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RESEARCH ARTICLE

RANDOM FIXED POINT THEOREMS FOR MULTI-VALUED CONTRACTION MAPPINGS IN COMPLETE METRIC SPACE

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ABSTRACT

"In this paper the concept of contraction mapping for multi-valued maps in complete metric space is introduced. The point to set map is also called multi-valued (multifunction) which guarantees the existence of fixed point generalization for some multi-valued contraction mappings such as Banach Contraction principle extended by Nadler (S. B. Nadler Jr., Multi-valued contraction mappings, pacific J. Math.30(1969)475-488), generalization of Banach and Caristi fixed point theorem for multi-valued maps was developed by many authors such as (Y. Feng, S. Liu, Fixed point theorems for multi-valued contractive mappings and multi-valued caristi type mappings, J.Math.Anal.Appl. 317(2006)103-112) and (N. Mizoguchi, W. Takahashi, Fixed point theorems for multi-valued mappings on complete metric space, J.Math.Anal.Appl.141(1989)177-188). Here we generalize N. Mizoguchi, W. Takahashi fixed point theorem and our result improves a latest result by Klim and Wadowski (D. Klim, D. Wardowski, Fixed point theorems for set valued contractions in complete metric space, J.Math.Anal.Appl.334(1)(2007)132-139).

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INTRODUCTION

The Multi-valued map first occurs in the inverse f^{-1} of a single valued map f from one set S to another set P, Kuratowski realized the importance of multi-valued maps also called set-valued maps or point to set map (multifunction). Other eminent mathematicians Hausdorff, Painleve and Bouligand have also visualized the role of multi-valued maps. It is popular among mathematicians working in the areas of game theory & economics. In 1969 Nadler (1969) extended Banach fixed point theorem (Suhas S. Patil and Dolhare, 2016) to multi-valued map which plays an important and vital role in the theory of variational inequalities, control theory, fractal geometry & differential inclusions. Since then the theory of multi-valued maps become an important part to mathematicians who are working in nonlinear analysis, Volterra integral equations, nonlinear fractional differentials equations and nonlinear integral differential equations. Lot of work has been done on multi-valued functions the Brouwer's fixed point theorem had been extended by Kakutani in 1941 (Kakutani, 1941). In 1946 Eilenberg and Montgomery generalized kakutani's result to acyclic absolute neighborhood retracts and upper semi continuous mappings.

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Preliminaries and Notations

In the first part of this article we combine the ideas of set-valued mapping using Lipchitz mapping (Suhas S. Patil and Uttam P. Dolhare, 2016) & proved some fixed point theorems in complete metric space. Also we use new multi-valued contractive conditions which are different from previous conditions. Using these conditions some random fixed point theorems for multi-valued self and non-self mappings are prove. In second part we generalize the Banach contraction principle (Suhas S. Patil and Dolhare, 2016) for single valued mappings, generalization of Nadler (1969), Feng and Liu (2006) Gordji and Ramezani (2010), and give some examples related to these conditions. Thought this paper S be a complete metric space with metric δ in complete metric space, R be the set of all real numbers and N be the set of positive integers. Let *CB(S)* be the set of all nonempty bounded closed subset of *S, Cl(S)* the class of all nonempty closed subset of S and k(S) be the set of all nonempty compact subsets of S respectively.

For any *P, Q* belongs to *CB(S)* then

$$\delta(P,Q) = \sup\{d(x,Q) : x \in P\}$$
$$H(P,Q) = \max\{\sup_{m\in q}\delta(m,P), \sup_{n\in q}\delta(n,Q)\}$$

Where $\delta(m,P)=\inf_{n\in p}\delta(m,n)$. Then the function H is a metric on *CB(S)* and is called Hausdorff metric.

The pair *(CB(S), H)* is called generalized Hausdrorff distance induced by *d*.

Example 2.1 Let *P = (1,2)* and *Q=(2,3),* where *S=R* set of all real number then

$$\delta(P,Q)=\sup_{m\in Q} d(m,P)=1 \text{ and}$$

$$\delta(Q,P)=\sup_{n\in P}\delta(n,Q)=1$$

$$\therefore\quad H(P,Q)=\max\{\delta(P,Q),\delta(Q,p)\}=1$$

Where the set distance δ is not symmetric.

