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RESEARCH ARTICLE

AN INTUITIONISTIC FUZZY CRITICAL PATH PROBLEM USING RANKING METHOD

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ABSTRACT

In this paper, an algorithm is presented to perform critical path analysis in an intuitionistic fuzzy environment. Here, the arc lengths are assigned triangular intuitionistic fuzzy numbers. Ranking procedures are applied on intuitionistic fuzzy numbers to find the critical path and an illustrative example is given to demonstrate the proposed method.

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INTRODUCTION

Network diagram plays a vital role to determine project completion time. Normally, a project will consists of a number of activities and some activities can be started only after finishing some other activities. There may be activities which are independent of some activities. Here, a new approach to find critical path is introduced. The base idea behind this method is the concept of using TrIFN in imprecise conditions. Fuzzy set was introduced by Zadeh in 1965. Ross has published an interesting book of fuzzy set theory and its applications in engineering. Several papers also published on applications of fuzzy sets in engineering. De and Amita Bhinchar [2010] developed an algorithm based on ranking method. In this paper, we present a method for finding critical path in an intuitionistic fuzzy project network. This paper is organized as follows. In section 2, some elementary concept and definitions are given. The procedure for finding CPM using TrIFN derived in section 3. An illustrative example is provided in section 4 to check the feasibility of the method. The last section draws some concluding remarks.

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Prerequisites

Intuitionistic Fuzzy Set: An Intuitionistic fuzzy set $\widetilde{A}^1$ in X is given by a set of ordered triples: $\widetilde{A}^1 = \{(x, \mu_{\widetilde{A}^1}(x), \nu_{\widetilde{A}^1}(x)) / x \in X\}$, where $\mu_{\widetilde{A}^1}, \nu_{\widetilde{A}^1} : X \to [0,1]$ are functions such that $0 \leq \mu_{\widetilde{A}^1}(x) + \nu_{\widetilde{A}^1}(x) \leq 1$ for all $x \in X$. For each x the numbers $\mu_{\widetilde{A}^1}(x) and \nu_{\widetilde{A}^1}(x)$ represent the degree of membership and degree of non-membership of the element $x \in X$ to $A \subset X$, respectively.

Intuitionistic Fuzzy Number (IFN)

An intuitionistic fuzzy subset $\widetilde{A}^1 = \{(x, \mu_{\widetilde{A}^1}(x), \nu_{\widetilde{A}^1}(x)) / x \in X\}$, of real line R is called an Intuitionistic Fuzzy Number (IFN) if the following holds:

(i) There exists $m \in R$, $\mu_{\widetilde{A}^1}(m) = 1 and \nu_{\widetilde{A}^1}(m) = 0$

(ii) μ_A is a continuous mapping from R to the closed interval [0,1] and for all $x \in R$, the relation $0 \leq \mu_{\widetilde{A}^1}(x) + \nu_{\widetilde{A}^1}(x) \leq 1$ holds.

The membership and non-membership function of $\widetilde{A}^1$ is of the following form:

$$\mu_{\tilde{A}^1}(x) = \begin{cases} 0, -\infty < x \le m-\alpha \\ f_1(x), x \in [m-\alpha, m] \\ 1, x = m \\ h_1(x), x \in [m, m+\beta] \\ 0, m+\beta \le x < \infty \end{cases}$$

where $f_1(x)$ and $h_1(x)$ are strictly increasing and decreasing function

$[m-\alpha, m] and [m, m+\beta]$ respectively.

$$v_{\tilde{A}^1}(x) = \begin{cases} 1, -\infty < x \le m-\alpha^1 \\ f_2(x), x \in [m-\alpha^1, m]; 0 \le f_1(x) + f_2(x) \le 1 \\ 0, x = m \\ h_{12}(x), x \in [m, m+\beta^1]; 0 \le h_1(x) + h_2(x) \le 1 \\ 0, m+\beta^1 \le x < \infty \end{cases}$$

Here m is the mean value of $\tilde{A}^1$. α, β are called left and right spreads of membership function $\mu_{\tilde{A}^1}$ (x) respectively. α^1, β^1 represents left and right spreads of non membership function $v_{\tilde{A}^1}(x)$ respectively.

