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RESEARCH ARTICLE

ON GREEN'S FUZZY ORTHODOX SEMIGROUPS

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ABSTRACT

In the field of research, especially in the logical algebra study of classes of regular semigroup are at most importance. Orthodox semigroups constitutes an important class of regular semigroups. An extension of these concepts was introduced by S. Madhavan on 1978 in his research article "Some results on generalized inverse semigroups" (Madhavan, 1978). A band is a semigroup in which every element is an idempotent. Inverse semigroups constitute the most important and promising classes of semigroups. Such a semigroup is certainly regular. But every regular semigroup need not be an inverse semigroup. If in a regular semigroup, any two idempotents commute, then it is an inverse semigroup. A regular semigroup in which idempotents form a semigroup is called an orthodox semigroup. In this study many results of pre algebra have extended the boarder framework of fuzzy setting. Following the formulation of Green's Fuzzy relations in the work "A study of fuzzy congruence on Green's fuzzy relations" (Hariprakash, 2016) here in this paper, the concept and results of orthodox semigroups are characterized using fuzzy properties. A general theory of logical algebra of fuzzy sets was introduced by Zadeh (Zadeh, 1965). In the paper 'A fuzzy approach to complete Upper Semilattice and complete lower semilattice" discussed the concept of fuzzy congruences relation on a semigroup (Hariprakash, 2016). During the course of this work the study concentrated in special classes of Green's fuzzy relations. It endeavours to find out a classes (quotient classes) of Green's Fuzzy Relations as a Fuzzy Orthodox Semigroup and called Green's Fuzzy Orthodox semigroup. Finally four lammas in Green's Fuzzy Orthodox semigroup are established.With the help of these lammas the study concludes by finding out a theorem stating a necessary and sufficient condition for quotient classes of regular semigroup to be a Green's Fuzzy Orthodox semigroup. In particular this work establishes Fuzzy Green's Fuzzy classes is an orthodox semigroup when the semigroup is a fuzzy congruence Green's Fuzzy antisimple orthodox semigroup.

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INTRODUCTION

Basic concepts

Definition 1.1 Idempotent in a semigroup *S*. An element $e \in S$ is said to be an idempotent in S if $e * e = e$ where $*$ is the binary operation in *S*.

Definition 1.2 Band: A semigroup in which every elements are idempotents is called a band.

Definition 1.3 Orthodox Semigroups. An orthodox semigroup *S* is defined as a regular semigroup in which the idempotents form a semigroup. In other words, the product of any two idempotents in *S* is an idempotent in *S*. In an orthodox semigroup *S*, the set Es of its idempotent is a band, since all its elements are idempotents.

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Definition 1.4 Fuzzy sets.

By a characteristic function, we can discriminate the members and non- members of a set under consideration by assigning values 0 and 1. Hence the characteristic function is a two valued function defined on a non - empty set X, which assigns values 0 or 1 to each member of the set. That is a characteristic function λ maps elements of a universal set in to $\{0,1\}$. If the function on X defines values in a particular range and indicates the membership grade of the elements, that is, larger values denote higher degrees of membership, the set defined by it is called a fuzzy set.

Definition 1.5 Fuzzy relation

In the discussion, about fuzzy relation, the association between elements of n- tuples are at different grades between 0 and 1. A fuzzy relation indicates the strength of the relation between n- tuple. The membership grade is represented by a real number in the closed interval [0,1].

Hence any function defined from × $X_{i\in N}$ to [0,1] is called a fuzzy relation on X. Fuzzy relation with valuation set [0,1] is called an ordinary fuzzy relation.

Definition 1.7. Fuzzy Greens equivalence classes

Let $\hat{\mathcal{L}}$ and $\hat{\mathcal{R}}$ are fuzzy congruences Green's Fuzzy relations on a semigroup S. For a ∈S, $\hat{\mathcal{L}}_a$ denote the fuzzy L-classes containing a and $\hat{\mathcal{R}}_a$ denote the fuzzy R - classes containing *a* .

Similarly S/$\hat{\mathcal{D}}$ and S/$\hat{\mathcal{H}}$ are defined.

