$ ,  $\deg v_i = 0$ , then A itself is an independent<br> *L*  $V(G) - A$ *Strong line domination in graphs*<br> **Proof:** Suppose  $A = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be the<br>
set of vertices which covers all the vertices in  $G$ . Further, if<br>  $\forall v_i \in A$ ,  $\deg v_i = 0$ , then  $A$  isself is an independent<br>  $I_{\subseteq$ *Strong line domination in graphs*<br> **Proof:** Suppose  $A = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be the<br>
ret of vertices which covers all the vertices in G. Further, if<br>  $\forall v_i \in A$ ,  $\deg v_i = 0$ , then A itself is an independent<br>
lominatin *Brandion in graphs*<br>  $B = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be the which covers all the vertices in G. Further, if  $v_i = 0$ , then A itself is an independent of G. Otherwise  $S = A' \cup I$ , where  $A' \subseteq A$  and forms a minimal independent Strong line domination in graphs<br> **Proof:** Suppose  $A = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be the<br>
te of vertices which covers all the vertices in G. Further, if<br>  $\forall v_i \in A$ ,  $\deg v_i = 0$ , then  $A$  itself is an independent<br>
folominati *Strong line domination in graphs*<br> *Yroof:* Suppose  $A = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be the<br> *Let* of vertices which covers all the vertices in  $G$ . Further, if<br>  $\forall v_i \in A$ ,  $deg \, v_i = 0$ , then  $A$  itself is an independent<br>
co *u*<sub>n</sub>  $\in$   $V(G)$  be the<br>
ces in *G*. Further, if<br>
is an independent<br> *J1*, where  $A \subseteq A$  and<br>
ent dominating set of<br> *G*)) be the set of all<br>
set of vertices<br>  $1 \deg (u_i) \ge \deg (v_i)$ ,<br>  $1 \le j \le m$ . Then<br>
nating set of *G*.<br>
the s (G) be the<br>Further, if<br>dependent<br> $A \subseteq A$  and<br>tting set of all<br>vertices<br> $\ge \deg(v_i)$ ,<br> $m$ . Then<br>t of G.<br>of vertices<br> $\forall u_k \in$ <br> $\forall u_i \in D$ <br> $\forall u_j \in D$ .<br>it of G.<br> $L(G)$ ) – D<br> $D \cup \{u\}$ <br> $D \cup \{u\}$ <br> $|V(G)|$  and<br> $X_{SL}(G) = 1$ <br>of degree *Som* in graphs<br>  $A = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be the<br>
inch covers all the vertices in  $G$ . Further, if<br>  $G$  otherwise  $S = A' \cup I$ , where  $A \subseteq A$  and<br>
ms a minimal independent dominating set of<br>
set there exists a set of ve *p*, *a*, *a*, *b*, *p*, *f* = *v* (*b*) be the e vertices in *G*. Further, if itself is an independent  $= A' \cup I$ , where  $A' \subseteq A$  and dependent dominating set of all ts a set of vertices  $B$  and deg  $(u_i) \ge \deg(v_i)$ ,  $\le n$ ,  $1 \$ Example the same vertex  $\{u\} \in V(L(G))$ <br>  $\{u\} \in B$ ,  $\omega$   $\omega$  and  $\omega$   $\{u\} \in E$   $\{u\}$   $\{u\}$   $\{u\} \in E$   $\{u\}$   $\{u\}$   $\{u\} \in E$   $\{u\}$   $\{u\}$   $\{u\} \in E$   $\{u\}$   $\{v_1, v_2, \ldots, v_m\} \subseteq V(L(G))$  be the set of all<br>
see t Iominating set of G. Otherwise  $S = A' \cup I$ , where  $A \subseteq A$  and<br>  $I \subseteq V(G) - A$  forms a minimal independent dominating set of<br>  $G$ . Now let  $B = \{v_1, v_2, ..., v_n\} \subseteq V(L(G))$  be the set of all<br>  $\forall E, B_1 = \{u_1, u_2, ..., u_n\} \subseteq V(L(G)) - B$  and  $\deg(u_i) \$  $V(L(G)) - B$  and deg  $(u_i) \ge \deg(v_i)$ ,<br>  $B$ ,  $1 \le i \le n$ ,  $1 \le j \le m$ . Then<br> *L* Strong line dominating set of  $G$ .<br>  $L(G)$ ), then select the set of vertices<br>  $N[D] = V(L(G))$  and  $\nabla u_k \in$ <br> **deg**  $(u_k) \le \deg(u_j)$  where  $\nabla u_j \in D$ .<br> *L* Strong  $B_i = \{u_1, u_2, ..., u_n\} \subseteq V(L(G)) - B$  and deg  $(u_i) \ge \deg(v_i)$ ,<br>  $\forall u_i \in B_1$ ,  $v_j \in B$ ,  $1 \le i \le n$ ,  $1 \le j \le m$ . Then<br>  $D = B \cup B_i$  forms a Strong line dominating set of G.<br>
Otherwise if  $B \subset V(L(G))$ , then select the set of vertices<br>  $S = B_1$  s  $j \le m$ . Then<br>g set of  $G$ .<br>set of vertices<br>) and  $\forall u_k \in$ <br>there  $\forall u_j \in D$ .<br>g set of  $G$ .<br> $\vdots V(L(G)) - D$ <br>rly  $D \cup \{u\}$ <br> $\{u\} \le |V(G)|$  and<br> $G, X_{SL}(G) = 1$ <br>tices of degree<br>following two<br>ne vertex  $V$ ,<br> $D = \{v\}$  is a<br>in  $V(L(G)) - D$ <br>a 1 ≤ *i* ≤ *n*, 1 ≤ *j* ≤ *m*. Then<br> *i* line dominating set of *G*.<br> *W*<sub>*P*</sub> l = *V*(*L*(*G*)) and  $\forall u_k \in B$ <br> *y* where  $\forall u_j \in D$ <br> *g* line dominating set of *G*.<br> *t* one vertex {*u*} ∈ *V*(*L*(*G*)) – *D*<br> *p*<sub>j</sub>),  $\$ et of *G*.<br>
of vertices<br> *Vu<sub>k</sub>* ∈<br>
e *Vu<sub>j</sub>* ∈ *D*.<br>
let of *G*.<br>  $L(G)) - D$ <br>  $D \cup \{u\}$ <br>  $\leq |V(G)|$  and<br>  $X_{SL}(G) = 1$ <br>
s of degree<br>
lowing two<br>
vertex *V*,<br>  $= \{v\}$  is a<br>  $V(L(G)) - D$ <br>
one vertex<br>
ms a Strong  $D = B \cup B$ , forms a Strong line dominating set of  $G$ .<br>
Otherwise if  $B \subset V(L(G))$ , then select the set of vertices<br>  $S = B_1$  such that  $N[D] = V(L(G))$  and  $\forall u_k \in$ <br>  $\langle V(L(G)) - D \rangle$ , then deg  $(u_k) \le \deg (u_j)$  where  $\forall u_j \in D$ .<br>
Clearly  $D$  *N* = *B*<sub>1</sub> such that  $N[D] = V(L(G))$  and  $\forall u_k \in$ <br>  $V(L(G)) - D$ , then deg  $(u_k) \le \text{deg } (u_l)$  where  $\forall u_l \in D$ <br>
Clearly *D* forms a Strong line dominating set of *G*.<br>
Otherwise there exists at least one vertex  $\{u\} \in V(L(G)) - D$ <br>
uch *D* D =  $V(L(G))$  and  $Vu_k \in$ <br>  $\leq$  deg  $(u_j)$  where  $Vu_j \in D$ .<br> *wertex*  $\{u\} \in V(L(G)) - D$ <br> *wertex*  $\{u\} \in V(L(G)) - D$ <br> *wertex*  $\{u\} \in V(L(G)) - D$ <br> *wi*  $\in D$  Clearly  $D \cup \{u\}$ <br> *Wi*  $\in D$  Clearly  $D \cup \{u\}$ <br> *Wi*  $\in D$  *Wii Mi* 

