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RESEARCH ARTICLE

EDGE DEGREE SET OF A FUZZY GRAPH

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ABSTRACT

In this paper edge degree set of a fuzzy graph is introduced and some of its properties are studied. It is proved that any set of real numbers can be a degree set of some fuzzy graph.

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INTRODUCTION

The phenomena of uncertainty in real life situation were described in a mathematical framework by Zadeh in 1965. Fuzzy graph is the generalization of the ordinary graph. Therefore it is natural that though fuzzy graph inherits many properties similar to those of ordinary graph, it deviates at many places. A. Nagoor gani and K. Radha introduced incidence sequence of a fuzzy graph in [6]. K. Radha and A. Rosemine introduced degree sequence of fuzzy graph in [9] and discussed degree set in [10]. In this paper we defined the edge degree set of a fuzzy graph and also we proved that any set of two positive real numbers can be realized by a fuzzy graph.

Preliminaries

A summary of basic definitions is given, which can be found in [1] - [10]. A fuzzy graph G is a pair of functions G: (σ, μ) where σ is a fuzzy subset of a non empty set V and μ is a symmetric fuzzy relation on σ (i.e.) $\mu(xy) \leq \sigma(x) \wedge \sigma(y) \forall x, y \in V$ .The underlying crisp graph of G: (σ, μ) is denote by G*: (V, E) where $E \subseteq V \times V$

In a fuzzy graph G: (σ, μ) degree of vertex μ∈V is d (u) $=\sum_{u\neq v}\mu(uv)$, The minimum degree of G is $\delta(G) = \wedge \{ d_G(u) / u \in V\}$, the maximum degree of G is $\Delta(G)= \vee \{d_G(u) / u \in V\}$.

A sequence of real numbers $(d_1,d_2,d_3,\ldots,d_n)$ with $d_1 \geq d_2 \geq d_3 \geq \ldots\ldots\ldots\ldots \geq d_n$, where d_i is equal to $d(v_i)$, is the degree sequence of a fuzzy graph G. A sequence S= $(d_1,d_2,d_3,\ldots,d_n)$of real numbers is said to be fuzzy graphic sequence if there exists a graph G whose vertices have degree d_i and G is called realization of S.A degree sequence of real numbers in which no two of its elements are equal is called perfect degree sequence. In crisp graph theory there is no perfect degree sequence. But fuzzy graphs may have perfect degree sequence. A degree sequence of real numbers in which exactly two of its elements are same is called quasi- perfect. A homomorphism of fuzzy graphs h: G→ G' is a map h: V → V' such that σ(x) ≤ σ'(h(x) ∀ x∈ V, μ(xy) ≤ μ'(h(x) h(y)) ∀ x ,y ∈ V. A weak isomorphism of fuzzy graphs h: G→ G' is a map
h:V → V' which is a bijective homomorphism that satisfies σ(x) = σ'(h(x) ∀ x∈ V,
μ(xy) ≤ μ'(h(x) h(y)) ∀ x,y ∈ S. A co-weak isomorphism of fuzzy graphs h: G→ G' is a map
h: V → V' which is a bijective homomorphism that satisfies σ(x) ≤ σ'(h(x) ∀ x∈ V,
μ(xy) = μ'(h(x) h(y)) ∀ x,y ∈ S.

An isomorphism h: G→ G' is a map h: V → V' which is a bijective that satisfies

$\sigma(x) = \sigma'(h(x)\ \forall x \in V$, $\mu(xy) = \mu'(h(x)\, h(y))\ \forall x,y \in S$. . The union of two fuzzy graphs

G_1: (σ_1, μ_1) and G_2 :(σ_2, μ_2) is G_1 defined to be a fuzzy graph $G = G_1 \cup G_2$:($\sigma_1 \cup \sigma_2$, $\mu_1 \cup \mu_2$) where $V=V_1\cup V_2$ and $E = E_1\cup E_2$ with

$$\sigma_1\cup\sigma_2(u) = \begin{cases} \sigma_1(u) & if\ u \epsilon V_1 - V_2 \\ \sigma_2(u) & if\ u \epsilon V_2 - V_1 \\ \sigma_1(u) \vee \sigma_2(u) & if\ u \epsilon V_1 \cap V_2 \end{cases}$$

and $$\mu_1\cup\mu_2(e) = \begin{cases} \mu_1(e) & if\ e \epsilon E_1 - E_2 \\ \mu_2(e) & if\ e \epsilon E_2 - E_1 \\ \mu_1(e) \vee \mu_2(e) & if\ e \epsilon E_1 \cap E_2 \end{cases}$$

The join of two fuzzy graphs G_1: (σ_1, μ_1) andG_2 :(σ_2, μ_2) is G_1 defined to be a fuzzy graph

$G_1 + G_2$:($\sigma_1 + \sigma_2$, $\mu_1 + \mu_2$) where $V=V_1\cup V_2$ and $E = E_1\cup E_2 \cup E'$ where E' is the set of all edges joining the vertices of V_1 with vertices of V_2 such that

$(\sigma_1 + \sigma_2)(u) = (\sigma_1\cup\sigma_2)(u)$ for all $u\in V_1\cup V_2$ and $$(\mu_1 + \mu_2)(u) = \begin{cases} (\mu_1 \cup \mu_2)(uv) & if\ uv \in E_1 \cup E_2 \\ \sigma_1(u) \wedge \sigma_2(u) & if\ uv \in E' \end{cases}$$

Let G and H be two graphs. The corona product G∘H is obtained by taking one copy of G and $|V(G)|$ copies of H; and by joining each vertex of the i-th copy of H to the i-th vertex of G, where $1 \leq i \leq |V(G)|$.

III. Edge Degree Set of a Fuzzy Graph

Definition 3.1

The set of distinct positive real numbers occurring in a edge degree sequence of a fuzzy graph is called its degree set.

Definition 3.2

A set of positive real numbers is called a edge degree set if it is the edge degree set of some fuzzy graph. The fuzzy graph is said to realize the edge degree set.

Example3.3

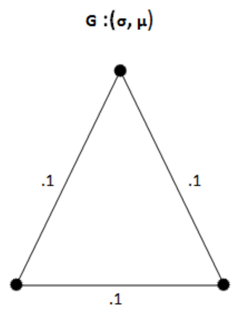


Fig. 1.