Triangular Intuitionistic Fuzzy Number (TrIFN)

A (TIFN) $\tilde{A}^{1\ 1}$ is an intuitionistic fuzzy set in R with the following $\mu_{\tilde{A}^1}(x) and v_{\tilde{A}^1}(x)$

$$\mu_{\tilde{A}^1}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$$

$$v_{\tilde{A}^1}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1^1}, a_1^1 \le x \le a_2 \\ \frac{x-a_2}{a_3^1-a_2}, a_2 \le x \le a_3^1 \\ 1, otherwise \end{cases}$$

where $a_1^1 \le a_1 \le a_2 \le a_3 \le a_3^1$ and $\mu_{\tilde{A}^1}(x) + v_{\tilde{A}^1}(x) \le 1$

Arithmetic Operations of two TrIFNs

The additions of two TrIFN are as follows.

For two triangular intuitionistic fuzzy numbers

A= $(\langle a_1, b_1, c_1 \rangle : \mu_A, \langle e_1, f_1, g_1 \rangle : \gamma_A)$ and B= $(\langle a_2, b_2, c_2 \rangle : \mu_B, \langle e_2, f_2, g_2 \rangle : \gamma_B)$ with $\mu_A \ne \mu_B$ and $\gamma_A \ne \gamma_B$, define

A+B =
$$(\langle a_1+a_2, b_1+b_2, c_1+c_2 \rangle : Min(\mu_A, \mu_B), \langle e_1+e_2, f_1+f_2, g_1+g_2 \rangle : Max(\gamma_A, \gamma_B))$$

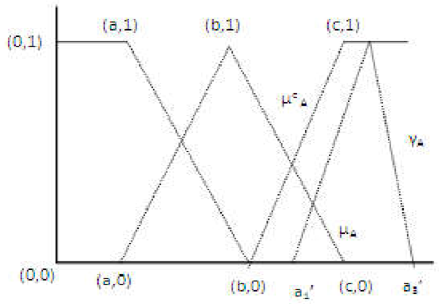

Figure 1 Triangular Intuitionstic fuzzy number

Forward pass calculations

Forward pass calculations are employed to calculate the Triangular Intuitionistic

Earliest Start $TrIFES_j$ in the project network.

$TrIFES_j$ = Max {$TrIFES_i \oplus TrIFt_{ij}$ }, i = number of preceding nodes.

$TrIFES_1 = TrIFLF_1 = 0$

Triangular Intuitionistic Fuzzy Earliest Finish TrIFEF in the project network.

$TrIFEF_j = TrIFES_i \oplus$ Fuzzy activity time

Backward pass calculations

Backward pass calculations are employed to calculate the Triangular Intuitionistic Fuzzy

Latest Finish $TrIFES_j$ in the project network.

$TrIFLF_i$ = Min {$TrIFLF_j \ \Theta \ TrIFt_{ij}$ }, j = number of succeeding nodes.

$TrIFLF_n = TrIFES_n$

Triangular Intuitionistic Fuzzy Latest Start TrIFEF in the project network.

TrIFLS = TrIFLF Θ Fuzzy activity time

Triangular Intuitionistic Fuzzy Total Float (TrIFTF)

TrIFTF = TrIFLF – TrIFEF (or) TrIFTF = TrIFLS – TrIFES

Mean and Centroid Index

$TrIFN_{mean} = \frac{A+2C-B}{2}$, $TrIFN_{centroid} = \frac{A+3C-B}{3}$,

Where $A = a_1+a_2$; $B = a_2+b_2$; $C = a_3+b_3$

Ranking Function

A ranking function $\Re : F(R) \rightarrow R$, where F(R) is the set of all fuzzy numbers defined on set of real numbers, maps each fuzzy number into a real number. Let A and B be the two TrIFN, then

