

*Available online at http://www.journalcra.com*

*International Journal of Current Research Vol. 9, Issue, 10, pp.58521-58529, October, 2017*

**INTERNATIONAL JOURNAL**<br> *OF CURRENT RESEARCH*<br>
<u>**INTERNATIONAL JOURNAL**</u> **OF CURRENT RESEARCH**

# **RESEARCH ARTICLE**

# **CANONIZATION OF THE HYPERSURFACES OF THE FIRST AND THE SECOND DEGREE CANONIZATION OF SECOND DEGREEAND APPLICATION FOR THE MAXIMAL ABSOLUTE AND RELATIVE INACCURACIES**

# **<sup>1</sup>Kiril Kolikov, 2,\*Radka Koleva and <sup>1</sup>Yordan Epitropov Epitropov**

1Plovdiv University "Paisii Hilendarski", Tzar Assen str. 24, Plovdiv, Bulgaria <sup>2</sup>University of Food Technologies, 26 Maritsa Blvd., Plovdiv, Bulgaria 2

#### **ARTICLE INFO ABSTRACT ARTICLE INFO**

*Article History: Article History:*Received  $06<sup>th</sup>$  July, 2017 Received in revised form  $23<sup>rd</sup>$  August, 2017 Accepted 28<sup>th</sup> September, 2017 Published online 17<sup>th</sup> October, 2017

#### *Key words:*

Linear Homogeneous form of *n* Unknowns, Orthogonal Transformation of the Unknowns, Canonical forms of the Surfaces of the first and of the second degree, Affine Euclidean space, Maximal Absolute and Relative Inaccuracies.

In this paper we reduce a linear homogeneous form of *n* unknowns with an orthogonal transformation of the unknowns in a linear homogeneous form of one unknown with an exactly definite positive coefficient. As an application of this result we find the canonical form of an arbitrary hyperplane in the real *n*-dimensional affine Euclidean space  $E_n$ . This method for the obtaining of the canonical forms of the hyperplanes is new, since until nowa canonical form of hyperplanes is not defined and is not considered.Besides we find an effective canonical form of the surfaces of the second degree in paper we reduce a linear homogeneous form of  $n$  unknowns with an orthogonal transformation unknowns in a linear homogeneous form of one unknown with an exactly definite positive ient. As an application of this result we

*n*. Our methodfor the canonization of the surfaces of the second degree in  $E_n$  is effective since it

gives the exact coefficients of the canonical form of the surface in a dependent ofthe coefficients of the givensurface equation. As an application we give a canonization of the hypersurfaces of the maximal absolute and relative inaccuracies (errors). Besides this method is different from the known approach in the case of the obtaining of a cylinder. a-dimensional affine Euclidean space  $E_n$ . This method for the obtaining of the canonical<br>he hyperplanes is new, since until nowa canonical form of hyperplanes is not defined and is<br>dered.Besides we find an effective cano

Copyright©2017, Kiril Kolikov et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, *distribution, and reproduction in any medium, provided the original work is properly cited. distribution,* the Unknowns, Canonical forms of the maximal absolute and relative inaccuracies (errors). Be<br>
Surfaces of the first and of the second<br>
degree, Affine Euclidean space, Maximal<br>
Absolute and Relative Inaccuracies.<br>
Copyright

Citation: Kiril Kolikov, Radka Koleva and Yordan Epitropov. 2017. "Canonization of the hypersurfaces of the first and the second degree and application for the maximal absolute and relative inaccuracies", *International Journal of Current Research*, 9, (10), 58521-58529.

# **INTRODUCTION INTRODUCTION**

The canonization of the surfaces of the second degree in the real *n*-dimensional affine Euclidean space  $\frac{1}{n}$  is well known ((Efimov, 2005; Konstantinov, 2000), and (Shafarevich, 2013)).In the known canonizations geometric interpretations are made while in our paper we offer an algebraic approach. In this paper we reduce with orthogonal transformation of the unknowns a real linear non-zero form  $f = a_1x_1 + ... + a_nx_n$  of *n* unknowns  $x_1,...,x_n$  in a linear homogenous form  $g = d_nu_n$  of a unknown  $u_n$ with a positive coefficient  $d_n = \sqrt{a_1^2 + ... + a_n^2}$ . Giving a definition of a canonical form of a planes in  $\pi$ , we apply the above result and we find the canonical form g of a surface of these hyperplanes. Until now a canonical form of hyperplanes is not defined and it is not considered. We obtain a effective canonical forms of the surfaces of the second degree in  $\pi$ . Our approach for an obtaining of the canonical form  $g$  in  $\pi$  of a surface  $f$  of the second degree is different from the known methods in a case of a cylinder. Besides this method is effective since it gives the exact coefficients of the canonical form g in a dependent of the coefficients of  $f$ . As an application of the obtained results we give the canonical forms of the maximal absolute and relative inaccuracies (errors). **1.** *This is an open access article distributed under ty medium, provided the original work is properly cit.<br> Koleva and Yordan Epitropov. 2017. "Canonizatitive inaccuracies", <i>International Journal of Current*<br>
rfaces In this paper we reduce a linear homogeneous form of o unknown with an encopy<br>all this methods in the such a application of this result to the distantive of the such an application of the such a control of this result of is an open access article distributed under<br>
um, provided the original work is properly and<br>
and **Yordan Epitropov. 2017.** "Canoniza<br>
accuracies", International Journal of Currence<br>
of the second degree in the rea<br>
(00), on the tentows an enforce and the original time. The section is the motion of the motion of the properties in the and a dimensional of the Burlin and the Burlin and the David and the David and the Burlin and considered Be ((Efimov, 2005; Konstantinov, 2000), and (Shafarevich, 2013)).In the known canonizations geometric interpretations are made<br>while in our paper we offer an algebraic approach. In this paper we reduce with orthogonal transf incar fon-2010 form  $f = a_1x_1 + \dots + a_nx_n$  of *n* dikinowns  $x_1, ..., x_n$  in a fined form of a<br>with a positive coefficient  $d_n = \sqrt{a_1^2 + \dots + a_n^2}$ . Giving a definition of a canonical form of a<br>and we find the canonical form *g* of inear homogenous form of one unknown with an exact<br>cartion of this result we find the canonical form of an<br>affine Euclidean space  $E_n$ . This method for the obts<br>is new, since until nowa canonical form of hyperplan<br>we find *n* an exactly definite positive<br>
rm of an arbitrary hyperplanes in<br>
r the obtaining of the canonical<br>
hyperplanes is not defined and is<br>
surfaces of the second degree in<br>
legree in  $E_n$  is effective since it<br>
dependent o  $n<sub>n</sub>$ , we apply the above results

# 1.Reduction of a linear homogenous form in a linear homogenous form of one unknown

In the following result with an orthogonal transformation of the unknowns we reduce a linear homogenous form of  $n$  unknownsin a linear homogenous formof one unknown. a linear homogenous formof one unknown.