(0.2, 0.2, 0.2) is the edge degree sequence of G in Fig.1 and the corresponding edge degree set is {0.2}.

Theorem 3.4:

Let G: (σ, μ) be a regular fuzzy graph on a cycle. Then $|S| \leq 2$.

Proof:

Consider a regular fuzzy graph G:(σ , μ) on a cycle $v_1e_1v_2e_2v_3....v_{n-1}e_{n-1}v_nv_1$ by its characterization either μ is a constant function or alternate edges have same value.If μ is a constant function then $|S| = 1$. If alternate edges have same value say m_1, m_2 , let $\mu(e_{2k}) = m_1$ and $\mu(e_{2k+1}) = m_2$ .Then $d(e_{2k}) = 2m_2$, $d(e_{2k+1}) = 2m_1$. Hence $|S| = 2$.
$\therefore |S| \leq 2$.

Theorem 3.5

If G and G' are isomorphic fuzzy graphs then their edge degree sets are identical.

Proof:

Since G:(σ,μ) and G':(σ',μ') are two isomorphic fuzzy graphs, there exists a bijective map h: V→ V' such that σ(x) =σ'(h(x)) ∀ x ∈V, μ(xy) = μ'(h(x) h(y)) ∀ x,y ∈ V.

Hence $d_G(u) = \sum_{u\neq v} \mu(uv) = \sum_{h(u)\neq h(v)} \mu'(h(u)h(v)) = d_{G'}(h(u))$.
Since u ∈ V is arbitrary, $d_G(u) = d_{G'}(h(u))$ ∀ u ∈ V.
Therefore $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv)$
$= d_{G'}(h(u)) + d_{G'}(h(v)) - 2\mu'(h(u)h(v))$
$= d_{G'}(h(u)h(v))$ ∀ uv ∈ E

Thus the edge degree sequence of G and G' are same and therefore their corresponding edge degree sets are identical.

Remark 3.6

The converse of theorem 3.5 need not be true. (i.e.) Two fuzzy graphs with same degree set need not be isomorphic. It can be verified by the following example.

Example 3.7

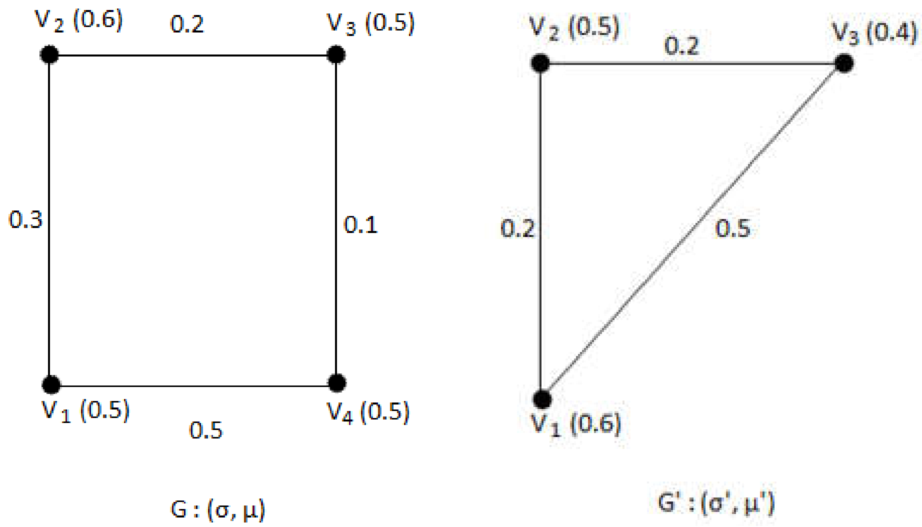


Fig. 2.

Here the degree set of both G and G' are {0.4, 0.7} but G and G' are not isomorphic.

Theorem 3.8

If G and G' are co-weak isomorphic fuzzy graphs, then their edge degree sets are identical.

Proof:

By the definition of co-weak isomorphism we have μ(uv) = μ'(h(u) h(v)) ∀ u,v∈ V. Therefore as in the proof of the theorem 3.5, G and G' have identical edge degree sequences so their edge degree sets are same.

Remark 3.9

The converse of theorem 3.8 need not be true. (i.e.) Two fuzzy graphs with same edge degree set need not be co-weak isomorphic. For example, the fuzzy graphs G and G' in Fig.2 are of same degree set but they are not co-weak isomorphic.

Remark 3.10

If G and G' are weak isomorphic fuzzy graphs, then their edge degree sets need not be identical. For example the fuzzy graphs in fig.3 are weak isomorphic to each other but their degree sets are not same.

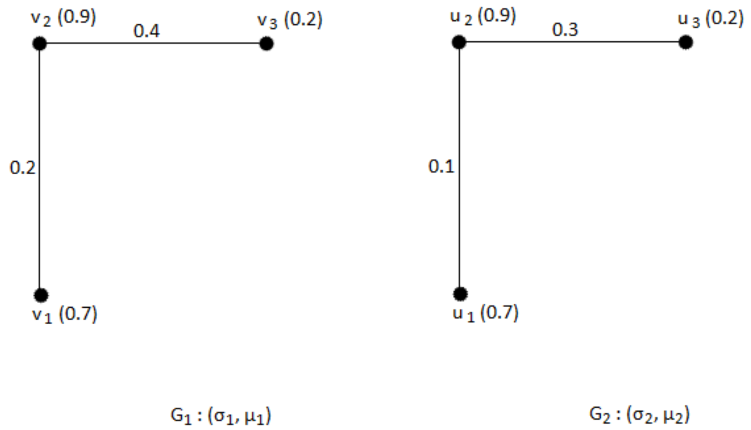


Fig. 3.

Also two fuzzy graphs with same degree set need not be weak isomorphic. For example, the fuzzy graphs G and G' in Fig.4 are of same degree set but they are not weak isomorphic.