 $\left[\frac{a}{2}\right]$  **Theorem 13:** For any connected  $\left[\frac{p}{q}\right]$   $G, \left[\frac{a}{q}\right]$ if and only if  $L(G)$  has at least one vertices of degree .

> **Proof:** To prove this result we consider the following two cases.

**Case 1:** Suppose  $L(G)$  has exactly one vertex  $V$ , . Then in this case  $\mathcal{L} = \{y \}$  is a minimal  $X_{SL} - \mathcal{S}et$  If  $D^{\dagger} = \{u\} \in N(v)$  in  $V(L(G)) - D$  $deg(u) \le v(L(G)) - 2$ . Then there exists at least one vertex in  $L(G)$  such that  $D_1 = D' \cup \{w\}$  forms a Strong line dominating set in  $L(G)$  a contradiction. deg  $(u_k) \le \deg (u_j)$  where  $\forall u_j \in D$ .<br> *Strong* line dominating set of *G*.<br> *L* at least one vertex  $\{u\} \in V(L(G)) - D$ <br> *Let*  $\{u\} \in D$ . Clearly  $D \cup \{u\}$ <br> *Let* of *G*. Hence  $|D \cup \{u\}| \le |V(G)|$  and<br> *Connected*  $(P,q)$  graph xists at least one vertex  $\{u\} \in V(L(G)) - D$ <br>
(i)  $> \deg(u_i)$ ,  $\forall u_i \in D$ , Clearly  $D \cup \{u\}$ <br>  $x_x - set$  of  $G$ . Hence  $|D \cup \{u\}| \le |V(G)|$  and<br>  $\{G\}$ .<br>  $L(G)$  has at least one vertices of degree<br>
this result we consider the followin

**Case 2:** Suppose  $L(G)$  contains at least two vertices  $U$  and  $V$ *T*. Otherwise if with  $deg(u) = |V(L(G))| - 1 = deg(v)$  and  $v \notin N(u)$ . Then  $D = \{u\}$ dominates all the vertices in  $L(G)$ . Since  $deg(u) = |V(L(G))|^{-1}$  and  $L = V(L(G)) - \{u\}$ . Hence  $D_1 = \{v\} \cup V_1$ , where *S* forms a minimal Strong line dominating set of  $T$   $V_1 \subseteq V(L(G)) - D$  forms a  $X_{SL} - set$  again a contradiction. Conversely, suppose  $deg(u) = |V(L(G))|-1 = deg(v)$  *u* and  $V$  are adjacent to all the vertices in  $L(G)$ . Then where  $u \in D$ ,  $v \in V(L(G)) - D$  and vice – versa. In any case we obtain  $|D_1| = 1$ . **EVALUATE:**<br>
From 13: For any connected  $V^{k,q}$  graph  $G$ ,  $\alpha_{k}(V)$ <br>
if and only if  $L(G)$  has at least one vertices of deg<br>
nal Strong  $|V(L(G))|^{-1}$ .<br>
Then the  $P(G) = \frac{1}{2}$ <br>  $P(D) = \frac{1}{2}$ <br>  $P(D) = \frac{1}{2}$ <br>  $P(D) = \frac{1}{2}$ <br> rwise there exists at least one vertex  $\{u\} \in V(L(G)) - D$ <br>
that deg  $(u) > \deg(u_i)$ ,  $\forall u_i \in D$ . Clearly  $D \cup \{u\}$ <br>
s a minimal  $X_{2k} - set$  of  $G$ . Hence  $|D \cup \{u\}| \leq |V(G)|$  and<br>
ly  $Y_{3k}(G) \leq i(G)$ .<br> **orem 13:** For any connected  $(P,q$ *D*  $\forall u_i \in D$  Clearly  $D \cup \{u\}$ <br> *G* . Hence  $|D \cup \{u\}| \le |V(G)|$  and<br>
ted  $(P,q)$  graph *G*,  $X_{SL}(G) = 1$ <br>
it least one vertices of degree<br>
we consider the following two<br>
has exactly one vertex  $V$ ,<br>  $P = \{v\}$  is a<br>  $D' = \{u\}$ Learly *Y*<sub>SL</sub>(G) ≤ *t*(G)<br> **C** Theorem 13: For any connected  $(P \cdot q)$  graph G,  $X_x(G) = 1$ <br>
f and only if  $L(G)$  has at least one vertices of degree<br>  $V(L(G))|-1$ <br> **Proof:** To prove this result we consider the following two<br> **C** Lettrix and *X*<sub>SL</sub> - set of *G*. Hence  $|D \cup \{u\}| \le |V(G)|$  and  $L(G) \le i(G)$ .<br> **13:** For any connected  $(P,q)$  graph *G*, *X*<sub>SL</sub>  $(G) = 1$ <br>
bly if  $L(G)$  has at least one vertices of degree<br>  $D = 1$ .<br>  $D = \{V(G)\}$  has exactly one Learly *Y*<sub>3£</sub> (*G*) ≤ *i*(*G*)<br> **Theorem 13:** For any connected  $(P,q)$  graph *G*,  $X_x(G) = 1$ <br> *f* and only if  $L(G)$  has at least one vertices of degree<br>  $|V(L(G))| - 1$ <br> **Proof:** To prove this result we consider the following g connected  $(p,q)$  graph  $G$ ,  $X_x(G)=1$ <br>
i) has at least one vertices of degree<br>
s result we consider the following two<br>  $L(G)$  has exactly one vertex  $V$ ,<br>
1. Then in this case  $D = \{v\}$  is a<br>
. If  $D' = \{u\} \in N(v)$  in  $V(L(G)) - D$ **r** and only if  $\vee$  has at least one vertices of degree<br> *v*  $(L(G))[-1]$ <br> **Proof:** To prove this result we consider the following two<br>
asses.<br> **Consection: Consectively**  $L(G)$  has exactly one vertex  $\vee$ ,<br>  $deg(v) = |v(L(G))|$ Tand only if  $V$  has at least one vertices of degree<br> *P(L(G))*|-1.<br> **C** and only if  $V$  has at least one vertices of degree<br> **C** abses.<br> **C** abses 1: Suppose  $L(G)$  has exactly one vertex  $V$ ,<br>  $\deg(v) = |V(L(G)|) - 1$ . Then in t *x* and least one vertices of degree<br> *ve* consider the following two<br>
has exactly one vertex  $V$ ,<br>
lead in this case  $D' = \{v\}$  is a<br>  $D' = \{u\} \in N(v)$  in  $V(L(G)) - D$ <br>
in  $V(L(G)) - D$ <br>
in a strong a contradiction.<br>
a contradictio