(i) $\widetilde{A} > \widetilde{B}$ if $\Re(\widetilde{A}) > \Re(\widetilde{B})$

(ii) $\widetilde{A} < \widetilde{B}$ if $\Re(\widetilde{A}) < \Re(\widetilde{B})$

(iii) $\widetilde{A} = \widetilde{B}$ if $\Re(\widetilde{A}) = \Re(\widetilde{B})$

For a TrIFN $A = (\langle a_1, b_1, c_1 \rangle, \langle a_2, b_2, c_2 \rangle)$, Ranking function is given by R(A) = $a_1 - ¼ (b_1 - c_1) + a_2 - ¼ (b_2 - c_2)$

Notations

- $TrIFES_j$: Triangular Intuitionistic Fuzzy Earliest Start
- $TrIFt_{ij}$: Triangular Intuitionistic Fuzzy activity time of nodes i and j
- TrIFLS : Triangular Intuitionistic Fuzzy Latest Start
- TrIFLF : Triangular Intuitionistic Fuzzy Latest Finish
- TrIFEF : Triangular Intuitionistic Fuzzy Earliest Finish
- TrIFCP : Triangular Intuitionistic Fuzzy Completion time of Path

Procedure

Step 1: Construct a network G(V,E) where V is the set of vertices and E is the set of edges. Arc lengths or edges are taken as TrIFN.

Step 2 : Calculate Triangular Intuitionistic Fuzzy Earliest Start TrIFES according to (2.5)

Step 3 : Calculate Triangular Intuitionistic Fuzzy Latest Finish TrIFLF according to (2.6)

Step 4 : Calculate Triangular Intuitionistic Fuzzy Total Float TrIFTF according to (2.7)

Step 5 : Find all the possible paths and calculate TrIFCP in a project network

Step 6: Find the ranking value of of $TrIFCP(P_i)$, i = 1,2,3,4 and compute the critical path.

Numerical Example

Intuitionistic Fuzzy Project Network

Step 1:

Table 1. Activities and their Intuitionistic Fuzzy durations

Activity	Intuitionistic Fuzzy Activity Time
1-2	$(\langle 2,3,4 \rangle, \langle 5,6,7 \rangle)$
1-3	$(\langle 4,5,6 \rangle, \langle 7,8,9 \rangle)$
1-4	$(\langle 10,11,12 \rangle, \langle 13,14,15 \rangle)$
2-5	$(\langle 3,5,6 \rangle, \langle 8,9,10 \rangle)$
3-5	$(\langle 2,5,7 \rangle, \langle 6,7,8 \rangle)$
3-6	$(\langle 3,4,5 \rangle, \langle 5,7,9 \rangle)$
4-6	$(\langle 12,13,14 \rangle, \langle 15,16,17 \rangle)$
5-7	$(\langle 6,7,8 \rangle, \langle 9,10,11 \rangle)$
6-7	$(\langle 7,8,9 \rangle, \langle 10,11,12 \rangle)$

Step 2: To calculate TrIFES

Set $TrIFES_1 = (\langle 0,0,0 \rangle, \langle 0,0,0 \rangle)$

Calculate $TrIFES_j$, j = 2,3,4,5,6,7 by using (2.5)

$TrIFES_2 = (\langle 2,3,4 \rangle, \langle 5,6,7 \rangle); TrIFES_3 = (\langle 4,5,6 \rangle, \langle 7,8,9 \rangle);$

$TrIFES_4 = (\langle 10,11,12 \rangle, \langle 13,14,15 \rangle)$

$TrIFES_5 = (\langle 6,10,13 \rangle, \langle 13,15,17 \rangle); TrIFES_6 = (\langle 22,24,26 \rangle, \langle 28,30,32, \rangle);$

$TrIFES_7 = (\langle 29,32,35 \rangle, \langle 38,41,44 \rangle)$

Step 3: To calculate TrIFLF

Set $TrIFLF_7 = TrIFES_7$

Calculate $TrIFLF_j$, j = 6,5,4,3,2,1 by using (2.6)