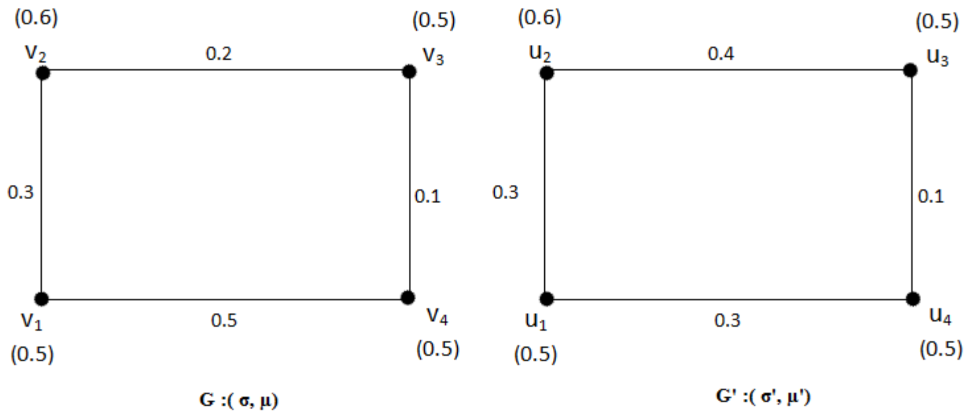


Fig. 4.

IV. Realization of any finite set of real numbers as edgedegree set

Theorem 4.1

Any singleton set of non negative real number is the edge degree set of a fuzzy graph.

Proof:

Let d be any real number and n be the least positive integer such that $(d/2n) \leq 1$.Now consider a complete graph K_{n+2}.
Assign $\mu(e) = d/2n$ for every edge e.

Then $d(uv) = d(u) + d(v) -2\mu(uv) =(d/2n)\ (n+1+n+1-2) = d$, for every edge e.
Take $\sigma(v) = 1$ for every vertex v
Thus $d(e) =d\ \forall\ e \in E(G)$ for every edge e.Hence G realizes $\{d\}$ as its edge degree set.

Theorem4.2

Any finite set $\{d_1.d_2,d_3\ldots\ d_k\}$ of positive real numbers is the edge degree set of a fuzzy graph.

Proof:

By theorem 3.11, there exists a fuzzy graph G_i realizing the edge degree set $\{d_i\}\ \forall\ i=1,2,3\ldots,k$. Then the fuzzy graph $G=G_1 \cup G_2 \cup\ \ldots\ \cup G_k$ realizes the edge degree set $\{d_1.d_2,d_3\ldots d_k\}$.

The fuzzy graph obtained in theorem 4.2 is a disconnected. The following theorem 4.3 shows that any set of two positive real numbers can be a degree set of a connected fuzzy graph.

Theorem 4.3

If d_1 and d_2 are two positive real numbers , then $\{d_1, d_2\}$ is an edge degree set of a fuzzy graph.

Proof:

Let d_1 and d_2 are two positive real numbers. Without loss of generality assume that $2d_2 - d_1 > 0$. For if $2d_2 - d_1 \leq 0$ then $2d_1 - d_2 \geq 4d_2 - d_2 = 3d_2 > 0$.

Choose a positive integer n such that $n > \frac{d_1}{2d_2-d_1} + 2$ ------ (1)

Then $n-2 > \frac{d_1}{2d_2-d_1} \Rightarrow \frac{1}{n-2} > \frac{2d_2-d_1}{d_1}$

$\Rightarrow \frac{1}{n-2} > \frac{2d_2}{d_1} - 1$

$\Rightarrow \frac{1}{n-2} + 1 > \frac{2d_2}{d_1}$

$\Rightarrow \frac{n-1}{n-2} > \frac{2d_2}{d_1}$

$\Rightarrow (n-1)\, d_1 > (n-2)\, 2d_2$

$\Rightarrow (n-1)\, d_1 - (n-2)\, 2d_2 > 0$ -------(2)

Equation (1) holds for the positive integer n, it holds for every positive integer greater than n.

Equation (2) holds for the positive integer greater than n.

Choose appositive integer m such that $m < \min\{(n-2)^2, \frac{d_1}{2d_2-d_1}(n-2)\}$.

Then $m < (n-2)^2$ and $m < \frac{d_1}{2d_2-d_1}(n-2)$.

Now $m < \frac{d_1}{2d_2-d_1}(n-2) \Rightarrow \frac{m}{(n-2)} < \frac{d_1}{2d_2-d_1}$

$\Rightarrow \frac{(n-2)}{m} < \frac{2d_2-d_1}{d_1}$

$\Rightarrow \frac{2d_2}{d_1} - 1 < \frac{(n-2)}{m}$

$\Rightarrow \frac{2d_2}{d_1} < \frac{(n-2)}{m} + 1$

$\Rightarrow \frac{2d_2}{d_1} < \frac{(n-2)+m}{m}$

$\Rightarrow 2md_2 < (n-2+m)\, d_1 > 0$

Also $m < (n-2)^2 \Longrightarrow (n-2)^2 - m > 0$.

Choose n large enough so that $0 < \frac{2(n-2)d_2 - (n-1)\, d_1}{2((n-2)^2 - m)} \leq 1$ and $0 < \frac{(n+m-2)d_1 - 2m\, d_2}{(n-2)^2 - m} \leq 2$

Let $C_1 = \frac{(n+m-2)d_1 - 2m\, d_2}{(n-2)^2 - m}$ and $C_2 = \frac{2(n-2)d_2 - (n-1)\, d_1}{(n-2)^2 - m}$. Then $0 < C_1 \leq 1$ and $0 < C_2 \leq 1$

Consider a fuzzy graph $G_1 : (\sigma_1, \mu_1)$ on a complete graph with n vertices $v_1, v_2, v_3, v_4, \ldots, v_n$ with $\mu_1(v_i v_j) = C_1\ \forall\ i \neq j$ and $\sigma_1(v_i) = 1\ \forall\ i = 1, 2, 3, \ldots, n$

Let $G_2 : (\sigma_2, \mu_2)$ be a fuzzy graph on m vertices $u_1, u_2, u_3, u_4, \ldots, u_m$ with $\mu_2(u_i v_j) = 0\ \forall\ i \neq j$ and $\sigma_2(u_j) = C_2\ \forall\ j = 1, 2, 3, \ldots, m$.

Let $G:(\sigma, \mu)$ be the join of G_1 and G_2. Then $\sigma(v_i) = \sigma_1(v_i)\ \forall\ i = 1, 2, 3, \ldots, n$, and $\sigma(u_j) = \sigma_2(u_j)\ \forall\ j = 1, 2, 3, \ldots, n$, $\mu(v_i v_j) = \mu_1(v_i v_j) = C_1\ \forall\ i \neq j$; $\mu(u_k u_l) = \mu_2(u_k u_l) = 0\ \forall\ k \neq l$. Also each vertex of G_1 is adjacent to each vertex of G_2 in G with $\mu(v_i u_j) = \sigma_1(v_i) \wedge \sigma_2(u_j)$

$= 1 \wedge C_2$
$= C_2$
$\forall\ i = 1, 2, 3, \ldots, n$
$j = 1, 2, 3, \ldots, m$.