Therefore  $x_{\text{SL}}(G)=1$ 

**Theorem 14:** For any connected  $(p,q)$  graph  $G, \gamma_{SL}(G) \leq \gamma_{ss}(G)$ 

**Proof:** let  $S'$  be a maximum independent set of vertices in  $G$ and  $S \subset S$  be the of all isolated vertices in  $G$ . Then  $(V - S') \cup S'$  is a Strong split dominating set of G. Since for each vertex  $v \in (V - S') \cup S''$  either  $v$  is an isolated vertex in  $\langle (V-S') U S'' \rangle$  or there exists a vertex  $u \in S' - S'$  and  $v$ is adjacent to  $u \cdot (V - S') \cup S'$  is minimal. Since S is maximum  $(V - S') \cup S'$  is minimum. Thus  $| (V-S') \cup S' | = \gamma_{ss}(G)$ 

Let  $F = \{e_1, e_2, e_3, \dots, e_n\}$  be set of edges in  $\mathcal{C}$  and  $F \subset E(G)$ . Then in  $L(G)$ ,  $D = \{v_1, v_2, v_3, \dots, w_n\}$  which<br>which  $\forall a \in F$  dex  $(a)$ ,  $\forall a \in F$  Theorem corresponds to  $\forall e_i \in F$  Let  $\deg(e_i)$   $\forall e_i \in F$  and Theorem 16: F<br>dec(c)  $\Rightarrow$  dec(c)  $\Rightarrow$  dec(c)  $\Rightarrow$  dec(c)  $\Rightarrow$  dec(c)  $\deg(e_j)$   $\forall e_j \in E(G) - F$  such that  $\deg(e_i) \geq \deg(e_j)$ . Suppose  $D = \{v_1, v_2, v_3, ..., ..., v_i\}$   $D$  and ,  $\forall u_k \in D$ ,  $1 \leq k \leq i$ . Then  $D$  forms a edge It follows that  $|D| \le |(V-S') \cup S''|$  Hence dist  $\gamma_{SL}(G) \leq \gamma_{ss}(G)$ Therefore  $X_x(G) = 1$ .<br> **Controllent 14:** For any connected  $(\mathcal{P}, q)$  graph with  $d\mathbf{i} \mathbf{m}(a, b) \geq 3$ ,<br> **Proof:** let  $S'$  be a maximum independent set of vertices in  $G$   $N(F) \cup N(I)$  and  $I$ <br> **Proof:** let  $S'$  be the of a