$TrIFLF_6 = (\langle 22,24,26 \rangle, \langle 28,30,32 \rangle); TrIFLF_5 = (\langle 23,25,27 \rangle, \langle 29,31,33 \rangle);$

$TrIFLF_4 = (\langle 10,11,12 \rangle, \langle 13,14,15 \rangle)$

$TrIFLF_3 = (\langle 19,20,21 \rangle, \langle 23,23,23 \rangle);$

$TrIFES_2 = (\langle 20,20,22 \rangle, \langle 21,22,23, \rangle); TrIFES_1 = (\langle 0,0,0 \rangle, \langle 0,0,0 \rangle)$

Step 4: To calculate TrIFTF

$TrIFTF_{12} = (\langle 18,17,18 \rangle, \langle 16,16,16 \rangle); TrIFFT_{13} = (\langle 15,15,15 \rangle, \langle 16,15,14 \rangle);$

$TrIFFT_{14} = (\langle 0,0,0 \rangle, \langle 0,0,0 \rangle);$

$TrIFTF_{25} = (\langle 18,17,17 \rangle, \langle 16,16,16 \rangle); TrIFFT_{35} = (\langle 17,15,14 \rangle, \langle 16,16,16 \rangle);$

$TrIFFT_{36} = (\langle 15,15,15 \rangle, \langle 16,15,14 \rangle);$

$TrIFTF_{46} = (\langle 0,0,0 \rangle, \langle 0,0,0 \rangle); TrIFFT_{57} = (\langle 17,15,14 \rangle, \langle 16,16,16 \rangle);$

$TrIFFT_{67} = (\langle 0,0,0 \rangle, \langle 0,0,0 \rangle);$

Step 5: Find all the possible paths and calculate TrIFCP in a project network.

P = { (1,2,5,7), (1,3,5,7), (1,3,6,7), (1,4,6,7)}

$TrIFCP(P_1) = (\langle 53,49,50 \rangle, \langle 48,48,48 \rangle); TrIFCP(P_2) = (\langle 49,45,43 \rangle, \langle 48,47,46 \rangle);$

$TrIFCP(P_3) = (\langle 30,30,30 \rangle, \langle 32,30,28 \rangle); TrIFCP(P_4) = (\langle 0,0,0 \rangle, \langle 0,0,0 \rangle);$

Step 6:

To obtain the critical path using the ranking function (2.9),
R[$TrIFN(P_1)$] = 101.25 ; R[$TrIFN(P_2)$] = 96.25;
R[$TrIFN(P_3)$] = 61.5 ; R[$TrIFN(P_4)$] = 0
Since $R(P_4) < R(P_3) < R(P_2) < R(P_1)$, I
ntuitionistic fuzzy critical path is **1-4-6-7**

Table 2. Results of the network based on Ranking function

Paths	TrIFTF	Ranking function based on (2.9)	Rank
P_1	$(\langle 53,49,50 \rangle, \langle 48,48,48 \rangle)$	101.25	4
P_2	$(\langle 49,45,43 \rangle, \langle 48,47,46 \rangle)$	96.25	3
P_3	$(\langle 30,30,30 \rangle, \langle 32,30,28 \rangle)$	61.5	2
P_4	$(\langle 0,0,0 \rangle, \langle 0,0,0 \rangle)$	0	1

Table 3. Results of the network based on Mean and Centroid Index

Paths	Mean Index	Centroid Index	Rank
P_1	100	99.3	4
P_2	91.5	90.6	3
P_3	59	58.6	2
P_4	0	0	1

Conclusion

In this paper, an algorithm is developed for solving critical path on a network with Intuitionistic fuzzy arc lengths. The proposed method can be used to find critical path of a project network that exists in real life situation. Critical Path Method is commonly used with all forms of projects, including construction, software development, research projects, project development, engineering and plant maintenance. Fuzzy models are more effective in determining the critical path in a real project network.

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