More over $\hat{\mathcal{L}}_a(x) = \hat{\mathcal{L}}(a, x)$; $R_a(x) = R(a, x) \forall x \in S$

$\hat{\mathcal{L}}_a = \{(a,x) \in S \times S : \hat{\mathcal{L}}(a,x) > 0\}$

$\hat{\mathcal{R}}_a = \{(a,x) \in S \times S : \hat{\mathcal{R}}(a,x) > 0\}$

Definition 2.4 Green's fuzzy set

Let *S* be a semigroup $\hat{\mathcal{L}}$, $\hat{\mathcal{R}}$, $\hat{\mathcal{D}}$, $\hat{\mathcal{H}}$ are Green's fuzzy relations on *S*. *S* is said to be a L -fuzzy set if $L(a,b) = 1$ and *S* is a R-fuzzy set if $R(a,b) = 1$. $\forall a,b \in S$.

Similarly we define D-fuzzy set and H -fuzzy set.

Definition 1.8. Green's Fuzzy equivalence classes

If $\hat{\mathcal{L}}$ and $\hat{\mathcal{R}}$ are Greens fuzzy congruences, $\hat{\mathcal{L}}_a$ is a fuzzy equivalence $\hat{\mathcal{L}}$-class and $\hat{\mathcal{R}}_a$ is a fuzzy equivalence $\hat{\mathcal{R}}$-class and $S/\hat{\mathcal{L}} = \{\hat{\mathcal{L}}_a : a \in S\}$ and $S/\hat{\mathcal{R}} = \{\hat{\mathcal{R}}_a : a \in S\}$

Definition 2.6 Simple semigroup

A semigroup *S* is simple, if the only ideal of S is itself.

Definition 1.9. Fuzzy self simple, Fuzzy simple and Fuzzy antisimple semigroups

Let *S* be semigroup. $\hat{\mathcal{L}}$ and $\hat{\mathcal{R}}$ are $\hat{\mathcal{L}}$- Green's fuzzy congruences and $\hat{\mathcal{R}}$ -fuzzy congruences on *S*. Then

(i) *S* is $\hat{\mathcal{L}}$-fuzzy self simple if $L_a(a) = 1 \ \forall a \in S$.

(ii) *S* is $\hat{\mathcal{R}}$-fuzzy self simple if $R_a(a) = 1 \ \forall a \in S$.

(iii) *S* is $\hat{\mathcal{L}}$-fuzzy simple if $\hat{\mathcal{L}}_a(x) = 1 \ \forall a,x \in S$.

(iv) *S* is R -fuzzy simple if $\hat{\mathcal{R}}_a(x) = 1 \ \forall a,x \in S$.

(v) *S* is L-fuzzy antisimple if $L_a(x) = 1 \Rightarrow x = a$ not for all but some $a\in S$.

2. Fuzzy Orthodox Semigroups

2.1 Introduction

Orthodox semigroups constitutes an important class of regular semigroups. The main idea of this paper is to characterize some notions on orthodox semigroup using fuzzy properties.

Definition 2.2 (Fuzzy orthodox semigroups)

An orthodox semigroup (definition 1.1) in which every element has a weight is called a fuzzy orthodox semigroups.

Lemma 2.3. Let $\hat{\mathcal{L}}$ be a $\hat{\mathcal{L}}$- Green's fuzzy congruences and $\hat{\mathcal{R}}$ be a $\hat{\mathcal{R}}$ -fuzzy congruences on *S*.. If a is an idempotent in a semigroup *S*,

1. $\hat{\mathcal{L}}_a$ is an idempotent in $S/\hat{\mathcal{L}}$ and the converse is true when *S* is $\hat{\mathcal{L}}$-fuzzy anti-simple.

2. $\hat{\mathcal{R}}_a$ is an idempotent in $S/\hat{\mathcal{R}}$ and the converse is true when *S* is $\hat{\mathcal{R}}$-fuzzy antisimple.

Proof. Since a is an idempotent in *S*, $aa = a$. But $a \in S \Rightarrow \hat{\mathcal{L}}_a \in S/\hat{\mathcal{L}}$. We have $\hat{\mathcal{L}}_{a} * \hat{\mathcal{L}}_a = \hat{\mathcal{L}}_{aa} = \hat{\mathcal{L}}_{a}$. Hence $\hat{\mathcal{L}}_a$ is an idempotent in $S/\hat{\mathcal{L}}$.

Conversely, assume $\hat{\mathcal{L}}_a$ is an idempotent in $S/\hat{\mathcal{L}}$ and *S* is $\hat{\mathcal{L}}$-fuzzy

antisimple. Since $\hat{\mathcal{L}}_a$ is an idempotent in $S/\hat{\mathcal{L}}$;

$\hat{\mathcal{L}}_{a} * \hat{\mathcal{L}}_a = \hat{\mathcal{L}}_a$ Ĝ $\hat{\mathcal{L}}_{aa} = \hat{\mathcal{L}}_{a}$.

then $\hat{\mathcal{L}}_a(a) = 1 \Rightarrow \hat{\mathcal{L}}_{aa}(a) = 1$.