Definition 2.1 A mapping $f:S\to R$ is said to be a fixed point semi-continuous if for any sequence $\{t_n\}$ in S and $t\in M$ such that $t_n\to t$ we have $f(t)\le\liminf_{n\to\infty}f(t_n)$

Hence Nadler (1969) extended the Banach contraction principle (Kamran, 2009) to point to set map in various ways. The following generalization given by N. Mizoguchi & W. Takahashi (Mizoguchi and Takahashi, 1989)

Theorem 2.1 (Mizoguchi and Takahashi, 1989**)** Let a mapping $T:S\to CB(S)$ in a complete metric space *(S, δ)* and if there exist a function γ of *(0,∞)* into *(0,1)* such that $\limsup_{r\to x^+}\gamma(r)<1$ for each $x\in[0,\infty)$ and $H(T_m,T_n)\le\gamma(d(m,n))d(m,n)$ for each $m,n\in S$, then *T* has a fixed point in *S*. The proof of above theorem 2.1 given by many authors but Suzuki (2007) give some examples which disprove the claim and prove that Mizoguchi & Takahashi's fixed point theorem is a real generalization of Nadler's fixed point theorem. In this paper we also generalize Mizoguchi & Takahashi's (1989) fixed point theorem as well as Klim and Wardowki (2007) fixed point theorem for multi-valued mappings.

Definition 2.2 (Suhas Patil and Uttam. Dolhare 2016**)** A function $f:S\to R$ is said to be lower semi-continuous, if for any $\{t_n\}$ subset of S and t belongs to S, $t_n\to t$ implies $f(t)\le\lim_{n\to\infty}f(t_n)$.

Definition 2.3 (Feng and Liu, 2006**)** Let $T:S\to N(S)$ be multi-valued mapping and is said to be upper semi-continuous, a neighborhood $J\ni T(t)$ for any $t\in S$ there is a neighborhood M of t such that $T(t_1)$ subset of J for any $t_1\in M$.

Proposition 2.1 let *(S, δ)* be a metric space and let *D, E* and *F* belongs to *CB(S)* then the following properties exists

i) $\delta(D,E)=0\Leftrightarrow D\subset C$

ii) $E\subset F\Rightarrow\delta(D,F)\le\delta(D,E)$

iii) $\delta(D\cup E,F)=\max\{\delta(D,F),\delta(E,F)\}$

iv) $\delta(D,E)\le\delta(D,F)+\delta(F,E)$

Proposition 2.2 Let *(S, δ)* be a metric space then *H* is a metric on *CB(S).*

Remark 2.1 The completeness of *(S,δ)* implies the completeness of *(CB(S), H)* and *(k(S), H).*

1.Fixed Point theorem for multi-valued contraction mappings

Definition 3.1 (Nadler.Jr., 1969**)** Let $f:S\to CB(S_1)$ is said to be multi-valued Lipchitz mapping *(m. v. l .m)* of S into S_1 in a metric space (S,δ_1) and (S,δ_2) if and only if $H(f(t),f(t_1)\le\beta\delta_1(t,t_1)$ for all $t, t_1\in S$, where $\beta\ge 0$. And the constant β is called Lipchitz constant for *f.*

Definition 3.2 (Nadler.Jr., 1969**)** IF *f* has Lipchitz constant and $\beta<1$ then *f* is said to be Multi-valued contraction mapping *(m. v. c. m).*

Remark 3.1 Multi-valued contraction mapping is continuous. Let $T:S\to N(S)$ be Multi-valued mapping then a function $f:S\to R$ as $f(t)=d(t,T(t))$ for a positive constant v, define the set $I_v^t\subset S$ where $v\in(0,1)$

$$I_v^t=\{t_1\in T(t)/v\delta(t,t_1)\le\delta(t,T(t))\}$$

Theorem 3.1 Let $T:S\to CB(S)$ be a multi-valued mapping in a complete metric space *(S, δ)* if there exist a constant $c\in(0,1)$ such that for any $t\in S$ there is $t_1\in I_v^x$ satisfying $\delta(t_1,T(t))\le\delta(t,t_1)$, then T has a fixed point in S provided $c<v$ and f is lower semi-continuous.