**Theorem1.**If  $f = \{x_1 + \ldots + x_n\}$  is a non-zeroreal linear homogenous formof *n* unknowns  $x_1, \ldots, x_n$ , then there exists an orthogonaltransformation of the unknownswhich reduces f in linear homogenous form  $g = d_n u_n$ , where  $d_n = \sqrt{\frac{2}{1} + \frac{2}{2} + \dots + \frac{2}{n}}$  is a positive real number. likov et al. Canonization of the hypersurfaces of the nad relation<br>  $1^x 1^x + \dots + n^x n^x$  is a non-zeroreal linear<br>
ation of the unknownswhich<br>  $\frac{1}{1 + \frac{2}{n}}$  is a positive real number. *f firil Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and application for the maximal absolute and relative inaccuracies<br>*  $f = {}_1x_1 + ... + {}_nx_n$  *is a non-zeroreal linear homogenous for* for the maximal absolute<br>  $x_1, ..., x_n$ , then thereexists an<br>
mous form  $g = d_n u_n$ , where<br>
and the theorem is proved. If *for the second degree and application for the maximal absolute<br>
<i>f* in time in the maximal absolute<br> *f* in time in the more is torm  $g = d_n u_n$ , where<br>  $f(x_1 = d_1 x_1, d_1 = \sqrt{a_1^2} > 0$  and the theorem is proved. If<br>  $f(x_1 = d_$ **Kiril Kolikov et al. Canonization of the hypersi**<br> **1.**If  $f = {}_1x_1 + ... + {}_nx_n$  is a non-zerorial<br>
differentiation of the unknown<br>  $\frac{2}{1} + \frac{2}{2} + ... + \frac{2}{n}$  is a positive real number.<br>  $\therefore n = 1$ , i.e.  $f = {}_1x_1$ . If  ${}_1 >$ S22 **Kiril Kolikov et al.** Canonization of the hypersurfaces of the first and the second degree and application for the maximal absocant relative inaccuracies<br> **Cheorem1.**If  $f = {}_{1}x_{1} + ... + {}_{n}x_{n}$  is a non-zeroreal linea *Kiril Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and approximate and relative inaccuracies<br>
.If*  $f = {}_1x_1 + ... + {}_nx_n$  *is a non-zeroreal linear homogenous formof <i>n* unkn<br>
transformat be first and the second degree and application for the maximal absolute<br>
ive inaccuracies<br>
1 **homogenous** form of *n* unknowns  $x_1, ..., x_n$ , then there exists and<br>
reduces f in linear homogenous form  $g = d_n u_n$ , where<br>  $f = \sqrt{\frac$ be and application for the maximal absolute<br>
1 *n* unknowns  $x_1, ..., x_n$ , then there exists an<br>
ear homogenous form  $g = d_n u_n$ , where<br>  $d_1 = \sqrt{a_1^2} > 0$  and the theorem is proved. If<br>  $= (-1)u_1 = \sqrt{\frac{2}{1}}u_1 = d_1u_1$ ,  $d_1 = \sqrt{a_$ Riril Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and application for the maximal absolute<br> **notice** and relative inaccuracies<br> **notice** and relative inaccuracies<br> **notice** and rela tegree and application for the maximal absolute<br>
mof *n* unknowns  $x_1, ..., x_n$ , then there exists an<br>
linear homogenous form  $g = d_n u_n$ , where<br>  $d_1 = \sqrt{a_1^2} > 0$  and the theorem is proved. If<br>  $f = (-1)u_1 = \sqrt{\frac{2}{1}}u_1 = d_1u_1$ , solute<br>
there<br>
there exists an<br>  $= d_n u_n$ , where<br>
m is proved. If<br>  $d_1 = \sqrt{a_1^2} > 0$ .<br>
the linear form

**Proof.** Let  $n = 1$ , i.e.  $f = \frac{x_1}{1}$ . If  $\frac{1}{1} > 0$ , thenobviously  $f = \sqrt{\frac{2}{1}}x_1 = d_1x_1$ ,  $d_1 = \sqrt{a_1^2} > 0$  and the theorem is proved. If , then we apply the orthogonal transformation  $x_1 = -u_1$  and we obtain  $f = (-1)u_1 = \sqrt{\frac{2}{\pi}}u_1 = d_1u_1$ ,  $d_1 = \sqrt{a_1^2} > 0$ . Therefore, the theorem holds for  $n = 1$ .  $n=1$ .

Suppose that  $n \ge 2$ . Let, for determination, it holds  $\frac{1}{1} \ne 0$ .