Now $d_G(v_i\, v_j) = \sum_{\substack{k=1\\k\neq j\\k\neq i}}^{n} \mu(v_i\ v_k) + \sum_{\substack{k=1\\k\neq j\\k\neq i}}^{n} \mu(v_j\ v_k) + \sum_{k=1}^{m} \mu(v_i\ u_k) + \sum_{k=1}^{m} \mu(v_j\ u_k)$

$= (n-2)\, C_1 + (n-2)\, C_1 + m\, C_2 + m\, C_2.$

$= 2(n-2) \left[\frac{(n+m-2)d_1 - 2m\, d_2}{2(n-2)^2 - m}\right] + 2m \left[\frac{(2n-4)d_2 - (n-1)\, d_1}{2(n-2)^2 - m}\right]$

$= \frac{(n^2 - 4n+4-m)d_1}{(n-2)^2 - m}$

$d_G(v_i\, v_j) = d_1$

$d_G(v_i\, u_j) = \sum_{\substack{k=1\\k\neq i}}^{n} \mu(v_i\ v_k) + \sum_{\substack{k=1\\k\neq j}}^{n} \mu(v_j\ u_k) + \sum_{\substack{k=1\\k\neq i}}^{m} \mu(u_j\ v_k)$

$= (n-1)\, C_1 + (m-1)\, C_2 + (n-1)\, C_2.$

$= (n-1)\, C_1 + (n+m-2)\, C_2$

$= (n-1) \left[\frac{(n+m-2)d_1 - 2m\, d_2}{2(n-2)^2 - m}\right] + (n+m-2) \left[\frac{2(n-2)d_2 - (n-1)\, d_1}{(n-2)^2 - m}\right]$

$d_G(v_i\, v_j) = d_2.$

Then (σ, μ) is a fuzzy graph with the edge degree set $\{d_1, d_2\}$.

Example 4.4

Let us illustrate the procedure described in the above theorem by the following example.

Consider $d_1 = 1.5$, $d_2 = 6.5$, then $2d_2 - d_1 = 2(6.5) - 1.5 = 13 - 1.5 = 11.5 > 0$ and $\frac{d_1}{2\ d_2\ - d_1} = \frac{1.5}{11.5} = 0.1304$.
Since $2 + \frac{d_1}{2\ d_2\ - d_1} = 2.1304$, $n \geq 3$. Also $m < \frac{d_1}{2\ d_2\ - d_1}\ (n-2)$

R.H.S exceeds one only when $(n-2) = 7 \Rightarrow n = 9$.

At n= 9 we have m=1 then $(n+m-2)\, d_1 - 2md_2 = (8 \times\ 1.5) - (2 \times 6.5) = 12 - 13 < 1$.

Therefore n=10 and m=1, $(n-2)^2 - m = 8^2 - 1 = 63$ and $(n+m-2)\, d_1 - 2md_2 = (9 \times\ 1.5) - (2 \times 6.5) = 13.5 - 13 = 0.5$,
$2(n-2)d_2 - (n-1)d_1 = (2 \times 8 \times\ 6.5) - (9 \times 1.5) = 104 - 13.5 = 90.5$.

$\Rightarrow C_1 = \frac{0.5}{2 \times 6\ 3} = \frac{5}{126\ 0} = 0.03968254$, $C_2 = \frac{90.5}{126\ 0} = 0.71825397$.

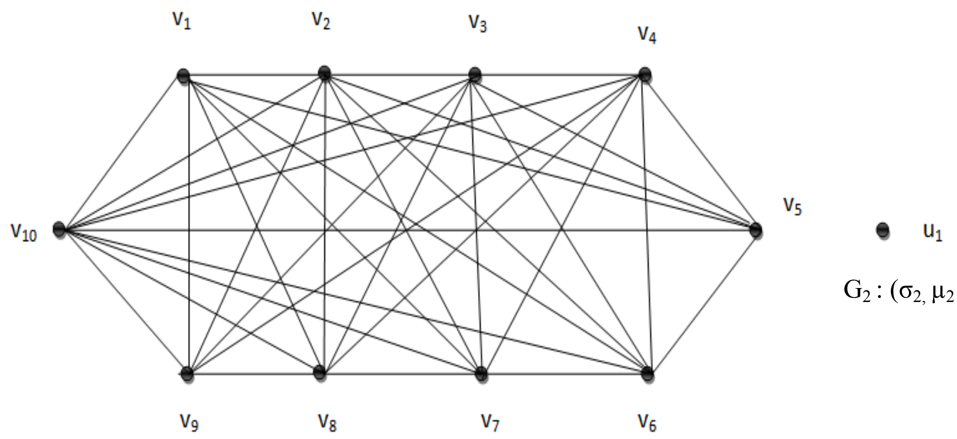
Consider a fuzzy graph $G_1 : (\sigma_1, \mu_1)$ on a complete graph with 10 vertices $v_1, v_2, v_3, v_4, \ldots, v_{10}$ with $\mu_1(v_i v_j) = C_1 = 0.03968254$ $\forall\ i \neq j$ and $\sigma_1(v_i) = 1\ \forall\ i = 1, 2, 3\ldots, 10$

Let $G_2 : (\sigma_2, \mu_2)$ be a fuzzy graph on vertex u_1 with $\mu_2(u_1 v_j) = 0\ \forall\ i \neq j$ and $\sigma_2(u_1) = C_2 = 0.71825397$ (Fig.5).