Theorem 3.2 (Feng and Liu, 2006**)** Let $T:S\to CB(S)$ on a complete metric space (S, δ) if there exist $m, n\in(0,1)$ such that $n<m$ and for any $t\in S$ there is $t_1\in T_t$ satisfying the following

i) $m\delta(t,t_1)\le\delta(t,T_t)$

ii) $\delta(t_1,Tt_1)\le n\delta(t,t_t)$

Then T has a fixed point in S such that the function $D(t)=\delta(t,T_t)$ is lower semi-continuous. Also D. klim and D. wardowski (Feng and Liu, 2006) extended the above theorem as follows.

Theorem 3.3 (Bonsall, 1962**)** Let $T:S\to Cl(S)$ in a complete metric space (S, δ) assuming that for each $t\in S$ there exist $t_1\in T_t$ such that $m\delta(t,t_1)\le\delta(t,T_t)$ and $\delta(t_1,Tt_1)\le\gamma(d(t,t_t))\delta(t,t_t)$ where γ is a map from $(0,\infty)$ to $(0,m)$ such that $\limsup_{r\to x^+}\gamma(r)<m$ for all $x\in[0,\infty)$ then T has a fixed point in S provided $D(t)=\delta(t,T_t)$ is lower semi-continuous.

Lemma 3.1 let *(S, δ)* be a metric space and $P,Q\in CB(S)$ then for each $p\in P$ and $\epsilon>0$ there exist an $q\in Q$ such that $\delta(p,q)\le H(P,Q)+\in$

Theorem 3.4 Let *(S, δ)* be a metric space and $\xi\ge 0$ and $T:S\to CB(S)$ be a generalized *(γ, ξ)* contraction, i.e. a mapping for which there exist a function $\gamma:[0,\infty)\to[0,1)$

satisfies $\limsup_{r\to x^+}\gamma(r)<1$ for every $x \in (0, \infty)$ such that $H(T_t, Tt_1) \le \gamma(d(t,t_t))d(t,t_t)+\xi(t_1,Tt)$ for all t, t_1 belongs to *S*, then *T* has at least one fixed point.

Proposition 3.1 Let *(S, δ)* be a metric space and *C* be any nonempty subset then let the mapping $T: C \to F(S)$ is an upper semi-continuous at $t_0 \in C$. then the mapping $\chi: C \to R^+$ defined by $\chi(t)=\delta(t,Tt)$, $t \in C$ is lower semi-continuous at t_0.

2.Generalized result of Nadler's Fixed Point Theorem

Now we introduce the class of multi-valued contraction mapping and obtain a fixed point theorem. Let *T* be a mapping from a metric space *(S, δ)* into *CB(S)* then *T* is said to be Lipchitz an if there exist a constant *k>0* such that $H(Tt, Tt_1) \le k\delta(t,t_1)$ for all $t, t_1 \in S$.

Definition 4.1 (Suhas S. Patil and Uttam P. Dolhare, 2016) A Multi-valued Lipchitz an mapping *T* is said to be contractive (Non expansive) if $k < 1$ $(k=1)$.

Here *F(T)* denote the set of fixed points of *T* there fore $F(T)=\{t \in S : t \in Tt)$

Theorem 4.1 Let *(S, δ)* is a complete metric space and $T: S \to CB(S)$ be a multi-valued contraction mapping then *T* has a fixed point in *S*.

Proof: Let k in *(0, 1)* be the Lipchitz constant of *T*, and $t_0 \in S$ and $t_1 \in Tt_0$. By result (1.1) there must exist $t_2 \in Tt_1$ such that,

$$\delta(t_1,t_2) \le H(Tt_0, Tt_1)+k$$

Similarly there exist $t_3 \in Tt_2$, such that

$$\delta(t_2,t_3) \le H(Tt_1, Tt_2)+k^2$$

Hence there exist a sequence $\{t_n\}$ in S such that $t_{n+1} \in Tt_n$ and $d(t_n,t_{n+1}) \le H(Tt_{n-1}, Tt_n)+k^n$ for all $n \in N$.