At first we reduce the form  $f = \frac{1}{1}x_1 + \frac{1}{2}x_2 + \dots + \frac{1}{n}x_n$  withan orthogonal transformation of  $x_1, x_2, \dots, x_n$  in the linear form  $f = d_2y_2 + a_3y_3 + ... + a_ny_n$ , where  $y_2,..., y_n$  are unknowns, i.e. we except the unknown  $x_1$ . Namely, we use the normed vector  $\left(\frac{1}{d_1}, \frac{2}{d_2}\right), \ d_2 = \sqrt{\frac{2}{1} + \frac{2}{2}}$ the unknownswhich reduces  $f$  in linear<br>  $x_1 > 0$ , thenobviously  $f = \sqrt{\frac{2}{1}}x_1 = d_1x_1$ ,  $d_1$ <br>
onal transformation  $x_1 = -u_1$  and we obtain  $f = ($ <br>
1.<br>
mination, it holds  $\frac{1}{1} \neq 0$ .<br>  $x_1 + \frac{2}{1}x_2 + ... + \frac{2}{n}x_n$  wit *fanonization of the hypersurfaces of the first and the second degree and application for the maximal absolute*<br>  $\binom{n}{n}$  is a non-zeroreal linear homogenous form of *n* unknowns  $x_1, ..., x_n$ , then thereexists an<br>  $f$  the u the maximal absolute<br>
...,  $x_n$ , then there<br>
xists and to the theorem is proved. If<br>
d the theorem is proved. If<br>  $\overline{z}u_1 = d_1u_1$ ,  $d_1 = \sqrt{a_1^2} > 0$ .<br>  $x_2, ..., x_n$  in the linear form<br>
y, we use the normed vector  $\int_{1}^{2} \frac{1}{1} + \frac{2}{2} + ... + \frac{2}{n}$  is a positive real number.<br>
Let  $n = 1$ , i.e.  $f = \frac{1}{1}x_1$ . If  $\frac{1}{1} > 0$ , thenobviour<br> **1**, then we apply the orthogonal transformation x<br>
ore, the theorem holds for  $n = 1$ .<br>
e that *Farit Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and application for the maximal absolution for the maximal absolution and relative inaccuracies<br> for the coronical of f = \sqrt{x\_1 + ... n of the hypersurfaces of the first and the second degree and application for the maximal absolute<br>
and relative inaccuracies<br>
a non-zerorcal linear homogenous formof <i>n* unknowns  $x_1, ..., x_n$ , then there<br>
exists an unkno of.Let  $n = 1$ , i.e.  $f = \binom{1}{1}x_1$ . I<br>
< 0, then we apply the orthogorefore, the theorem holds for  $n =$ <br>
pose that  $n \ge 2$ . Let, for deter<br>
first we reduce the form  $f =$ <br>  $= d_2y_2 + \binom{1}{3}x_3 + ... + \binom{1}{n}y_n$ , when<br>  $\frac{1}{2$ of.Let  $n = 1$ , i.e.  $f = _1x_1$ . I<br>  $< 0$ , then we apply the orthogo<br>
refore, the theorem holds for  $n =$ <br>
pose that  $n \ge 2$ . Let, for deter<br>
first we reduce the form  $f =$ <br>  $= d_2y_2 + _3y_3 + ... + _ny_n$ , whe<br>  $\frac{1}{2}, \frac{2}{d_2}$ ,  $d$ **EXECT:**<br> **Example 12.1**<br> **COOf.Let**  $n = 1$ , i.e.  $f = \frac{1}{1}x_1$ . If  $\frac{1}{1} > 0$ , then<br> **COOf.Let**  $n =$ S222 **Kiril Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and application for the the canonization of the hypersurfaces of the first and the second degree and application for the the**  $f = \frac{1}{2}x_1$ . If  $\frac{1}{2} > 0$ , thenobvion<br>y the orthogonal transformation.<br>holds for  $n = 1$ .<br>et, for determination, it holds<br>form  $f = \frac{1}{2}x_1 + \frac{1}{2}x_2 + ... + \frac{1}{2}x_n$ , where  $y_2$ ,  $y_n$  are unknown and  $\frac{2}{3} + \frac{2}{3}$ *Kiril Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and application for the maximal absolute and relative inaccurates:<br>*  $f = {}_1X_1 + ... + {}_{n}X_n$  *is a non-zeroroal linear homogenous formof* ie theorem holds for  $n = 1$ .<br>  $x + n \ge 2$ . Let, for determination, it holds  $x + \ne 0$ .<br>
Figure the form  $f = x_1x_1 + x_2x_2 + ... + x_nx_n$  with an orthogoral theorem  $x_1x_2x_3 + ... + x_nx_n$ , where  $y_2,..., y_n$  are unknowns, i.e. we excently uppose that  $n \ge 2$ . Let, for determination, it holds  $_1 \ne 0$ .<br>
t first we reduce the form  $f = _{1}x_{1} + _{2}x_{2} + ... + _{n}x_{n}$  withan orthogonal<br>  $f = d_{2}y_{2} + _{3}y_{3} + ... + _{n}y_{n}$ , where  $y_{2},..., y_{n}$  are unknowns, i.e. we exc t  $n \ge 2$ . Let, for determination, it holds  $_1 \ne 0$ .<br>
reduce the form  $f = _1x_1 + _2x_2 + ... + _nx_n$  withan orthogon<br>  $x_3y_3 + ... + _ny_n$ , where  $y_2,..., y_n$  are unknowns, i.e. we exce<br>  $d_2 = \sqrt{\frac{2}{1} + \frac{2}{2}}$ <br>
at the linear transforma **Example 11.**  $f = {}_{1}x_{1} + ... + {}_{n}x_{n}$  is a non-zeroreal linear homogenous form<br>of *n* unknownsuration of the unknownswhich reduces  $f$  in linear homogenous<br>  $= \sqrt{\frac{2}{1} + \frac{2}{2} + ... + \frac{2}{n}}$  is a positive real number.<br> **of L Theorem1.1(**  $f = \frac{1}{r^2} + \dots + \frac{1}{s}x_n$  is a non-zeroreal linear homogeneos form f *n* unknowns  $x_1, ..., x_n$ , then thereasists an orthogonalitransformation of the unknownswhich reduces f in linear homogeneous form  $g = d_n u_n$ , nogonal transformation of  $x_1, x_2, ..., x_n$  in the linear for<br>except the unknown  $x_1$ . Namely, we use the normed vector<br>..., *n*.<br>( $\therefore$ <br> $\therefore$ <br> $\frac{1}{1} = d_2 y_2 + \frac{1}{3} y_3 + ... + \frac{1}{n} y_n$ , i.e. the indicated orthogor<br>where  $1 \le k$ *f* in linear homogenous form  $g = d_n u_n$ , where<br>  $\int_{0}^{2} x_1 dx_1 dx_1 dx_2 dx_3 dx_4 dx_5 dx_6 dx_7 dx_8 dx_9 dx_9 dx_1 dx_1 dx_2 dx_3 dx_4 dx_5 dx_7 dx_8 dx_9 dx_9 dx_1 dx_1 dx_2 dx_3 dx_4 dx_5 dx_7 dx_8 dx_9 dx_9 dx_1 dx_1 dx_2 dx_3 dx_4 dx_5 dx_7 dx_8 dx_9 dx_1 dx_1 dx_2 dx_3 dx_4 dx_4 dx_5 dx_7 dx_8 dx_9 dx_1 dx_1 dx_2 dx_3$ thenobviously  $f = \sqrt{\frac{3}{4}}x_1 = d_1x_1$ ,  $d_1 = \sqrt{a_1^2} > 0$  and the theorem is proved. If<br>
formation  $x_1 = -u_1$  and we obtain  $f = (-1)u_1 = \sqrt{\frac{3}{4}}u_1 = d_1u_1$ ,  $d_1 = \sqrt{a_1^2} > 0$ .<br>
it holds  $x_1 \neq 0$ .<br>  $x_2 + \ldots + x_nx_n$  with an 1 ... O. then we apply the orthogonal transformation  $x_i = -u_i$  and we obtain  $f = (-1)u_i = \sqrt{\frac{2}{3}}u_i = d_iu_i$ .  $d_i = \sqrt{a_i^2}$ <br>herefore, the theorem holds for  $n = 1$ .<br>uppose that  $n \ge 2$ . Let, for determination, it holds  $x_i = -u_i$  se that  $n \ge 2$ . Let, for determination, it holds  $_1 \ne 0$ .<br>
t we reduce the form  $f = _1x_1 + _2x_2 + ... + _nx_n$  withan orthogonal<br>  $I_2y_2 + _3y_3 + ... + _ny_n$ , where  $y_2,..., y_n$  are unknowns, i.e. we exceptth<br>  $\frac{2}{d_2}$ ,  $d_2 = \sqrt{\frac{2}{1$ =1.<br> *k*  $x_1x_2x_3 + \cdots + x_nx_n$  with<br>  $x_1x_2x_3 + \cdots + x_nx_n$  with a orthogonal transformation of  $x_1, x_2, \ldots, x_n$  in the linear form<br> *kech*  $x_2, \ldots, x_nx_n$  are unknowns, i.e. we except<br> *kech d*<sub>2</sub>  $y_1, \ldots, y_n$  are unknowns *a* d<sub>2</sub> =  $\sqrt{\frac{2}{1} + \frac{1}{2}}$ <br> *a* d<sub>2</sub> =  $\sqrt{\frac{2}{1} + \frac{2}{2}}$ <br> *a* d<sub>2</sub> =  $\sqrt{\frac{2}{1} + \frac{2}{2}}$ <br> *a* d<sub>2</sub> y<sub>1</sub> +  $\frac{1}{d_2}$  y<sub>2</sub>, x<sub>2</sub> =  $-\frac{1}{d_2}$  y<sub>1</sub> +  $\frac{2}{d_2}$  y<sub>2</sub>, x<sub>i</sub> = y<sub>i</sub>, i = 3<br> *a* to verify, that this tr  $d_2y_2 + 3y_3 + ... + y_ny_n$ , where  $y_2,..., y_n$  are u<br>  $\left(\frac{2}{d_2}\right)$ ,  $d_2 = \sqrt{\frac{2}{1} + \frac{2}{2}}$ <br>
we form the linear transformation<br>  $x_1 = \frac{2}{d_2}y_1 + \frac{1}{d_2}y_2$ ,  $x_2 = -\frac{1}{d_2}y_1 + \frac{2}{d_2}y_2$ ,<br>
easily to verify, that this tr  $\left(\frac{1}{d_2}, \frac{2}{d_2}\right), d_2 = \sqrt{\frac{2}{1} + \frac{2}{2}}$ <br>
and we form the<br>
linear transformation<br>  $f_1: x_1 = \frac{2}{d_2}y_1 + \frac{1}{d_2}y_2, x_2 = -\frac{1}{d_2}y_1 + \frac{2}{d_2}y_2, x_i = y_i, i = 1$ <br>
It is easily to verify, that<br>
this transformation is or we reduce the form  $f = \frac{1}{r}X_1 + \frac{1}{2}X_2 + \dots + \frac{1}{n}X_n$  withian orthogonal transformation of  $X_1, X_2, ..., X_n$  in<br>  $y_2 + \frac{1}{3}y_3 + \dots + \frac{1}{n}y_n$ , where  $y_2, ..., y_n$  are unknowns, i.e. we except the unknown  $X_1$ . Namely, we us mation<br>  $c_2 = -\frac{1}{d_2} y_1 + \frac{2}{d_2} y_2$ ,  $x_i = y_i$ ,  $i = 3,...,n$ .<br>
transformation is orthogonal and that  $f_1 = d_2 y_2$ .<br>
nown  $x_1$  in  $f$ .<br>
I transformation  $\{\ }$ <sub>-1</sub> from the form (1), where  $1 \le k$ <br>  $\ast$   $\ast$   $\ast$   $\ast$ ,  $\ast$  $y_1 + \frac{2}{d_2} y_2$ ,  $x_i = y_i$ ,  $i = 3,...,n$ .<br>
mation is orthogonal and that  $f_1 = d_2 y_2 + 3y_3 + ... + n$ <br> *a* f.<br> *d d d*  $f_1$  from the form (1), where  $1 \le k - 1 \le n - 2$ , it is<br>  $d \left(\frac{2}{k} + \frac{2}{k} + ... + \frac{2}{k}\right)$ .<br>  $d \left(\frac{d}{dx} + \frac{1$  $i_1x_1 + 2x_2 + \ldots + x_n$ , withan orthogonal transformation of  $x_i$ ,  $x_2$ ,  $\ldots$ ,  $x_n$  in the linear form<br>ere  $y_2$ ,  $\ldots$ ,  $y_n$  are unknowns, i.e. we except<br>the unknown  $x_i$ . Namely, we use the normed vector<br>on<br> $-\frac{1}{d_2}y_$ 