**Corollary:** For a tree  $T = K_{1,n}$  with  $n \ge 2$  vertices  $y_{51}(T) = (n + 1) - (y'(T) + 1)$ 

**Theorem 15:** For any connected  $(p,q)$  graph .

**Proof:** Let  $S = \{v_1, v_2, ..., v_n\} \subseteq V(G)$  be the set of vertices with  $deg(v_i) \geq 2$ . Suppose exists a set  $S_i \subseteq S$  of vertices with  $dist(u, v) \ge 3$ , which covers all the vertices in G Then  $S_1$  forms a dominating set of G. Otherwise if Suppose the set  $V_2$  dominates  $V_0$ . Then  $S = V_1 \cup V_2$  $diam(u, v) < 3$ , then there exists at least one vertex  $x \notin S_1$ such that  $S = S_1 \cup \{x\}$  forms a minimal  $X$  - set of G. Hence  $S' = \gamma(G)$ . Let  $C_1 = \{v_1, v_2, ..., v_n\} \subseteq V(L(G))$  be the set of vertices with  $dist(u, v) \ge 3$ . Suppose there exists a set  $D_1 \subseteq C_1$  which covers all the vertices in  $L(G)$ . Then  $D_1$ itself is a line dominating set. If  $dist(u, v) < 3$  and , then  $D^{\dagger} = D_1 \cup \{w\}$ , where  $W \notin \mathcal{P}$  $v \in D_1$  forms a minimal dominating set of  $L(G)$ . Hence  $\left|D_1 \cup \{w\}\right| = x_L(G)$ . The edges which are incident with the vertices of S in G corresponds to the set of vertices  $S =$  $\{v_1, v_2, ..., v_m\} \subseteq V(L(G))$ . Let F be the set of vertices with  $\deg(v)=1$ ,  $\forall v \in F'$ . *diam u v* , 3 <sup>1</sup> *x S*  $f(G) - F$  such that deg  $(e_i) \ge \deg(e_i)$ <br>  $D' = \{v_1, v_2, v_2, \dots, v_r\}$  $\ge \emptyset$  and **Proof:** Let *J*<br>  $f'(x) = 0$  and **Proof:** Let *J*<br> Suppose  $D^2 = \{v_1, v_2, v_3, ..., w_n\} \subseteq D$  and **Proof:** Let  $J = \{e_1, e_2, ..., e_n\} \subseteq E(G)$  be<br>  $V = S^*$  **L** follows that  $|D| \subset |(V-S) \cup S'|$  Hence distributive the longest particle is the set of<br>  $V = S^*$  **L** follows  $V_{\mathcal{R}}(G) \leq V_{\mathcal$ *N*  $V_{\rm R}(G) \leq Y_{\rm R}(G)$ <br> *N*  $V_{\rm R}(G) \leq Y_{\rm R}(G)$ <br> **N**  $W_{\rm R}(F) = 0$ <br> **N**  $W_{\rm R}(F) =$  $u(G) \leq \gamma_H(G)$ <br>  $u(G) = \gamma_H(G)$ <br>  $u(G) = (u + 1) - (y'(T) + 1)$ <br> **b**  $u(G) = (u + 1) - (y'(T) + 1)$ <br> **correspondently:** For a tree  $T = K_{LR}$  with  $n \geq 2$  vertices of  $I$ . Suppose  $D_C = B$ <br>  $v_R(T) = (n + 1) - (y'(T) + 1)$ <br> **becoming the transform of \gamma** <sup>1</sup> ( ) *D w G<sup>L</sup>* **Theorem 15:**  $\overline{Y}_1 \times \overline{Y}_2 \times \overline{Y}_3 \times \overline{Y}_4 \times \overline{Y}_5$ <br> **Theorem 15:**  $\overline{Y}_0 \times \overline{Y}_4 \times \overline{Y}_5$ <br> **Controllarity**  $\overline{Y}_0 \times \overline{Y}_4 \times \overline{Y}_5$ <br> **Theorem 15:**  $\overline{Y}_{\text{ref}}(G) + \gamma_E(G) \leq q + 1 + \gamma(G)$ <br> **Theorem 17:** For any rem 15: For any connected  $(\mathfrak{p}, q)$  graph<br>  $(G) + \gamma_{\text{ct}}(L(G)) + \gamma_{\text{c}}(G) \leq q + 1 + \gamma(G)$ <br>  $\vdots$   $G = {\nu_1, \nu_2, ..., \nu_n} \subseteq V(G)$  be the set of<br>
es with  $\frac{deg(v_i)}{\geq 2}$ , Suppose exists a set  $S_i \subseteq S$  of<br>
es with  $\frac{dist(u, v) \geq 3}{i}$ , w For any connected  $(p,q)$ <br> **For any connected**  $(p,q)$ <br>  $\downarrow (L(G)) + \gamma_L(G) \le q + 1 + \gamma(G)$ .<br> **For any connected**  $(p,q)$ <br>  $\downarrow (L(G)) + \gamma_L(G) \le p + \Delta(G)$ .<br>  $\downarrow (L$ 