Let G:(σ, μ) be the join of G_1 and G_2. Then $\sigma(v_i) = \sigma_1(v_i)\ \forall\ i = 1, 2, 3\ldots, 10$, and $\sigma(u_1) = \sigma_2(u_1)\ \ \forall\ i = 1, 2, 3\ldots, n$,
$\mu(v_i v_j) = \mu_1(v_i v_j) = C_1\ \forall\ i \neq j$;

Now $d_{G_1 \oplus G_2}(v_i v_j) = 2(n-2)\, C_1 + 2\, m\, C_2$
$= (2 \times 8 \times\ 0.03968254) - (2 \times 0.71825397)$
$= 0.6352 + 1.4366 = 1.5$
$d_{G_1 \oplus G_2}(u_i v_j) = (n-1)\, C_1 + C_2(n + m - 2)$
$= (9 \times\ 0.03968254) - (9 \times 0.71825397)$
$= 0.3573 + 6.4647 = 6.5$

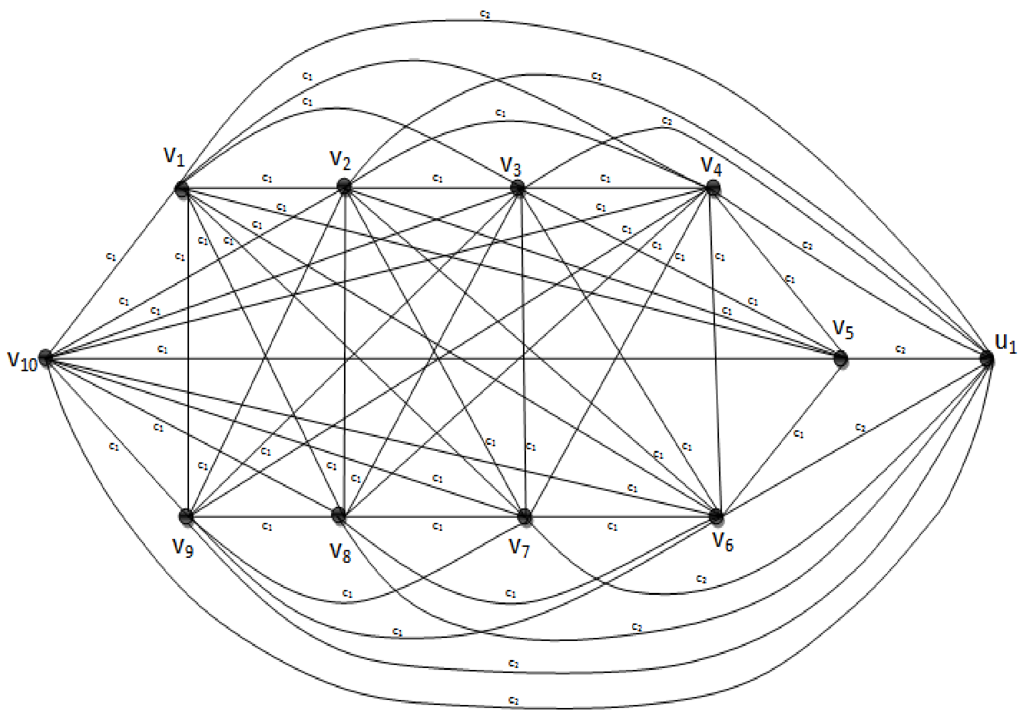
Then G: (σ, μ) (fig.6) is a fuzzy graph with degree set {1.5, 6.5}.



$G_2 : (\sigma_2, \mu_2)$

$G_1 : (\sigma_1, \mu_1)$

Fig. 5.



G: (σ, μ)

Fig. 6.

Theorem 4.5:

Let d be any positive real number then $\{d\}$ is a edge degree set of a fuzzy graph on a cycle if and only if $0 < d \leq 2$.

Proof :

Let C_n be a cycle $v_1v_2v_3v_4\ldots\ldots\ldots v_n v_1$ on n vertices where n is any positive integer.

Suppose that $\{d\}$ is the degree set of a fuzzy graph (σ, μ) on C_n.

Then $d(v_i v_{i+1}) = \mu(v_{i-1} v_i) + \mu(v_{i+1}v_{i+2})\ \forall\ i=1,2,\ldots,n.$
$\leq 1+1$
$\leq 2.$

Also $\mu(v_i v_{i+1}) > 0 \forall\ i=1,2,\ldots,n.$

Hence $0 < d \leq 2$.

Conversely assume that $0 < d \leq 2$. Then $0 < d/2 \leq 1$. Assign $\mu(v_i v_{i+1}) = d/2\ \forall\ i=1,2,\ldots,n$ where $v_{n+1}= v_n$. Assign any value satisfying the condition of fuzzy graph as $\sigma(v_i)$ for all i. Therefore $d(v_i v_{i+1}) = \mu(v_{i-1} v_i) + \mu(v_{i+1}v_{i+2})\ \forall\ i=1,2,\ldots,n.$

$= \frac{d}{2} + \frac{d}{2}$
$= d, \qquad \forall\ i=1,2,\ldots,n.$

Thus {d} is a edge degree set of a fuzzy graph(σ, μ) on C_n .

Theorem 4.6 :

Let d be any positive real number then{d} is a edge degree set of a fuzzy graph on a path P_n if and only if $0 < d \leq 1$ and n=3.

Proof :

Let P_n be a path on n vertices say, $v_1, v_2, v_3, v_4, \ldots, v_n$.

Suppose that {d} is a degree set of a fuzzy graph (σ, μ) on the path P_n .

Then $d(v_i v_{i+1}) = d\ \forall\ i=1, 2, ..,n, n \geq 3$.

Therefore $d= d(v_{n-1}v_n) = \mu(v_{n-2} v_{n-1}) \Rightarrow 0 < d \leq 1$.

If possible assume that $n \geq 3$.

Then $d(v_{n-3}v_{n-2}) = \mu(v_{n-4} v_{n-3}) + \mu(v_{n-2} v_{n-1})$
$= d+ \mu(v_{n-4} v_{n-3}$
$> d$ which is a contradiction.

Hence n =3.

Conversely, assume that $0 < d \leq 1$ and n=3.
Consider a path P_3 on three vertices v_1, v_2, v_3 and assign $\mu(v_1 v_2) = \mu(v_2 v_3) = d$.

Also assign any value satisfying the condition of fuzzy graph as the membership values of the vertices.

Then $d(v_1v_2) = \mu(v_2 v_3)$ and $d(v_2v_3) = \mu(v_1 v_2)$

Thus {d} is a edge degree set of a fuzzy graph (σ, μ) on the path P_3.

Theorem 4.7:

Let d be any positive real number then{d} is an edge degree set of a fuzzy graph on a complete graph K_n if and only if $0 < d \leq 2(n-2)$.

Proof :

Let K_n be a complete graph on n vertices say, $v_1, v_2, v_3, v_4, \ldots, v_n$.