and we form thelinear transformation

$$
\{ \zeta_1 : x_1 = \frac{2}{d_2} y_1 + \frac{1}{d_2} y_2, \quad x_2 = -\frac{1}{d_2} y_1 + \frac{2}{d_2} y_2, \quad x_i = y_i, \quad i = 3, \dots, n. \tag{1}
$$

It is easily to verify, that this transformation is orthogonal and that  $f_1 = d_2 y_2 + a_3 y_3 + ... + a_n y_n$ , i.e. the indicated orthogonal transformation excepts theunknown  $x_1$  in  $f$ . Insformation is orthogonal and that  $f_1 =$ <br>
wn  $x_1$  in  $f$ .<br>
ansformation  $\left\{\int_{-1}^{1} f(t) dt\right\}$  from the form (1), when<br>  $n z_n$ ,<br>
d  $d_k = \sqrt{\frac{2}{1} + ... + \frac{2}{k}}$ .<br>
the unknown  $z_k$  with theorthogonal transf<br>  $d_{k+1} = -\frac{d_k}{d_{$ tion is orthogonal and that  $f_1 = d_2 y_2$ <br>  $f$ .<br>
ation { <sub>-1</sub> from the form (1), where  $1 \le h$ <br>  $\frac{2}{h} + ... + \frac{2}{k}$ .<br>
own  $z_k$  with theorthogonal transformation<br>  $k + t_{k-1} + \frac{k+1}{d_{k+1}}t_k$ ,  $z_i = t_i$ ,<br>  $\frac{1}{d_k + \frac{2}{k+1}}$ ,  $\cd$ ransformation is orthogonal and that  $f_1 = d_2 y_2$ <br>
own  $x_1$  in  $f$ .<br>
transformation  $\left\{\int_{-1} f$ rom the form (1), where  $1 \leq$ <br>  $\int_{R} z_n$ ,<br>
and  $d_k = \sqrt{\frac{2}{1} + ... + \frac{2}{k}}$ .<br>  $\int_{k+1}^{2} = -\frac{d_k}{d_{k+1}} t_{k-1} + \frac{k+1}{d_{k+1}} t_k$ , ansformation is orthogonal and that  $f_1 = d_2y_2 + 3y_3 + ... + n y_n$ <br>
wm  $x_1$  in  $f$ .<br>
ransformation  $\left\{\int_{-1}^{1}$  from the form (1), where  $1 \le k - 1 \le n - 2$ , it is o<br>  $n z_n$ ,<br>
dd  $d_k = \sqrt{\frac{2}{1} + ... + \frac{2}{k}}$ .<br>
the unknown  $z_k$  with t 2),  $i = 3, ..., n$ .<br>
(1)<br>
and that  $f_1 = d_2y_2 + a_3y_3 + ... + a_3y_n$ , i.e. the indicated<br>orthogonal<br>
form (1), where  $1 \le k - 1 \le n - 2$ , it is obtained the linear form<br>
begonal transformation<br>  $i_3 = t_1$ ,<br>  $d_{k+1} = \sqrt{\frac{2}{k} + \frac{2}{k} + ... + \frac$ 

Suppose, that bythe orthogonal transformation  $\{-1$  from the form (1), where  $1 \leq k-1 \leq n-2$  , it is obtained the linear form

$$
f_{k-1} = d_k z_k + \, k_{k+1} z_{k+1} + \ldots + \, k_n z_n,
$$

where  $z_1, ..., z_n$  are unknowns and  $d_k = \sqrt{\frac{2}{1} + ... + \frac{2}{k}}$ .