 $X_{SL}(G)=1$ <br>Suppose  $I = \{v_1, v_2, ..., v_j\} \subseteq S^{\dagger}$  be the set of vertices with  $diam(a,b) \ge 3$ , where  $a \in F, b \in I$ . Then  $D = F'$ covers all the vertices in  $L(G)$ . Hence  $D$  forms a  $X_{ct}$  - set of  $L(G)$ . Otherwise there exists a vertex  $z \in$  $(F') \cup N(I)$  and  $D = F' \cup I \cup \{z\}$  forms a minimal cototal dominating set of  $L(G)$ . Hence  $|D| = X_{\alpha}(L(G))$ . We consider  $A = \{e_1, e_2, ..., e_k\}$  be the set of all edges which are incident to the vertices of  $F'$ . Since  $V(L(G)) = E(G)$ , then  $\mathbf{b} = \{u_1, u_2, ..., u_i\} \subseteq A$  be the set of vertices which covers all the vertices in  $L(G)$ . Clearly  $\overline{D}$  forms a minimal Strong line dominating set of  $L(G)$ . Therefore it implies that  $|D| \cup |D| \cup |D_1 \cup \{w\}| \le |E(G)| \cup |S'| + 1$  Thus  $\gamma_{SL}(G) + \gamma_{ct}(L(G)) + \gamma_L(G) \leq q + 1 + \gamma(G)$ *<sup>k</sup> u D* <sup>1</sup> *k i pp.39782-39787, October, 2016*<br> *I* = { $v_1, v_2, ..., v_j$ }  $\subseteq S$  be the set of vertices<br>  $n(a,b) \ge 3$ , where  $a \in F, b \in I$ . Then  $D = F \cup I$ <br>
the vertices in  $L(G)$ . Hence  $D$  forms a  $x_{ct}$ - set<br> *N*(*I*) and  $D = F \cup I \cup \{z\}$  forms a ctober, 2016<br>
...,  $v_j$   $\subseteq$  *S*<sup>\*</sup> be the set of vertices<br>  $e^{at} = e^{at}F$ ,  $b \in I$ . Then  $D = F \cup I$ <br>  $L(G)$ . Hence  $D$  forms a  $X_{ct}$  - set<br>
there exists a vertex  $z \in F' \cup I \cup \{z\}$  forms a minimal cototal<br>
Hence  $|D| = X_{ct}(L(G))$  *e, 10, pp.39782-39787, October, 2016*<br>  $\sum_{k=0}^{\infty} I = \{v_1, v_2, \dots, v_j\} \subseteq S^*$  be the set of vertices<br>  $diam(a,b) \ge 3$ , where  $a \in F, b \in I$ . Then  $D = F \cup I$ <br>
all the vertices in  $L(G)$ . Hence  $D$  forms a  $X_{ct}$ - set<br>  $L(G)$ . Otherw *a F b I* , of vertices<br>  $D = F' \cup I$ <br>
a  $X_{ct}$  – set<br>
ex  $Z \in$ <br>
mal cototal<br>
e consider<br>
incident to *Letober, 2016*<br>  $\ldots, v_j$   $\subseteq$  *S*<sup>*v*</sup> be the set of vertices<br>  $\text{ere}$  *a* ∈ *F*, *b* ∈ *I*. Then *D* = *F'* ∪ *I*<br> *L*(*G*). Hence *D* forms a <sup>*x*</sup> *a* - set<br>
there exists a vertex  $z \in$ <br> *F'* ∪ *I* ∪ {*z*} forms a mi *Le, 10, pp.39782-39787, October, 2016*<br>  $\sum_{\substack{0 \leq x \\ y \neq 0}}$  *L*  $\left\{V_1, V_2, \ldots, V_j\right\} \subseteq S$  be the set of vertices<br>  $\text{diam}(a, b) \geq 3$ , where  $a \in F, b \in I$ . Then  $D = F \cup I$ <br>
s all the vertices in  $L(G)$ . Hence  $D$  forms a 8, *Issue, 10, pp.39782-39787, October, 2016*<br>
Suppose  $I = \{v_1, v_2, \dots, v_j\} \subseteq S^{\dagger}$  be the set of vertices<br>
with  $diam(a,b) \ge 3$ , where  $a \in F, b \in I$ . Then  $D = F \cup I$ <br>
covers all the vertices in  $L(G)$ . Hence  $D$  forms a  $X_{a}$ - 787, October, 2016<br>
<sup>2</sup><sub>2</sub>, ...,  $v_j$   $\subseteq$   $S$ <sup>3</sup> be the set of vertices<br>
where  $a \in F$ ,  $b \in I$ . Then  $D = F \cup I$ <br>
is in  $L(G)$ . Hence  $D$  forms a  $X_{ct}$  - set<br>
vise there exists a vertex  $z \in$ <br>  $D = F \cup I \cup \{z\}$  forms a minimal **Propagation**  $v_2$ ,  $..., v_j$   $\subseteq$  *S* be the set of vertices<br> *L*(*G*) Hence *D* forms a  $X_{\alpha}$  set<br>
wise there exists a vertex  $z \in$ <br> *L*(*G*) Hence *D* forms a  $X_{\alpha}$  set<br>  $D = F \cup I \cup \{z\}$  forms a minimal cototal<br>  $\mu(\sigma)$ 16<br>  $\equiv S^{\dagger}$  be the set of vertices<br>  $F^{\dagger}, b \in I$  Then  $D = F^{\dagger} \cup I$ <br>
Hence  $D$  forms a  $X_{ct}$  set<br>
exists a vertex  $z \in$ <br>  $\{z\}$  forms a minimal cototal<br>  $D|\!=\!X_{\alpha}(L(G))$  We consider<br>
edges which are incident to<br>
ce  $V$ *As, Issue, 10, pp.39782-39787, October, 2016*<br>
Suppose  $I = \{v_1, v_2, ..., v_j\} \subseteq S$  be the set of vertices<br>
with  $diam(a,b) \ge 3$ , where  $a \in F, b \in I$ . Then  $D = F \cup I$ <br>
covers all the vertices in  $L(G)$ . Hence  $D$  forms a  $X_{\alpha}$ -set<br> 8, *Issue, 10, pp.39782-39787, October, 2016*<br>
Suppose  $I = \{v_1, v_2, ..., v_j\} \subseteq S$  be the set of vertices<br>
sith  $diam(a,b) \ge 3$ , where  $a \in F, b \in I$ . Then  $D = F \cup I$ <br>
covers all the vertices in  $L(G)$ . Hence  $D$  forms a  $X_{\alpha}$ -set<br>
o 8, *Issue, 10, pp.39782.39787, October, 2016*<br>
uppose  $I = \{v_1, v_2, \ldots, v_j\} \subseteq S$  be the set of vertices<br>
ith  $diam(a,b) \ge 3$ , where  $a \in F, b \in I$ . Then  $D = F \cup I$ <br>
overs all the vertices in  $L(G)$ . Hence  $D$  forms a  $x_{\alpha}$ - set<br> 1.39782-39787, October, 2016<br>  $\begin{aligned}\n &\text{(1, 1)} &\text{(2, 2)} &\text{(2, 2)} \\
 &\text{(3, 2)} &\text{(3, 2)} \\
 &\text{(4, 2)} &\text{(4, 2)} \\
 &\text{(4$ *v*<sub>1</sub>, *v*<sub>2</sub>, ..., *v*<sub>1</sub>,  $y \text{ }\subseteq S$  be the set of vertices<br>  $\geq 3$ , where  $a \in F, b \in I$ . Then  $D = F \cup I$ <br>
tices in  $L(G)$ . Hence  $D$  forms a  $x_{a}$ -set<br>
herwise there exists a vertex  $z \in$ <br>
and  $D = F \cup I \cup \{z\}$  forms a mini

**16:** For any connected  $(p,q)$ <sub>graph</sub> .<br>.<br>.

**Proof:** Let  $J' = \{e_1, e_2, ..., e_n\} \subseteq E(G)$  be the minimal set of edges which constitute the longest path between any two  $D \le |(V - S) \cup S'|$  Hence distinct vertices  $u, v \in V(G)$  with  $u, v = u, u, v = u$  $H = \{u_1, u_2, u_3, \dots, u_n\} \subseteq V[L(G)]$  be the set of vertices such that  ${u_i} = {e_i} \in E(G)$ ,  $1 \le i \le n$ , where  ${e_i}$  are incident with the vertices of  $\vec{l}$ . Suppose  $\vec{v} \in \vec{H}$  be the set of vertices with deg  $(w) \ge 3$  for every  $w \in D$  such that  $N[D] = V(L(G))$  and  $\forall v_i \in V[L(G)] - L$ . Then  $[D] \cup \{v_i\}$  forms a Strong line dominating set. It follows that  $|D \cup \{v_i\}| \leq diam(G)$ . Hence  $\gamma_{SL}(G) \leq diam(G)$ .  $\epsilon(V - S) \cup S'$  since *V* is an isolated ventex in the set of will be the set of vertices of  $S'$  is minimum. Thus vertices of  $V$  is minimum in the set of vertices  $\sqrt{V}(G) = E(G)$ , then  $\sqrt{V} - S$ )  $\cup S'$  is minimum. Thus domi *v* (*U* - 5) US is minimal. Since 5 is<br>  $(U - S')U$  is minimal. Since 5 is<br>  $[(U - S')U]$  is minimal. Since 5 in  $U(S)$  collearly *D* forms a minimal Norong is<br>  $\frac{1}{2} \times F_{20}$  and  $F_{10} = F(G)$   $\frac{1}{2} \times F_{21}$  (*D*)  $|\sqrt{D}| \times |\sqrt$ ( $V = S$ ) US is minimum. Thus commute of  $\pi$  ( $V = S$ )  $V = \pi/2$  ( $V = \pi/2$ ) domination and  $V = \pi/2$ ) domination and  $V = \pi/2$  ,  $V = \pi/2$  , *A* = { $e_1, e_2, ..., e_k$ } be the set of all edges which are incident to<br> *B* = { $u_1, u_2, ..., u_k$ }  $\subseteq A$  be the set of vertices which covers all the<br> *D* = { $u_1, u_2, ..., u_k$ }  $\subseteq A$  be the set of vertices which covers all the<br>
vertic *f f* (*C*). Clearly  $D$  forms a minimal Strong line<br>
(*C*). Clearly  $D$  forms a minimal Strong line<br>
set of  $L(G)$ . Therefore it implies that<br>  $D_1 \cup \{w\} \le |E(G)| \cup |S'| + 1$ . Thus<br>
(*G*)) +  $\gamma_L(G) \le q + 1 + \gamma(G)$ <br> **16:** For any *Vertices* in  $L(\theta)$ . Clearly  $V(\theta)$  forms a minimal Strong line<br>  $V_{PL}(G) = V(\theta)$   $V_{PL}(V(\theta)) = V(\theta)$   $V_{PL}(G) = V(\theta)$  Thus<br>  $V_{PL}(G) + V_{ee}(L(G)) + V_{te}(G) \le q + 1 + \gamma(G)$ <br> **Theorem 16:** For any connected  $\left(\frac{\theta}{\theta}\right)$   $\frac{\theta}{\theta}$  and<br> **Proof** 