Suppose that {d} is an edge degree set of a fuzzy graph on a complete graph K_n .

Then $d(v_i v_j) = \sum_{\substack{k=1 \\ k\neq j}}^{n-2} \mu(v_i\, v_k) + \sum_{\substack{k=1 \\ k\neq i}}^{n-2} \mu(v_j\, v_k)$

$\leq \sum_{\substack{j=1 \\ j\neq i}}^{n-2} 1 + \sum_{\substack{j=1 \\ j\neq i}}^{n-2} 1$

$\leq n-2 + n-2 = 2n-4$

Also $\mu(v_i v_{i+1}) > 0\ \forall\ i=1,2,\ldots,n$.Hence $0 < d \leq 2(n-2)$.

Conversely assume that $0 < d \leq 2(n-2)$. Then $0 < \frac{d}{2(n-2)} \leq 1$.

Assign $\mu(v_i v_j) = d/(2(n-1))\ \forall\ i\neq j$ Assign any value in[$\frac{d}{2(n-2)}$, 1]as $\sigma(v_i)$ for all i.

Then $d(v_i v_j) = \sum_{\substack{k=1 \\ k\neq j}}^{n-2} \mu(v_i\, v_k) + \sum_{\substack{k=1 \\ k\neq i}}^{n-2} \mu(v_j\, v_k)$

$= \sum_{\substack{j=1 \\ j\neq i}}^{n-2} \frac{d}{2(n-2)} + \sum_{\substack{j=1 \\ j\neq i}}^{n-2} \frac{d}{2(n-2)}$

$= 2(n-2)\frac{d}{2(n-2)}$

$=$ d. for all $i \neq j$.

Thus {d} is an edge degree set of the fuzzy graph (σ, μ) on K_n .

Theorem 4.8:

Let d be any real number then{d} is an edge degree set of a fuzzy graph on a star $K_{1,n}$ if and only if $0 < d \leq 1$ and n=2.

Proof:

Let $K_{1,n}$ be star $v_0, v_1, v_2, v_3, \ldots, v_n$ with n+1 vertices and v_0 as its center.
Let {d} is an edge degree set of a fuzzy graph (σ, μ) on the path $K_{1,n}$.
Then $d = d(v_i v_0) = \mu(v_j v_0)$, $\forall$ j=1, 2, ..,n, $j \neq i$
$\Rightarrow 0 < d \leq 1$.

Also $d(v_i v_0) = \sum_{j=1}^{n-1} \mu(v_j v_0)$
$= \sum_{i=1}^{n-1} d$
$= (n-1)d$.

Since {d} is the degree set of a fuzzy graph on a star $K_{1,n}$, all the edges must receive same degree d. Therefore $(n-1)d = d \Rightarrow n=2$.
Conversely assume that $0 < d \leq 1$ and n=2.

Consider $K_{1,2}$ on three vertices v_0, v_1, v_2 with v_0 as its center.

Assign $\mu(v_0 v_1) = d$, $\mu(v_0 v_2) = d$, so that $d(v_0 v_1) = d(v_0 v_2) = d$.

Assign any value satisfying the condition of fuzzy graph as $\sigma(v_i)$ for i = 0, 1, 2.

Thus {d} is an edge degree set of the fuzzy graph (σ, μ) on $K_{1,2}$.

Theorem 4.9:

Let d be nay nonnegative real number. If there exists a positive integer m> 1 such that
$0 < d \leq 2m$ and $0 < (m+1)d \leq 2m$, then {d} is the edge degree set of fuzzy graph on bistar with 2m+2 vertices.

Proof:

Consider a bistar graph G on '2m+2' vertices $v_0, v_1, v_2, v_3, \ldots, v_m, u_0, u_1, u_2, u_3, \ldots, u_m$ which is obtained by joining the apex vertices v_0 and u_0 of the two stars with the vertex sets $\{v_0, v_1, v_2, v_3, \ldots, v_m\}$ and $\{u_0, u_1, u_2, u_3, \ldots, u_m\}$.

By hypothesis $0 < \frac{d}{2m} \leq 1$ and $0 < \frac{(m+1)d}{2m} \leq 1$

Assign $\mu(u_0 v_0) = \frac{(m+1)d}{2m}$, $\mu(v_i v_0) = \mu(u_i u_0) = \frac{d}{2m}$.for i=1,2,3,…..m. Assign any value satisfying the condition of fuzzy graph as the membership values of the vertices.

Then $d(v_0 u_0) = \sum_{i=1}^{m} \mu(v_i v_0) + \sum_{i=1}^{m} \mu(u_i u_0)$
$= \sum_{i=1}^{m} \frac{d}{2m} + \sum_{i=1}^{m} \frac{d}{2m}$
$= m\frac{d}{2m} + m\frac{d}{2m}$
$=$ d.
For i=1,2,….,m

$d(v_i v_0) = \sum_{\substack{j=1 \\ j\neq i}}^{m} \mu(v_j v_0) + \mu(u_0 v_0)$,
$= (m-1)\frac{d}{2m} + (m+1)\frac{d}{2m}$
$= 2m\frac{d}{2m}$
$=$ d.
Similarly $d(u_i u_0) = d$, for i=1,2,….,m
Thus {d} is the edge degree set of fuzzy graph (σ , μ) on bistar with 2m+2 vertices.

Theorem 4.10

If $0 < d_1 \leq 2$ and $0 < d_2 \leq 2$ then $\{d_1, d_2\}$ is an edge degree set of a fuzzy graph on an even cycle.

Proof:

Since $d_i \leq 2$, $\frac{d_i}{2} \leq 1$, i=1,2 .Consider a cycle C_{2n} on 2n vertices $v_1e_1v_2e_2v_3....v_{2n-1}e_{2n-1}v_{2n}ev_1$. Assign $\mu(e_1) = \mu(e_3) = ...$
$= \mu(e_{2n-1}) = \frac{d_1}{2}$ and $\mu(e_2) = \mu(e_4) = ... \frac{d_2}{2}$.