Then for each $n \in N$, $t_{n+1} \in Tt_n$ and so we have,

$$\begin{aligned} d(t_n,t_{n+1}) &\le H(Tt_{n-1}, Tt_n)+k^n \\ &\le k\delta(t_{n-1}, t_n)+k^n \\ &\le k[k\delta(t_{n-2}, t_{n-1})+k^{n-1}]+k^n \\ &\le k^2\delta(t_{n-2}, t_{n-1})+2k^n \\ &\dots\dots \\ &\dots\dots \\ &\le k^n\delta(t_0, t_1)+nk^n \end{aligned}$$

Hence $\sum_{n=0}^{\infty} k^n < \infty$ and $\sum_{n=0}^{\infty} nk^n < \infty$,

We have, $\sum_{n=0}^{\infty}\delta(t_n,t_{n+1}) \le \delta(t_0, t_1)\sum_{n=0}^{\infty}k^n+\sum_{n=0}^{\infty}nk^n<\infty$

Hence $\{t_n\}$ is a Cauchy sequence.

By completeness of X, there exists $v \in S$, such that $\lim_{n\to\infty} t_n = v$

Again by the continuity of T, $\lim_{n\to\infty} H(Tt_n, Tv)=0$ since $t_{n+1} \in Tt_n$

$\therefore \lim_{n\to\infty}\delta(t_{n+1}, Tv)=0$

Which implies that $\delta(v,Tv)=0$. Since T_v is closed.

$\therefore v \in T_v$.

Remark 4.1 (Ciric, 2008) The fixed point of multi-valued contraction mapping is not necessarily unique.

The above remark proved in following examples

Examples 4.1 Let $S = (0,1)$ and $f:[0,1]\to[0,1]$ be a map such that

$$f(t)=\{\tfrac{t}{2}+\tfrac{1}{2}, 0 \le t \le \tfrac{1}{2} \quad \text{And} \quad f(t)=\{-\tfrac{t}{2}+1, \tfrac{1}{2} \le t \le 1$$

Let us define $T: S \to 2^t$ by $Tt=\{f(t)\}\cup\{0\}$, $t \in S$.

Then T is a multi-valued contraction mapping with $F(T)=\{0, \tfrac{2}{3}\}$

Proposition 4.1 Let $A, T: S \to CB(S)$ be two contraction mappings in a complete metric space *(S, δ)* then each Lipchitz constant $k < 1$.

$H(At, At_1) \le k\delta(t,t_1)$ And $H(Tt, Tt_1) \le k\delta(t,t_1)$ for all $t, t_1 \in S$

Then $H(F(A), F(T)) \le (1-k)^{-1}\sup_{t\in S} H(At, Tt_1)$

Theorem 4.2 Let $f, g: S \to Cl(S)$ be a mapping in a complete metric space (S, δ). If there exist a constant v ∈ (0, 1) be a non negative real numbers β & μ such that $v+2\beta+2\mu<1$ and a function $\alpha: S \to [0,b)$ such that *f*, *g* satisfy the conditions,

$$\delta(t_1, gt_1) \le \alpha(\delta(t,t_1))d(t,t_1)+\beta\{\delta(t,ft)+d(t,gt_1)\}+\mu\delta(t,gt_1)$$

For all $t_1 \in I_v^t$

$$\delta(t_1, ft_1) \le \alpha(\delta(t,t_1))d(t,t_1)+\beta\{\delta(t,ft)+d(t,gt_1)\}+\mu\delta(t,ft_1)$$

For all $t \in gt_1$

Where, $I_v^t=\{t_1 \in ft : v\delta(t,t_1) \le \delta(t,ft)\}$.

Then f, g have a common fixed point provided $I(t)=\delta(t,ft)$ is lower semi-continuous.

Theorem 4.3 Let $T: S \to CB(S)$ in a complete metric space *(S, δ)* such that,

$$H(Tt, Tt_1) \le \alpha\delta(t,t_1)+\beta[d(t,Tt)+d(t_1,Tt_1)]+\mu[d(t,Tt_1)+d(t_1,Tt)$$

For all *t, t1* $\in S$, where *α, β & μ* greater than or equal to zero and $\alpha+2\beta+2\mu<1$, then *T* has a fixed point.