**Theorem 17:** For any connected  $(\varphi, q)$  graph  $\mathbb{G}$ .

**Proof:** Let  $f: V(L(G)) \to \{0,1,2\}$  and partition the vertex set  $V(L(G))$  into  $(V_0, V_1, V_2)$  induced by f with  $|V_i| = n_i$  for  $i = 0, 1, 2$ . Suppose the set  $V_2$  dominates  $V_0$ . Then  $S = V_1 \cup V_2$  forms a minimal roman dominating set of  $L(G)$ . Further, let  $F = \{v_1, v_2, ..., v_k\} \subseteq V(L(G))$  be the set of vertices with  $\deg(v_i) \geq 2$ . Suppose there exists a vertex set  $D \subseteq F$  with  $N[D] = V(L(G))$  and if  $|\deg(x) - \deg(y)| \le 1 \quad \forall x \in D$  $y \in V(L(G)) - D$ . Then *D* forms a Strong line dominating set in  $L(G)$ . Otherwise there exists at least one vertex  $\{w\} \subseteq F$  where  $\{w\} \notin D$  such that  $D \cup \{w\}$  forms a minimal  $X_{SL} - set$  in  $L(G)$ . Since for any graph G there exists at least one vertex  $v \in V(G)$  of maximum degree  $\Delta(G)$ , it follows that  $|D \cup \{w\}|\cup |S| \leq p \cup |\deg(v)|$  Clearly  $X_{SL}(G) + X_{R}(L(G)) \leq p + \Delta(G)$ <sup>1</sup> *S S x <sup>G</sup>* **C** v v i  $\psi$  i *D*  $\leq k \leq i$ , Then *D* forms a edges which constitute the longest path between any two<br>  $\ln |N| \leq |(V-S) \cup S'|$ . Hence distinct vertices  $H_*/\mathbb{R}^2$   $V(G)$  with  $dS(\omega,*) = d \omega m$ <br>  $\omega = \frac{1}{2} \int_{\omega}^{\omega} \log |P(\omega)| \leq 1$ . Let  $S(\omega,$ *<sup>V</sup>* <sup>2</sup> *<sup>V</sup>*<sup>0</sup> 1 2 *S V V* Thus<br>
ected  $(p,q)$ <sub>graph</sub><br>
e the minimal set of<br>
h between any two<br>  $u, v$ ) =  $diam(G)$ . Let<br>
of vertices such that<br>
incident with the<br>
set of vertices with<br>
at  $N[D] = V(L(G))$  and<br>
rong line dominating<br>
nce  $\gamma_{SL}(G) \leq diam(G)$ .<br>  $[$  ( *F v v v V L G* 1 2 , ,..., *<sup>k</sup>* **Cheorem 16:** For any connected  $(\mathbf{p}, \mathbf{q})$ <br> **C**,  $y_{\mathbf{SL}}(G) \leq \text{diam}(G)$ <br> **Proof:** Let  $J = \{e_1, e_2, ..., e_n\} \subseteq E(G)$  be the minimal set of deges which constitute the longest path between any two list<br>
listinct vertices  $u$ **Cheorem 16:** For any connected  $\left(\frac{p}{q}, \frac{q}{q}\right)$   $\text{graph}$ <br> **Proof:** Let  $J' = \{e_1, e_2, \ldots, e_n\} \subseteq E(G)$  be the minimal set of<br>
digises which constitute the longest path between any two<br>
listinct vertices  $u, v \in V(G)$  with *G*,  $y_{2k}(G) \leq diam(G)$ <br> **Proof:** Let  $J = \{e_1, e_2, ..., e_n\} \subseteq E(G)$  be the minimal set of<br>
ddges which constitute the longest path between any two<br>
listinct vertices  $H, V \in V(G)$  with  $dist(u, v) = diam(G)$ . Let<br>  $H = \{u_1, u_2, u_3, ..., u_n\} \subseteq V[L(G)]$ **Proof:** Let  $J' = \{e_1, e_2, ..., e_n\} \subseteq E(G)$  be the minimal set of diges which constitute the longest pain between any two  $H = \{u_1, u_2, u_3, ..., u_n\} \subseteq V[L(G)]$  with  $H = \{u_1, u_2, u_3, ..., u_n\} \subseteq V[L(G)]$  be the set of vertices such that  $\{u_i\$ edges which constitute the longest path between any two<br>
distribute vertices  $u, V \in V(G)$  with  $dist(u,v) = diam(G)$ . Let<br>  $H = (u_1, u_2, u_3, \dots, u_n) \in V(G)$  when  $\epsilon$  is et of vertices such that<br>  $(u_i) = [\epsilon_i] \in E(G)$ ,  $1 \le i \le n$ , where  $[\epsilon_i]$  ar Histinct vertices  $u, v \in V(G)$  with  $dist(u, v) = diam(G)$ . Let  $H = \{u_1, u_2, u_3, \ldots, u_n\} \subseteq V[L(G)]$  be the set of vertices such that  $|u_i| = [e_i] \in E(G)$ ,  $1 \le i \le n$ , where  $[e_i]$  are incident with the vertices of  $I$ . Suppose  $D \subseteq H$  be the se *n* –  $v_4$ ,  $w_3$ ,  $w_4$ ,  $w_5$  –  $v_6$  and  $v_7$  is the set of vertices such that  $w_1 = v_4$ ,  $w_2 = 0$ ,  $v_6 = E(G)$ ,  $1 \le t \le \pi$ , where  $\Phi$  and that  $wD1 = \psi(G)$  and  $deg(w) \ge 3$  for every  $w \in D$  such that  $wD1 = \psi(G)$  and  $w_7$  *P* = *Q*<sub>*M*</sub> *D<sub><i>M*</sub> *D*<sub>*M*</sub> *DM D* 