In the linear form  $f_{k-1}$  we except the unknown  $z_k$  with the orthogonal transformation

It is easily to verify, that this transformation is orthogonal and that 
$$
f_1 = d_2y_2 + 3y_1
$$
  
transformation excepts the unknown  $x_1$  in  $f$ .  
Suppose, that by the orthogonal transformation  $\{f_{-1}$  from the form (1), where  $1 \le k - 1 \le f_{k-1} = d_k z_k + \frac{1}{k+1} z_{k+1} + \dots + \frac{1}{n} z_n$ ,  
where  $z_1, ..., z_n$  are unknowns and  $d_k = \sqrt{\frac{2}{1} + \dots + \frac{2}{k}}$ .  
In the linear form  $f_{k-1}$  we except the unknown  $z_k$  with the orthogonal transformation  

$$
\{f_k : z_k = \frac{k+1}{d_{k+1}} t_{k-1} + \frac{d_k}{d_{k+1}} t_k, z_{k+1} = -\frac{d_k}{d_{k+1}} t_{k-1} + \frac{k+1}{d_{k+1}} t_k, z_i = t_i,
$$
 $i = 1, ..., k-1, k+2, ..., n$ , where  $d_{k+1} = \sqrt{d_k^2 + \frac{2}{k+1}}$ ,  $\dots$   $d_{k+1} = \sqrt{\frac{2}{1} + \frac{2}{2} + \dots + \frac{2}{k+1}}}$ .

 $i = 1,..., k-1, k+2,..., n$ , where  $d_{k+1} = \sqrt{d_k^2 + \frac{2}{k+1}}, \ldots, d_{k+1} = \sqrt{\frac{2}{k+1} + \frac{2}{k+1} + \frac{2}{k+1}}$ . In this way we obtain the linear form  $f = d + 1 + \ldots dt_{n,2} + \ldots + t_n t_n$ verny, that its data commutation is often<br>
a excepts the unknown  $x_1$  in f.<br>
by the orthogonal transformation  $\left\{\begin{array}{l} 1 \text{ ft} \\ -1 \text{ ft} \end{array}\right.$ <br>  $\left. + t_{k+1} z_{k+1} + ... + t_n z_n \right\}$ ,<br>  $z_n$  are unknowns and  $d_k = \sqrt{\frac{2}{1} + ... + \frac{2$ for verify, that this transformation is<br>
on excepts the unknown  $x_1$  in  $f$ .<br>
at by the orthogonal transformation {<br>  $x_k + \frac{1}{k+1}z_{k+1} + \dots + \frac{1}{n}z_n$ ,<br>  $z_n$  are unknowns and  $d_k = \sqrt{\frac{2}{1} + \dots}$ <br>
form  $f_{k-1}$  we except t on is ofthogonal and that  $J_1 - a_2 y_2$ <br>
f<br>  $\frac{1}{1}$ <br>  $\frac{1}{1} + ... + \frac{2}{k}$ <br>  $\frac{1}{1} + ... + \frac{2}{k}$ <br>  $\frac{1}{1} + ... + \frac{2}{k}$ <br>  $\frac{1}{1} + \frac{k+1}{d_{k+1}} t_k$ ,  $z_i = t_i$ ,<br>  $\sqrt{d_k^2 + \frac{2}{k+1}}$ ,  $\therefore d_{k+1} = \sqrt{\frac{2}{1} + \frac{2}{2} + ...}$ *f*.<br>
ation { -<sub>1</sub> from the form (1), where<br>  $\int_{-1}^{2} + ... + \int_{k}^{2}$ .<br>
nown  $z_k$  with theorthogonal transfor<br>  $\frac{d_k}{k+1}t_{k-1} + \frac{k+1}{d_{k+1}}t_k$ ,  $z_i = t_i$ ,<br>  $= \sqrt{d_k^2 + \frac{2}{k+1}}$ , ...  $d_{k+1} = \sqrt{\frac{2}{1} + \frac{2}{2}}$ . *i*  $\frac{1}{d_2} \cdot \frac{2}{d_2}$ ,  $\frac{1}{d_2} \cdot \frac{4}{d_2} = \sqrt{\frac{2}{4} + \frac{2}{2}}$ <br> *i*  $\frac{1}{d_2} \cdot \frac{2}{d_2} \cdot \frac{1}{d_2} \cdot \frac{$ *k*  $\frac{1}{d_2} y_i + \frac{2}{d_2} y_2$ ,  $x_i = y_i$ ,  $i = 3,...,n$ .<br>
(1)<br>
formation is orthogonal and that  $f_i = d_2 y_2 + y_3 + ... + y_n y_n$ , i.e. the indicated<br> *k*  $f_i$  in  $f$ .<br> *k* formation  $\left\{ \int_{-1}^{1} f(x) dx \right\} = \int_{0}^{1} f(x) dx$ <br> *k*  $\left\{ x - \sqrt{\frac{$ 

$$
f_k = d_{k+1}t_{k+1} + \quad_{k+2}t_{k+2} + \ldots + \quad_n t_n
$$

where  $t_1, ..., t_n$  are new unknowns.