Then $d(e_{2r}) = \mu(e_{2r-1}) + \mu(e_{2r+1}) = \frac{d_1}{2} + \frac{d_1}{2} = d_1$, for r=1,2,3,…,n , where $e_{2n+1}= e_1$ Also $d(e_{2r+1}) = \mu(e_{2r}) + \mu(e_{2r+2}) = \frac{d_2}{2} + \frac{d_2}{2} = d_2$, for r=1,2,3,…,n-1. Take $\sigma(v_i) = 1$, i = 1,2,3,4, …, 2n. Then (σ, μ) is a fuzzy graph on C_{2n} with degree set $\{d_1, d_2\}$.

Theorem 4.11

If $0 < d_1 \leq 2$ and $0 < d_2 \leq 4$ tthen $\{d_1, d_2\}$ is an edge degree set of a fuzzy graph on $C_n \circ K_1$.

Proof:

Since $d_1 \leq 2$, $\frac{d_1}{2} \leq 1$. Consider a cycle C_n on n vertices $v_1e_1v_2e_2v_3....v_ne_nv_1$. Assign $\mu(v_1v_2) = \mu(v_2v_3) = \mu(v_3v_1) = \frac{d_1}{2}$. Take n vertices $u_1, u_2, u_3, …, u_n$. Join u_i to v_i and assign $\mu(u_iv_i) = \frac{d_2 - d_1}{2}$ i = 1,2,3. Then $d(u_iv_i) = \mu(e_{i-1}) + \mu(e_i) = \frac{d_1}{2} + \frac{d_1}{2} = d_1$, i=1, 2, 3,…,n

Now $d(e_i) = \mu(v_iv_{i-1}) + \mu(v_{i+1}v_{i+2}) + \mu(u_iv_i) + \mu(u_{i+1}v_{i+1})$

$$= \frac{d_1}{2} + \frac{d_1}{2} + \frac{d_2 - d_1}{2} + \frac{d_2 - d_1}{2}$$

$$= d_1 + d_2 - d_1 = d_2.$$

Assign any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$. Then (σ, μ) is a fuzzy graph with degree set $\{d_1, d_2\}$.

Theorem 4.12

Let $\{d_1, d_2\}$ be the set of positive real numbers. If there exists two positive integers m and n and two real numbers r, s ∈ (0,1] such that $2(n-1)r + 2ms = d_1$ and $(m-1)s + nr = d_2$, then $\{d_1, d_2\}$is an edge degree set of a fuzzy graph on $K_{n+1} \circ \overline{K_m}$.

Proof:

Consider a complete graph K_{n+1} on n+1 vertices $v_1, v_2, v_3, v_4, …., v_{n+1}$.

Assign a membership value as 'r' to all its edges.

Now consider the corona product $K_{n+1} \circ \overline{K_m}$.

Take m(n+1) vertices u_{i_j}, i = 1,2,,…., n+1, j=1,2,,…., m. Join the m vertices $u_{i_1}, u_{i_2}, u_{i_3},…., u_{i_m}$ to v_i , i= 1,2,,…., n+1.The resultant graph is the corona product $K_{n+1} \circ \overline{K_m}$.

Assign $\mu(v_i\, u_{i_j}) = s$, ∀ i = 1,2,,…., n+1, j=1,2,,…., m.

Then $\mu(v_i v_j) = \sum_{k\neq j} \mu(v_i\ v_k) + \sum_{k\neq i} \mu(v_j\ v_k) + \sum_{k=1}^{m} \mu(v_i\, u_{i_k}) + \sum_{k=1}^{m} \mu(v_i\, u_{j_k})$

$$= (n-1)r + (n-1)r + ms + ms$$

$$= 2(n-1)r + 2ms$$

$$= d_1$$

$$\mu(v_i\, u_{i_j}) = \sum_{k\neq j} \mu(u_{i_k}) + \sum_{l=1}^{n} \mu(v_i\, v_l)$$

$$= (m-1)s + nr$$

$$= d_2$$

Assign any value satisfying the condition of a fuzzy graph as $\sigma(v_i)$. Then (σ, μ) is a fuzzy graph with degree set $\{d_1, d_2\}$.

Theorem 4.13

Let $\{d_1, d_2\}$ be the set of positive real numbers with $d_1 > d_2$. If there exists $c \in (0, 1]$ such that $\frac{d_2}{d_2 - f}$ is an integer where $f = \frac{d_1 - 2c}{d_2}$ and $d_2 - f < 1$ then $\{d_1, d_2\}$ is an edge degree set of a fuzzy graph on the corona product $K_{n+1} \circ K_1$.

Proof:

Let $n = \frac{d_2}{d_2 - f}$.

Consider a complete graph on n+1 vertices $v_1, v_2, v_3, v_4, \ldots, v_{n+1}$. Assign $\mu(v_i v_j) = d_2 - f, \forall\ i \neq j$.

Take another n+1 vertices $u_1, u_2, u_3, u_4, \ldots, u_{n+1}$. Join u_i and v_i by an edge , $\forall\ i = 1, 2, 3, \ldots, n+1$.

Since $v_i v_j$, $i \neq j$ are the only edges adjacent to the edge $u_i v_i$,

$$
\begin{aligned}
d(u_i v_i) &= \textstyle\sum_{i \neq j} \mu(v_i\ v_j) \\
&= \textstyle\sum_{i \neq j} (d_2 - f) \\
&= n(d_2 - f) \\
&= d_2. \quad , \ i = 1, 2, 3, \ldots, n+1
\end{aligned}
$$

$$
\begin{aligned}
d(v_i v_j) &= \sum_{\substack{i \neq j \\ k \neq i}} \mu(v_i\ v_k) + \sum_{\substack{k \neq i \\ k \neq j}} \mu(v_j\ v_k) + \mu(u_i\ v_i) + \mu(u_j\ v_j) \\
&= (n-1)(d_2 - f) + (n-1)(d_2 - f) + c + c \\
&= 2n(d_2 - f) - 2(d_2 - f) + 2c \\
&= 2d_2 - 2d_2 + 2f + 2c \\
&= 2f + 2c \\
&= d_1 .
\end{aligned}
$$

Assign any value satisfying the condition of fuzzy graph as $\sigma(u_i)$ and $\sigma(v_i)$, $i = 1, 2, 3, \ldots, n+1$. Then (σ, μ) is a fuzzy graph on $K_{n+1} \circ K_1$ with degree set $\{d_1, d_2\}$. In the following section, we obtain conditions for a given set of real numbers to be an edge degree set of some particular fuzzy graphs.