Proof: Let $t_0 \in S$, $t_1 \in Tt_0$ and define $r=\dfrac{\alpha+\beta+\mu}{1-(\beta+\mu)}$, then if $r=0$ the proof is over.

Now assume $r > 0$ then we get,

$\delta(t_1,t_2) \le H(Tt_0, Tt_1)+r$, there exist $t_2 \in Tt_1$

$\delta(t_2,t_3) \le H(Tt_1,Tt_2)+r^2$, there exist $t_3 \in Tt_2$

$\delta(t_3,t_4) \le H(Tt_2,Tt_3)+r^3$, there exist $t_4 \in Tt_3$

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$\delta(t_n,t_{n+1}) \le H(Tt_{n-1},Tt_n)+r^n$, there exist $t_{n+1} \in Tt_n$

Hence we get,

$$\delta(t_n,t_{n+1}) \le H(Tt_{n-1},Tt_n)+r^n$$

$$\le \alpha d(t_{n-1},t_n)+\beta[d(t_n,Tt_n)+d(t_{n-1},Tt_{n-1})]+\mu[d(t_n,Tt_{n-1})+d(t_{n-1},Tt_n)]+r^n \forall n \in N$$

$$\le \alpha\delta(t_{n-1},t_n)+\beta[\delta(t_n,t_{n+1})+\delta(t_{n-1},t_n)]+\mu[\delta(t_{n-1},t_n)+\delta(t_n,t_{n+1})]+r^n \forall n \in N$$

.

Then we get

$$\delta(t_n,t_{n+1}) \le r\delta(t_{n-1},t_n)+\frac{r^n}{1-(\beta+\mu)}$$

Then it can be conclude that

$$\delta(t_n,t_{n+1}) \le r^n\delta(t_0,t_1)+\frac{nr^n}{1-(\beta+\mu)} \forall n \in N$$

Since $r<1$ and $\sum_{n=1}^{\infty}\delta(t_n,t_{n+1})<\infty$

It follows that $\{t_n\}$ is a Cauchy sequence in S,

By completeness of S there exist $t' \in S$ such that $\lim_{n\to\infty} t_n = t'$

we have to show that t' is a fixed point of T.

$\therefore$

$$d(t',Tt') \le \delta(t',t_{n+1})+d(t_{n+1},Tt') \le \delta(t',t_{n+1})+H(Tt_n,Tt')$$

$$\le \delta(t',t_{n+1})+\alpha\delta(t_n,t')+\beta[d(t_n,Tt_n)+d(t',Tt')]+\mu[d(t_n,Tt')+d(t',Tt_n)] \forall n \in N$$

$\therefore$

$$d(t',Tt') \le d(t',t_{n+1})+\alpha d(t_n,t')+\beta[d(t_n,t_{n+1})+d(t',Tt')]+\mu[d(t_n,Tt')+d(t_{n+1},t')]$$

For all $n \in N$, the limit $n \to \infty$ then we have,

$d(t',Tt') \le (\beta+\mu)d(t',Tt')$

On the other hand $\beta+\mu<1$ then $d(t',Tt')=0$

$\Rightarrow t' \in Tt'$

Which show that t' is a fixed point of T.

Corollary 4.1 Let (S, δ) complete metric space and let T be a mapping from S into

(CB(S),H) Satisfies,

$$H(Tt,Tt_1) \le b_1\delta(t,t_1)+b_2\delta(t,Tt)+b_3\delta(t_1,Tt_1)+b_4\delta(t,Tt_1)+b_5\delta(t_1,Tt) \forall t,t_1 \in S$$

Where $b_i \ge 0$ for each $i \in \{1,2,3,......5\}$

and $\sum_{i=1}^{5} b_i < 1$ then T has a fixed point.