The induction is ended. Finally we obtain the linear form of the kind  $g = f_{n-1} = d_n u_n$ , where  $u_n$  is anunknownand  $i<sub>1</sub><sup>2</sup> + ... + i<sub>n</sub><sup>2</sup>$  is a positive real number. The linear form g is obtained with a sequential realizationof the transforms  $\{1, 1, 2, ..., \{1, 1, ..., k\}$ , i.e. with the product  $\{1, 2, ..., 1, ..., k\}$ . However it is well known, that the product of orthogonal transformations of the unknowns is an orthogonal transformation of the original unknowns. Therefore,  $\{\{\zeta_1,\zeta_2...\zeta_{n-1}\}$  is an orthogonal transformation of  $\sum_{i=1}^{n}$ ,...,  $x_n$ , which reduces  $f$  in  $g = d_n u_n$ , where  $d_n = \sqrt{\frac{2}{1} + ... + \frac{2}{n}}$ . The theorem is proved. ad we form the<br>linear transformation<br>  $\int_1 :X_i = \frac{2}{d_2} y_i + \frac{1}{d_2} y_2$ ,  $X_2 = -\frac{1}{d_3} y_i + \frac{2}{d_3} y_2$ ,  $X_i = y_i$ ,  $i = 3,...,n$ .<br>
is easily to verify, that<br>this transformation is orthogonal and that  $f_i = d_3 y_1 + \frac{1}{d_3} y_3 +$  $t = \frac{1}{d_2} y_i + \frac{1}{d_2} y_2$ ,  $x_2 = -\frac{1}{d_2} y_i + \frac{2}{d_2} y_2$ ,  $x_i = y_i$ ,  $i = 3,...,n$ .<br>
(1)<br> *tial*  $y_i$  to retriv, thaths transformation is orthogonal and that  $f_1 = d_2 y_2 + y_3 + ... + y_n$ , i.e. the indicated<br>
nring in vertry, that ear form  $f_{k-1}$  we except the unknowr<br>  $=\frac{k+1}{d_{k+1}}t_{k-1} + \frac{d_k}{d_{k+1}}t_k$ ,  $z_{k+1} = -\frac{d_k}{d_{k+1}}t_k$ <br>  $k-1, k+2, ..., n$ , where  $d_{k+1} = \sqrt{a_{k+1}}t_{k+1} + \frac{1}{b_{k+2}}t_{k+2} + ... + \frac{1}{b_n}$ ,<br>  $..., t_n$  are new unknowns.<br>
action is ended. t is easily to verify, that<br>this transformation is orthogonal and that  $f_1 = d_2y_2 + ... + z_xy_n$ , i.e. the indit<br>ransformation excepts the<br>unknown  $x_i$  in  $f$ .<br>iarprose, that hydre orthogonal ransformation  $\left\{ \int_{-1}^{1} f(\omega) \, d$ It is easily to verify, that<br>this transformation is orthogonal and that  $f_1 = d_2$ )<br>transformation excepts the<br>unknown  $x_i$  in  $f$ .<br>Suppose, that by the orthogonal transformation  $\left\{ \begin{array}{l} -_1$  from the form (1), where 1  $-1 \le n - 2$ , it is obtained the linear form<br>  $\frac{1}{2}$ ... In this way we obtain the linear form<br>  $g = f_{n-1} = d_n u_n$ , where  $u_n$  is anunknow<br>
tih a sequential realization the trans<br>
t the product of orthogonal transformation<br>  $f_{k-1} = d_k z_k + \sum_{k+1} z_{k+1} + \ldots + \sum_{n} z_n$ ,<br>
where  $z_1, \ldots, z_n$  are unknowns and  $d_k = \sqrt{\frac{3}{1} + \ldots + \frac{2}{k}}$ .<br>
in the linear form  $f_{k-1}$  we except<br>the unknown  $z_k$  with the<br>orthogonal transformation<br>  $\{\xi_k : z_k = \frac{k+1}{d_{k+1}} t_{$ 1 *from the torm* (1), where  $1 \leq k - 1 \leq n - 2$ , it is obtained the linear form<br>  $\frac{k+1}{k}I_k$ ,  $z_i = I_j$ ,<br>
with theorthogonal transformation<br>  $\frac{k+1}{\ell_{k+1}}I_k$ ,  $z_i = I_j$ ,<br>  $\frac{1}{d_{k+1}} \cdots d_{k+1} = \sqrt{\frac{2}{k} + \frac{2}{3} + ... + \frac{2}{k+1}}$ .

## **2.Canonical forms of the hypersurfaces of the first degree**

The surfaces of the first degree in the real affine Euclidean space  $n_{n}$  are given bythe equation

38523 International Journal of Current Research, Vol. 9, Is

\n
$$
f_1 = \sum_{i=1}^{n} a_i x_i + a = 0
$$

\nwhere  $a_i, a \in \text{and at least } a_i \neq 0$ .

\n**Definition1.** We say, that a plain has a canonical form (canonical form).

where  $a_i, a \in \text{and at least } a_i \neq 0$ .

**Definition1.** We say, that a plain has a canonical form (canonical equation) if this plain has the form  $z = 0$ , where z is an unknown. Let in  $\pi_n$  an orthogonal coordinate system  $Ox_1x_2...x_n$  is given. We note that the equation  $x_n = 0$  in  $\pi_n$  is the plain defined by the coordinate axes  $Ox_1, ..., Ox_{n-1}$ , since every point  $(x_1, x_2, ..., x_{n-1}, 0)$  of  $\pi$  satisfies this equation. 523 International Journal of Current Research, Vo.<br>  $f_1 = \sum_{i=1}^n a_i x_i + a = 0$ <br>
where  $a_i, a \in \text{and at least } a_i \neq 0$ .<br> **Definition1.** We say, that a plain has a canonical form (canon whown. Let in  $\sum_n$  an orthogonal coordinate sys *International Journal of Current Research, Vol. 9, Issue, 10, pp.58521-58529, October, 2017*<br>  $=\sum_{i=1}^{n} a_i x_i + a = 0$ <br>
Free  $a_i, a \in \text{and at least } a_i \neq 0$ .<br> **inition1.** We say, that a plain has a canonical form (canonical equation *International Journal of Current Research, Vol. 9, Issue, 10, pp.58521-58*<br>  $\sum_{i=1}^{n} a_i x_i + a = 0$ <br>  $a_i, a \in \text{and at least } a_i \neq 0.$ <br> **iion1.** We say, that a plain has a canonical form (canonical equation) if this<br>
wn. Let in  $\sum_{$ *International Journal of Current Research, Vol. 9, Issue, 10, pp.58521-58529, October, 2017*<br>
0<br>
d at least  $a_i \neq 0$ .<br>
, that a plain has a canonical form (canonical equation) if this plain has the form  $z = 0$ , where  $z$ 1 1 ,..., *Ox Oxn x x x* 1 2 1 , ,..., ,0 *<sup>n</sup> <sup>n</sup>*

In the following result we obtain the canonical form of the plain  $f_1$  given by (2).

**Theorem 2.**In the many-dimensional real affine Euclidean space  $\frac{1}{n}$  the following cases hold for the plain  $f_1$ .

1)If at least one coefficient  $a_i$  is non-zero, then the canonical form of  $f_i$  is given by the equation  $z_n = 0$ , i.e. this canonical form the plain, defined by the coordinate axes  $O_1z_1,...,O_1z_{n-1}$  of some coordinate system  $O_1z_1z_2...z_n$  of  $\theta$ . 2)If  $a_1 = ... = a_n = a = 0$ , then  $f_1$  is the space  $r_n$ . *nal of Current Research, Vol. 9, Issue, 10, pp.58521-58529, October, 2017*<br>
(2)<br>
canonical form (canonical equation) if this plain has the form  $z = 0$ , where Z is<br>
ordinate system  $OX_1X_2...X_n$  is given. We note<br>
that the

3)If  $a_1 = \dots = a_n = 0$   $a \neq 0$ , then  $f_1$  is the empty set.