**Theorem 18:** For any connected  $(\mathbf{p}, \mathbf{q})$  graph  $\mathbf{G}_1$  $\gamma_{SL}(G) \leq \gamma(G) + \gamma_L(G)$ .

**Proof:** Suppose  $C = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be the set of vertices with  $\deg(v_j) \geq 2$ . Then there exists a minimal set  $S_{\subseteq \mathcal{F}}$ and  $N[S] = V(G)$ . Clearly S forms a dominating set of G. Let  $C_1 = \{v_1, v_2, ..., v_n\} \subseteq V(L(G))$  be the corresponding to the set of vertices C with  $dist(u, v) \ge 3$ . Suppose there exists a set  $D_1 \subseteq C_1$ . which covers all the vertices in  $L(G)$ . Then  $D_i$  itself is a line dominating set. Further if  $dist(u, v) < 3$  and  $N[D_1] \neq V(L(G))$  then  $D = D_1 \cup \{w\}$ , where  $W \notin N[v]$ ,  $v \in D_1$  forms a minimal dominating set of  $L(G)$ . Hence  $|D| = x_L(G)$ . Let  $H = \{u_1, u_2, u_3, \dots, u_n\} \subseteq V[L(G)]$  be the set of vertices such that  $\{u_i\} = \{e_i\} \in E(G)$ ,  $1 \le i \le n$  where  $\{e_i\}$  are incident with the vertices of  $C_1$ . Suppose  $D \subseteq H$  be the set of vertices with  $deg(w) \ge 3$  for every  $w \in D$  and  $N[D] = V(L(G))$  and  $\forall v_i \in V[L(G)]$  has degree at most 2, and  $v_i \in V[L(G)] - D$ . Then  $\{D\} \cup \{v_i\}$  forms a Strong line dominating set. It follows that  $|D \cup \{v_i\}| \le |S| \cup |D|$  and hence Havnes TW ST Hedetiniemi  $\gamma_{SL}(G) \leq \gamma(G) + \gamma_L(G)$ *Muddebihal and Nawazoddin U. Patel, Strong line domination in graphs*<br> **Proof:** Suppose  $C = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(G)$  be the set of **Proof:** Let  $S = \{v_1, v_2, v_3, \ldots, \ldots, v_n\} \subseteq V(G)$  be an invertices with  $\deg(v_j) \ge 2$ . Th *Muddebihal and Nawazoddin U. Patel, Strong line domination in graphs*<br>  $\mathcal{L} = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(G)$  be the set of **Proof:** Let  $S = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(G)$  be an independent<br>  $\mathcal{L}(v_j) \geq 2$ . Then there exist *Muddebihal and Nawazoddin U. Patel, Strong line domination in graphs*<br> *L..., v<sub>n</sub>*}  $\subseteq V(G)$  be the set of **Proof:** Let  $S = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(G)$ <br>
re exists a minimal set  $\Im \subseteq \mathbb{Z}$  set of  $\mathbb{G}$ . Since  $\mathbb{G}$ *Muddebihal and Nawazoddin U. Patel, Strong line domination in graphs*<br> **Proof:** Suppose  $C = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(G)$  be the set of **Proof:** Let  $S = \{v_1, v_2, v_3, \ldots, v_n\} \subseteq V(G)$  be an independent<br>
rertices with  $\frac{deg(v_i)}$ 39787<br> **Muddebihal and Nawazoddin U. Patel, Strong line domination in graphs**<br> **Proof:** Suppose  $C = \{v_1, v_2, v_3, \dots, w_n\} \subseteq V(G)$  be the set of **Proof:** Let  $S = \{v_1, v_2, v_3, \dots, w_n\}$ <br>
vertices with  $\frac{\partial w_1}{\partial S} = V(G)$ . Clearl **19787**<br> **19787**<br> **19787**<br> **19787**<br> **19760**<br> **19760**<br> **19760**<br> **19760**<br> **19760**<br> **19760**<br> **1987**<br> **19870**<br> **19870**<br> **19870**<br> **19870**<br> **19970**<br> **19970**<br> **19971**<br> **19971**<br> **19971**<br> **19971**<br> **19971**<br> **19971**<br> **1998**<br> **1998**<br> **Muddehibal and Numezuddin U. Patd, Strong line dimination in graphs<br>**  $C = \{v_1, v_2, v_3, \ldots, \ldots, v_n\} \subseteq V(C)$  **be the set of <b>Proof:** Let  $S = \{v_1, v_2, v_3, \ldots, \ldots, v_n\} \subseteq V(C)$  be an independent<br>  $C = \{v_1, v_2, v_3, \ldots, \ldots, v_n\} \subseteq V(C$ pose  $C = \{v_1, v_2, v_3, \ldots, w_n\} \in V(G)$  be the set of **Proof.** Let  $S = \{v_1, v_2, v_3, \ldots, w_n\} \in V(G)$  be an independent  $\frac{4\pi (y_1)}{8} \ge 2$ . Then there exists a minimal set  $\delta \subseteq e$  set of  $\bar{G}$ . Since  $\bar{G}$  be these restri there exists a minimal set  $\delta \subseteq \mathbb{C}$  and solution of  $\epsilon$  cheap of  $\epsilon$  conceptoding to the set of  $\epsilon$  ( $\epsilon$ ). Then  $D_i$  $\{F_1v_1, ..., V_n\} \subseteq V(L(G))$  be the corresponding to the set of dominating set of  $\mathcal{L}_n v_1, ..., V_n$  is the expression of the vertice sins and  $P_1 \subseteq C$ ; be the minimal dominating set change the vertices in  $D_1 \neq V(L(G))$ , then  $D =$ 