Since *S* is $\hat{\mathcal{L}}$-fuzzy anti-simple and

$\hat{\mathcal{L}}_{aa}(a) = 1 \Rightarrow aa = a$.

Hence, a is an idempotent in S. Similary, we can prove (b).

Theorem 2.4. If $\hat{\mathcal{L}}$ is a $\hat{\mathcal{L}}$- Green's fuzzy congruence on an $\hat{\mathcal{L}}$-fuzzy antisimple orthodox semigroup *S*, $S/\hat{\mathcal{L}}$ is an orthodox semigroup.

Proof. Given *S* is an orthodox semigroup. Let E_s be the set of idempotents in *S*. Since *S* is an orthodox semigroup, for $e, f \in S \Rightarrow ef \in S$.

Let $\hat{\mathcal{L}}_{\sim}$ and $\hat{\mathcal{L}}$ be any two idempotents in $S/\hat{\mathcal{L}}$. Then by lemma 2.4

$\alpha, \beta \in Es \Rightarrow \alpha\beta \in S$. We have $\hat{\mathcal{L}}_{\sim} * \hat{\mathcal{L}} = \hat{\mathcal{L}}_{\sim}$. Since $\alpha\beta \in Es$ by lemma 2.3, $\hat{\mathcal{L}}_{\sim}$ is an idempotent in $S/\hat{\mathcal{L}}$.

Hence the product of any two idempotents in $S/\hat{\mathcal{L}}$ is an idempotent. That is $S/\hat{\mathcal{L}}$ is an orthodox semigroup.

Here $S/\hat{\mathcal{L}}$ is called $\hat{\mathcal{L}}$-fuzzy orthodox semigroup.

The above theorem can be restated as follows.

Theorem 2.5. If $\hat{\mathcal{L}}$ is a Green's fuzzy congruence on an $\hat{\mathcal{L}}$-fuzzy antisimple orthodox semigroup *S*, $S/\hat{\mathcal{L}}$. is an $\hat{\mathcal{L}}$-fuzzy orthodox semigroup.

Theorem 2.6. lf $S/\hat{\mathcal{L}}$ is an $\hat{\mathcal{L}}$-fuzzy orthodox semigroup and S is $\hat{\mathcal{L}}$-fuzzy antisimple, *S* is a fuzzy-orthodox semigroup where $\hat{\mathcal{L}}$ is a Green's fuzzy congruence.

Proof. Let $a,b \in E(S) \Rightarrow \hat{\mathcal{L}}_{a,}\ \hat{\mathcal{L}}_{b} \in E(S/\hat{\mathcal{L}}) \Rightarrow \hat{\mathcal{L}}_{a} * \hat{\mathcal{L}}_{b} \in E(S/\hat{\mathcal{L}})$

$\Rightarrow \hat{\mathcal{L}}_{ab} \in E(S/\hat{\mathcal{L}})$ Since $S/\hat{\mathcal{L}}$ is an $\hat{\mathcal{L}}$-fuzzy orthodox semigroup,

By lemma 2.3 $ab \in E(S)$

That is, *S* is a fuzzy orthodox semigroup,

Theorem 2.8. If S is an $\hat{\mathcal{R}}$-fuzzy antisimple orthodox semigroup and $\hat{\mathcal{R}}$ is a Green's fuzzy congruence on *S*, $S/\hat{\mathcal{R}}$ is a fuzzy orthodox semigroup.

Proof. Result follows from theorem 2.5, using the property of $\hat{\mathcal{R}}$ fuzzy antisimple.

Here $S/\hat{\mathcal{R}}$, is called $\hat{\mathcal{R}}$-fuzzy orthodox semigroup.

Theorem 2.9. If $S/\hat{\mathcal{R}}$ is an $\hat{\mathcal{R}}$-fuzzy orthodox semigroup and S is $\hat{\mathcal{R}}$-fuzzy antisimple, *S* is a fuzzy orthodox semigroup where $\hat{\mathcal{R}}$ is a Green's fuzzy congruence.

Proof. Result follows from preposition 2.7 using the propert6y of $\hat{\mathcal{R}}$-fuzzy orthodox semigroup and $\hat{\mathcal{R}}$- fuzzy antisimple.

All these results can be extended to fuzzy semigroups.

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