**Proof.**Statements 2) and 3) of the theorem are trivial. We shall prove the statement 1). We represent the surface  $f_1$  in the form  $\int_{1}^{c} = f + a$ , where  $f = a_1 x_1 + ... + a_n x_n$ . Theorem1 implies that there exists an orthogonal transformation which reduces in  $f = d_n y_n$ ,  $d_n = \sqrt{a_1^2 + ... + a_n^2}$ . In the last form of f we replace  $x_1,...,x_n$  with the unknowns . Consequently  $f_1$  obtains the form  $d_n y_n + a = 0$ . We make the transformation  $\frac{1}{x_n} + a$  which wesupplementwith International Journal of Current Research, Vol. 9, Issue, 10, pp.58521-58529, October, 2017<br>  $\sum_{i=1}^{n} a_i x_i + a = 0$ <br>  $\alpha, a_i \in \mathbb{R}$  and at least  $a_i \neq 0$ .<br> **atition1.**We say, that a plain has a canonical form (canonical International Journal of Current Research, Vol. 9, Issue, 10, pp.58521-58529, October, 2017<br>  $\sum_{i=1}^{n} a_i x_i + a = 0$ <br>  $\therefore a_i, a \in \mathbb{R}$  and at least  $a_j \neq 0$ .<br> **antion I.**We say, that a plain has a canonical form (canonica  $f_1 = \sum_{j=1}^{n} a_j x_j + a = 0$ <br>
there  $a_j, a \in A$  and at least  $a_i \neq 0$ .<br> **effinition 1.**We say, that a plain has a canonical form (canonical equation) if this plain has the form<br> **effinition 1.**We say, that a plain has a canon cast  $a_i \neq 0$ .<br> **f** a plain has a canonical form (canonical equation) if this plain has the form  $z = 0$ , where *z* is an<br> *f* a rothogonal coordinate system  $Ox_1x_2...x_n$  is given. We noted<br>at the equation  $x_n = 0$  in  $\pi$ **Example 12.1.** The many-dimensional real affine Euclidean space  $n_n$  the following c<br>least one coefficient  $a_i$  is non-zero, then the canonical form of  $f_1$  is given t<br>in, defined by the coordinate axes  $O_1z_1, ..., O_1z_{n$  $f_1 = \sum_{i=1}^{n} a_i x_i + a = 0$ <br>
there  $a_i, a \in A$  and at least  $a_i \neq 0$ .<br> **efinition 1.**We say, that a plain has a canonical form (canonical equation) if this plain has the form  $z_i$ <br>
defined by the coordinate axes  $Ox_1,..., Ox_{n$ ast  $a_i \neq 0$ .<br>
a plain has a canonical form (canonical equation)<br>
orthogonal coordinate system  $Ox_1x_2...x_n$  is given.<br>
vaxes  $Ox_1,..., Ox_{n-1}$ , since every point  $(x_1, x_2,...,x_{n-1}$ <br>
obtain the canonical form of the plain  $f$ (2)<br>
a canonical form (canonical equation) if this plain has the form  $z = 0$ , where  $z$  is an<br> *noordinate system*  $O[X_1x_2...x_n]$  is given. We note<br>that the equation  $x_n = 0$  in  $\alpha$  is the<br>  $Ox_{n-1}$ , since every point  $\left$ ,..., *n x x*  $1, \ldots, Y_n$ . Consequently  $J_1$ where  $a_i, a \in \text{ and } a_i \neq 0$ .<br> **Definition1.** We say, that a plain has a canonical for<br>
nknown. Let in  $n$  an orthogonal coordinate systen<br>
efined by the coordinate axes  $Ox_1, ..., Ox_{n-1}$ , since ev<br>
nthe following result we o as  $a_i \neq 0$ .<br>  $f_i \neq 0$ .<br>  $f_i$  a plain has a canonical form (canonical equation) if this plain has the form  $z = 0$ , where  $z$  is an<br> *f* throgonal coordinate system  $Ox_i x_i...x_n$  is given. We note<br>that the equation  $x_n = 0$  ion  $z_n = 0$ , i.e. this canonical for  $n_0$  of  $n_1$ .<br>
present the surface  $f_1$  in the fonal transformation which reduce  $x_1, ..., x_n$  with the unkno<br>  $x_n = y_n + \frac{a}{d_n}$  which wesupplements which in  $E_n$  and  $f_1$  obtains the f Equently  $f_1$  obtains the form  $d_n y_n + a = 0$ . We make the transformation  $z_n = y_n + \frac{a}{d_n}$  which we<br>supplementwith<br>  $= z_{n-1}$ , where  $z_1, ..., z_n$  are unknowns. In this way we obtain a translation in  $E_n$  and  $f_1$  obtains the<br> has the form  $z = 0$ , where  $z$  is an<br>the equation  $x_n = 0$  in  $n$  is the plain<br>fies this equation.<br>or the plain  $f_1$ .<br>tion  $z_n = 0$ , i.e. this canonical form<br> $z_n$  of  $n$ .<br>epresent the surface  $f_1$  in the form<br>gonal transf is the form  $z = 0$ , where z is an<br>
e equation  $x_n = 0$  in  $n$  is the plain<br>
es this equation.<br>
the plain  $f_1$ .<br>
on  $z_n = 0$ , i.e. this canonical form<br>
of  $n$ .<br>
on  $z_n$ .<br>
or  $f_1$  in the form<br>
nal transformation which reduc **Definition1.** We say, that a plain has a canonical form (canonical equation) if this plain has the form  $z = (x_0, x_0, \ldots, x_n)$  and  $C(x_1, \ldots, x_{n-1})$ ,  $D(x_2, \ldots, x_n)$  is given. We note<br>that the equation  $x_n = (x_0, x_0, \ldots, 0, x$ has a canonical form (canonical equation) if this plain has the form  $z = 0$ , where  $z$  is an<br>al coordinate system  $Ox_ix_j...x_n$  is given. We note<br>that the equation  $x_n = 0$  in  $z_i$  is the plain<br> $,..., Ox_{n-1}$ , since every point In this hown. Let in  $\alpha$  an orthogonal coordinate system  $OX_1X_2...X_n$  is<br>
defined by the coordinate axes  $OX_1,..., OX_{n-1}$ , since every point  $(X_1, X_2, ...)$ <br>
in the following result we obtain the canonical form of the plain least one coefficient  $a_i$  is non-zero, then the canonical form of  $f_1$  is given by the equation, defined by the coordinate axes  $O_1z_1,..., O_1z_{n-1}$  of some coordinate system  $O_1z_1z_2...z_n$ <br>  $= ... = a_n = a = 0$ , then  $f_1$  is t  $a_n x_n$  in  $f = d_n y_n$ ,  $d_n = \sqrt{a_1^2 + ... + a_n^2}$ . In the quently  $f_1$  obtains the form  $d_n y_n + a = 0$ <br>  $= z_{n-1}$ , where  $z_1,..., z_n$  are unknowns. If  $= 0$ .<br>  $\text{wed. } \Box$ <br> **ns of the hypersurfaces of the second deg**<br>
sional real affin  $\begin{aligned}\n&\vdots \dots + a_n x_n \text{ in } J = a_n y_n, & a_n = \sqrt{a_1^2 + \dots + a} \\
&\text{Consequently } f_1 \text{ obtains the form } d_n y_n + \\
&\text{, } y_{n-1} = z_{n-1}, \text{ where } z_1, \dots, z_n \text{ are unknown,} \\
&\text{i.e. } z_n = 0.\n\end{aligned}$ <br>
In is proved. $\Box$ <br> **all forms of the hypersurfaces of the secon**-dimensional real aff equently  $f_1$  obtains the form  $d$ <br>  $= z_{n-1}$ , where  $z_1, ..., z_n$  are<br>  $i_n = 0$ .<br>
roved. $\Box$ <br> **ims of the hypersurfaces of the ensional real affine Euclidean sp**<br> **i**,  $\sum_{i,j=1}^n a_{ij}x_ix_j + 2\sum_{i=1}^n a_ix_i + a = 0$ <br>
and at least  $a$ IIf at least one coefficient  $a_j$  is non-zero, then the canonical form of  $f_1$  is given by the exerchion, defined by the coordinate axes  $O_1z_1,..., O_1z_{n-1}$  of some coordinate system  $O_1z_1z_2$ <br>
If  $a_1 = ... = a_n = a = 0$ , then  $\mu$  a = 0<br>
and at least  $a_i \neq 0$ .<br>
and at least  $a_i \neq 0$ .<br>
and at least  $a_i \neq 0$ .<br>  $\sin A$ , that a plain has a canonical form (canonical equation) if this plain has the form  $z = 0$ , where  $z$  is an<br>  $\sin A$ , an orthogonal in, derined by the coordinate axes  $Q_1z_1,..., Q_1z_n$  of some coordinate system  $Q_1z_1z_2$ <br>  $= ... = a_n = a = 0$ , then  $f_1$  is the space  $\binom{n}{n}$ .<br>
Statements 2) and 3) of the theorem are trivial. We shall prove the statement 1). ..,  $O_1 z_{n-1}$  of some coordinate system  $O_1 z_1 z_2 ... z_n$ <br>  $\cdots$ <br>
arrivial. We shall prove the statement 1). We replace<br>
Theorem1 implies that there exists an orthogo<br>  $\frac{1}{2} + ... + a_n^2$ . In the last form of  $f$  we replace<br> **Proof.**Statements 2) and 3) of the theorem are trivial. We shall  $f_1 = f + a$ , where  $f = a_1x_1 + ... + a_nx_n$ . Theorem 1 implies  $f = a_1x_1 + ... + a_nx_n$  in  $f = d_1y_n$ ,  $d_n = \sqrt{a_1^2 + ... + a_n^2}$ . In the  $y_1,..., y_n$ . Consequently  $f_1$  obtains t  $a_1 = ... = a_n = 0$  a  $\neq 0$ , then  $f_1$  is the empty set.<br> **of.** Statements 2) and 3) of the theorem are trivial. We shall prove the statement 1). We represent the surf<br>  $F = f + a$ , where  $f = a_1x_1 + ... + a_nx_n$ . Theorem i implies<br>  $f$ *f*  $\theta$ ,  $\theta$ ,  $\sinh f_1$  *f* and entry, set.<br> *f*  $= a_1x_1 + ... + a_nx_n$ . Theorem 1 implies<br> *f*  $= d_ny_n$ ,  $d_n = \sqrt{a_1^2 + ... + a_n^2}$ . In the last form of<br> *f*  $\int f_1$  obtains the form  $d_ny_n + a = 0$ . We make the<br> *f*  $\int f_1$  obtains th  $a \neq 0$ , then  $f_i$  is the empty set.<br>
3) of the theorem are trivial. We shall prove the statement 1). We represent the surface  $f_i$  in the form<br>  $= a_1x_1 + ... + a_nx_n$ . Theoremal implies<br>  $f = d_ny_1 + ... + d_nx_n$ . Theoremal implies<br> *x<sub>n</sub>*. Theorem1 implies<br>that there exists an orthogonal transformation<br>which reduces  $\sqrt{a_i^2 + ... + a_n^2}$ . In the last form of  $f$  we replace  $x_1, ..., x_n$  with the unknowns<br>orm  $d_n y_n + a = 0$ . We make the transformation  $z_n = y_n + \frac$ +  $a_n x_n$ . Theorem1 implies<br>that there exists an orthogonal transformationwhich reduces<br> $d_n = \sqrt{a_1^2 + ... + a_n^2}$ . In the last form of  $f$  we replace  $x_1, ..., x_n$  with the unknowns<br>the form  $d_n y_n + a = 0$ . We make the transformation