Corollary 4.2 Let the two mapping $f,g: S \to Cl(S)$ in a complete metric space S with metric δ, If there exist a constant $v \in (0,1)$ and $\psi: S \to [0,v)$ and f,g satisfy the following conditions $\delta(t_1,gt_1) \le \psi(\delta(t,t_1))d(t,t_1)$ then for all $t_1 \in I_v^t$ where

$I_v^t = \{t_1 \in ft : v\delta(t,t_1) \le \delta(t,ft)\}$ and

$\delta(t,ft) \le \psi\delta(t,t_1)\delta(t,t_1)$ for all $t \in gt$

Then f, g have a common fixed point provided $d(t)=\delta(t,ft)$ is lower semi-continuous.

Corollary 4.3 (Nadler.Jr. 1969) Let T be a mapping from *(S, δ)* into *(CB(S),H)* in a complete metric spaces satisfies $H(Tt,Tt_1) \le \mu\delta(t,t_1)$ for all $t,t_1 \in S$ where $0 \le \mu < 1$, then T has a fixed point.

Theorem 4.4 Let $f,g: S \to Cl(S)$ be a mapping in a metric space *(S, δ)* and $\mu:[0,\infty)\to[0,1)$ be a function such that $\limsup_{x\to r^+}\gamma(x)<1$ for all $r \in (0,\infty)$ and f, g satisfy the following condition

$\delta(t,gt) \le \mu(\delta(t,t_1))\delta(t,t_1)$ - (1) for all $t \in ft$ and

$\delta(t,ft) \le \mu(\delta(t,t_1))\delta(t,t_1) \forall t \in gt_1$ - (2)

Then f, g have a common fixed point provided $d(t)=\delta(t,ft)$ is lower semi-continuous.

Proof: Let us define $\alpha:[0,\infty)\to[0,1)$ such that $\alpha(x)=\frac{1+\mu(x)}{2}$

Then,

$$\limsup_{r\to x^+}\alpha(r)<1 \quad \mu(x)<\alpha(x)<1 \forall x \in [0,\infty)$$

Let $t_0 \in S$ and $t_1 \in ft_0$ we get from first condition,

$\delta(t_1,gt_1) \le \mu(\delta(t_0,t_1))\delta(t_0,t_1)$ so there exist $t_2 \in gt_1$ such that

$\delta(t_1,t_2) \le \alpha(\delta(t_0,t_1))\delta(t_0,t_1)$ Since $t_2 \in gt_1$ hence from second condition we have,

$\delta(t_2,ft_2) \le \mu(\delta(t_0,t_1))\delta(t_0,t_1)$ Hence continuing this process we can choose an iterative sequence $\{t_n\}_{n=1}^{\infty}$ such that $t_{2n} \in gt_{2n-1}$ and $t_{2n+1} \in ft_{2n}$ and $\delta(t_{n+1},t_{n+2}) \le \alpha(\delta(t_n,t_{n+1}))\delta(t_n,t_{n+1})$.

Let $\alpha(t)<1$ for all $x \in [0,\infty)$ so $\{\delta(t_n,t_{n+1})\}_{n=0}^{\infty}$ is decreasing sequence in R and must converge to some non negative real number $\delta \in R$.

Since $\limsup_{x\to\delta^+}\alpha(x)<1$ choose $r \in (0,1)$ and $v \in N$ such that $\alpha(\delta(t_n,t_{n+1}))<r$ for every $n \ge v$.

Hence,

$$\sum_{n=1}^{\infty}(\delta(t_n,t_{n+1})) \le \sum_{n=1}^{v}(\delta(t_n,t_{n+1}))+\sum_{n=v+1}^{\infty} r^n(\delta(t_1,t_0))<\infty$$

$\therefore$ $\{t_n\}$ is a Cauchy sequence and S is complete, then it converges to some y ∈ S and $t_{2n+1} \in ft_{2n}$.

$$\therefore \delta(y,fy) \le \liminf_{n\to\infty}\delta(t_{2n},ft_{2n})$$

$$\le \liminf_{n\to\infty}(\delta(t_{2n},t_{2n+1})+\delta(t_{2n+1},ft_{2n}))$$

$= 0$

Hence $y \in f_y$ and by putting $t = t_1 = y$ in first condition we get,

$$\delta(y, gy) \le \mu(\delta(y, y))\delta(y, y)$$

$$\therefore\ y \in gy$$

This completes the proof.

Conclusion

In present paper we see some common random fixed point theorems for multi-valued contraction mapping in complete metric space.

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