**Theorem 19:** For any connected  $(p,q)$  graph G  $\gamma_{SL}(G) \leq \gamma_t(G) + \gamma(G)$ 

**Proof:** Let  $C' = \{v_1, v_2, ..., v_n\} \subseteq V(G)$  be the set of all non end vertices in G. Suppose  $C \subseteq C$  and  $\forall v_i \in V(G) - C$  are Dekker, Inc. adjacent to at least one vertex of  $C$ . Then  $C'$  forms a  $X$  - set of G. Further, let  $F = \{e_1, e_2, ..., e_k\}$  be the set of edges which are incident to the vertices of c<sup>o</sup>, and hence  $|C^{\dagger}| = x(G)$ . Let Mitchell S.L. and S.T. 1977. He  $S \subseteq C$  be the  $X_{t-}$  set of G. By the minimality for every Muddebihal H. *et al.* 2015. Strong vertex  $v \in S$ , the induced subgraph  $\langle S - v \rangle$  contains an isolated vertex. Let  $S_1 = \{v : v \in S\}$  and A be the set of isolated vertices in  $\langle S_1 \rangle$   $B = S_1 - A$ . Further let C be the minimum set of vertices of  $S - S_1$  and each vertex of A is adjacent to some vertex of C. Clearly  $|C| \le |A|$ . Suppose  $S' = S - \{S_1 \cup S_2\}$ and every  $u_i v_i \in (S')$ ,  $1 \le i \le k$ , clearly  $|S| = x$ ,  $(\langle S \rangle)$ . Then panfarosh, U. A., M. H. Muddebihal and Ar  $H = \{u_1, u_2, u_3, \dots, u_n\} \subseteq V[L(G)]$  be the set of vertices where  ${u_i} = {e_i} \in E(G)$   $1 \le i \le n$  and  ${e_i}$  are incident with the vertices of  $C$ . Further let  $D \subseteq H$  be the set of vertices with  $deg(w) \ge 3$  for every  $w \in D$  such that  $N[D] = V(L(G))$  and if  $\forall v_i \in V[L(G)]$  has degree at most 2 and  $v_i \in V[L(G)] - D$ . Then  $\{D\} \cup \{v_i\}$  forms a Strong line dominating set. Clearly it follows that  $|D \cup \{v_i\}| \le |C| \cup |S|$  and hence  $r_{st}(G) \le r_t(G) + r(G)$ . Sampathk set. Further if  $\mu(x) = \mu(x)$ , where  $W \notin N[v]$ , then  $\mu(x) = \mu(x)$ , then  $\mu(x) = \frac{1}{2}$  of  $\mu(x) = \frac{1}{2}$  forms a minimal dominating set of  $L(G)$ . Hence  $\pi_2(G) = \pi_2(G)$ .<br>
forms a minimal dominating set of  $L(G)$  Hence  $\pi_2(G) = \$ set. Function if and domination set of  $\{P_0\}$ , the set of all one and domination set of  $\{P_0\}$ , the set of  $\{P_1\}$ , the set of  $\{P_2\}$ ,  $\{P_1\}$ , the set of  $\{P_2\}$ ,  $\{P_3\}$ ,  $\{P_4\}$ ,  $\{P_5\}$ ,  $\{P_6\}$  $\mu(x) = \frac{1}{2}$  **C**  $B = \frac{1}{2}$  **EV**  $\mu(x) = \frac{1}{2}$  **C EV**  $\mu(x) = \frac{1}{2}$  1 *S S A* vertices of  $C$ . Suppose  $D' \subseteq H$  be the set of growth, No, 127-136.<br> *CA*  $\deg(w) \ge 3$  for every  $w \in D$  and L. KAMstay, 1992. Restrained domination in grap<br>  $\pi$  **C**  $D \cup \{v_1\} \le |v_1|/2$  and hence  $\pi$  **C** and  $\pi$  **C C EVALUE SET SET AND SURFAME TRANSITY (IT IS A SURFAME TO THE SET AND A SURFAME TO THE SET AND A SURFAME IN A SURFAME IN THE SURFAME** It the vertices of  $C_1$  **Exapple the set of all the set of a matrix** in the set of  $C_1$  **Exapple**  $C_2$  **i**  $C_3$  **C**  $C_4$  **i**  $C_5$  **C**  $C_6$  **i C**  $C_7$  **C i C**  $C_7$  **C i C i C i C i c i c** 1)  $\leq y_1(G) + \gamma(G)$ <br>
11. Let  $C = \{v_1, v_2, ..., v_s\} \leq v(G)$  be the set of all non end Thomass. The consistent and PJ. States in  $G$ , Suppose  $C \leq C$  and we will  $\sim C = C$  Detail with R, R, B. Janakianu and R, R, Iver, 1999. The

**Theorem 20:** For any connected  $(p,q)$  graph  $G$ ,  $\gamma_{51}(G) \leq \gamma_{9}(G)$ Where  $v_g(G)$  is a global domination number of G.

**Proof:** Let  $S = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be an independent set of G. Since G has no isolated vertices,  $v - s$  is dominating set of  $\mathcal G$ . Clearly for very vertex  $v \in \mathcal S$ ,  $(V - \mathcal S) \cup \{v\}$  is a global dominating set of  $\overline{G}$ .

Since  $|V-S\rangle\cup\{v\}|= \gamma_g(G)$  Let  $D=\{v_1,v_2,v_3,......,v_n\}\subseteq V(L(G))$ be the minimal dominating set of  $L(G)$  and  $deg(v_i) \geq 2 \forall v_i \in D$ with  $\deg(v_k) \leq 2 \forall v_k \in V[L(G)] - D$  Then D is a Strong dominating set of  $L(G)$ . It follows that  $|D| \leq |(V-S) \cup \{v\}|$  and hence  $\gamma_{SL}(G) = \gamma_g(G)$ ists a set  $D_i \subseteq C_i$  Since  $F_i \cong \{x \in \mathcal{L}: y \in \mathcal{$ g line domination in graphs<br>
Let  $S = \{v_1, v_2, v_3, \dots, v_n\} \subseteq V(G)$  be an independent<br>  $\overline{G}$ . Since  $\overline{G}$  has no isolated vertices,  $v - S$  is dominating<br>  $\overline{G}$ . Clearly for very vertex  $v \in S$ ,  $(V - S) \cup \{v\}$  is a globa

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