Theorem 4.14

If $\{d_1, d_2\}$ is an edge degree set of a fuzzy graph on a bistar then $0 < d_2 \leq \left(\frac{n-2}{2}\right)$ and $0 < d_1 \leq (n-2)$.

Proof:

Let $\{d_1, d_2\}$ is an edge degree set of a fuzzy graph on a bistar graph G on 'n' vertices $v_0, v_1, v_2, v_3, \ldots, v_k, u_0, u_1, u_2, u_3, \ldots, u_k$ which is obtained by joining the apex vertices v_0 and u_0 of the two stars with the vertex sets $(v_0, v_1, v_2, v_3, \ldots, v_k)$ and $(u_0, u_1, u_2, u_3, \ldots, u_k)$.

Then $d_2 = d(v_i\ v_0) = \sum_{i=1}^{\left(\frac{n-2}{2}\right)-1} \mu(v_i v_0) + \mu(u_0 v_0) \leq \left(\frac{n-4}{2}\right) + 1$

Hence $d(v_i\ v_0) \leq \left(\frac{n-2}{2}\right)\ \ \forall\ i=1, 2, .., \left(\frac{n-2}{2}\right)$. Similarly $d(u_i v_0) \leq \left(\frac{n-2}{2}\right)\ \forall\ i=1, 2, .., \left(\frac{n-2}{2}\right)$

Since $0 < \mu(v_i v_0)$, $\mu(u_i u_0) \leq 1$, $0 < d_2 \leq \left(\frac{n-2}{2}\right)$.

$$
\begin{aligned}
\text{Also } d_1 = d(v_0\ u_0) &= \sum_{i=1}^{\left(\frac{n-2}{2}\right)} \mu(v_i v_0) + \sum_{i=1}^{\left(\frac{n-2}{2}\right)} \mu(u_i u_0) \\
&\leq \left(\frac{n-2}{2}\right) + \left(\frac{n-2}{2}\right) \\
&\leq (n-2)
\end{aligned}
$$

Since $0 < \mu(v_i v_0)$, $\mu(u_i u_0) \leq 1$, $0 < d_1 \leq (n-2)$.Hence the theorem.

Theorem 4.15:

If $d_1$and d_2 are two real numbers such that $0 < d_1 \leq 4$ and $(n+1)\, d_1 = 4\, d_2$, for some positive integer n then $\{d_1, d_2\}$ is the edge degree set of a fuzzy graph on a wheel W_n.

Proof:

Let W_n be wheel on $v_0, v_1, v_2, v_3, \ldots, v_n$ with n+1 vertices and v_0 as its center.

Since $0 < d_1 \leq 4$, $0 < \frac{d_1}{4} \leq 1$.Assign $\mu(v_i v_{i+1}) = \frac{d_1}{4}\ \forall$ i=1,2,….,n where $v_{n+1}= v_1$ and $\mu(v_0v_i) = \frac{d_1}{4}\ \forall$ i=1,2,….,n. Assign any value satisfying the condition of fuzzy graph as the membership values of the vertices.

Then for i= 1,2,…..,n $d(v_i\ v_{i+1}) = \mu(v_i v_{i-1}) + \mu(v_i v_0) + \mu(v_{i+1} v_{i+2}) + \mu(v_{i+1} v_0)$

$$= \frac{d_1}{4} + \frac{d_1}{4} + \frac{d_1}{4} + \frac{d_1}{4}$$

$$= d_1.$$

and $d(v_0 v_i) = \sum_{\substack{j=1\\ j\neq i}}^{n} \mu(v_j v_0) + \mu(v_i v_{i-1}) + \mu(v_{i+1} v_{i+2})$

$$=\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{d_1}{4} + \frac{d_1}{4} + \frac{d_1}{4}$$

$$= (n+1)\frac{d_1}{4}.$$

$$= d_2.$$

Hence $\{d_1, d_2\}$is an edge degree set of a fuzzy graph (σ, μ) on a starW_n.

Theorem 4.16:

Let $d_1$and d_2 be two non negative real numbers. If there exists a positive integer m> 1 such that $0 < d_1 \leq 2m$, $0 < m(2d_2- d_1) +d_1 \leq 2m$ then $\{d_1, d_2\}$ is the edge degree set of fuzzy graph on bistar with 2m+2 vertices.

Proof:

Consider a bistar graph G on '2m+2' vertices $v_0, v_1, v_2, v_3, \ldots, v_m, u_0, u_1, u_2, u_3, \ldots, u_m$ which is obtained by joining the apex vertices v_0 and u_0 of the two stars with the vertex sets $(v_0, v_1, v_2, v_3, \ldots, v_m)$ and $(u_0, u_1, u_2, u_3, \ldots, u_m)$.

By hypothesis $0 < \frac{d_1}{2m} \leq 1$ and $0 < \frac{m(2d_2-d_1)+d_1}{2m} \leq 1$

Assign $\mu(u_0 v_0) = \frac{m(2d_2-d_1)+d_1}{2m}$, $\mu(v_iv_0) = \mu(u_iu_0) = \frac{d}{2m}$.for i=1,2,3,…..m.

Assign any value satisfying the condition of fuzzy graph as the membership values of the vertices.

Then $d(v_0 u_0) = \sum_{i=1}^{m} \mu(v_i v_0) + \sum_{i=1}^{m} \mu(u_i u_0)$

$$= \sum_{i=1}^{m} \frac{d_1}{2m} + \sum_{i=1}^{m} \frac{d_1}{2m}$$

$$= m\frac{d_1}{2m} + m\frac{d_1}{2m}$$

$$= d_1.$$

For i=1,2,….,m

$d(v_i v_0) = \sum_{\substack{j=1\\ j\neq i}}^{m} \mu(v_j v_0) + \mu(u_0 v_0)$, i=1,2,….,m

$$= (m-1)\frac{d_1}{2m} + \frac{m(2d_2-d_1)+d_1}{2m}$$

$$= \frac{1}{2m}(md_1 - d_1 + 2md_2 - md_1 + d_1)$$

$$= \frac{1}{2m}(2md_2) = d_2.$$

Similarly $d(u_i u_0) = d_2$

Thus $\{d_1, d_2\}$ is the edge degree set of fuzzy graph (σ, μ) on bistar with 2m+2 vertices.

Conclusion

In fuzzy graph theory degree of an edge is a parameter of a graph In this paper we made a study about the edge degree set of a fuzzy graph.

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