, where  $z_1, ..., z_n$  are unknowns. In this way we obtain a translation in  $E_n$  and  $f_1$  obtains the form  $d_{n}z_{n}=0$ , i.e.  $z_{n}=0$ .

The theorem is proved. $\square$ 

#### **4. Canonical forms of the hypersurfaces of the second degree**

In the many-dimensional real affine Euclidean space  $\bar{h}$  the surfaces of the second degree are given bythe equation

$$
y_1 = z_1, ..., y_{n-1} = z_{n-1}, \text{ where } z_1, ..., z_n \text{ are unknowns. In this way we obtain a translation in } E_n \text{ and } f_1 \text{ obtains the form}
$$
\n
$$
d_n z_n = 0, \text{ i.e. } z_n = 0.
$$
\nThe theorem is proved.
$$
\Box
$$
\n**4. Canonical forms of the hypersurfaces of the second degree**\nIn the many-dimensional real affine Euclidean space  $n$  the surfaces of the second degree are given by the equation\n
$$
f(x_1, ..., x_n) = \sum_{i,j=1}^n a_{ij} x_i x_j + 2 \sum_{i=1}^n a_i x_i + a = 0, \ a_{ji} = a_{ij}, i, j = 1, 2, ..., n,
$$
\nwhere  $a_{ij}, a_i, a \in \Box$  and at least  $a_j \neq 0$  and  $x_1, ..., x_n$  are unknowns.\nThe forms\n
$$
f_1 = 2 \sum_{i=1}^n a_i x_i + a, \ f_2 = \sum_{i,j=1}^n a_{ij} x_i x_j, \quad a_{ji} = a_{ij}, i, j = 1, 2, ..., n,
$$
\nare called a linear and a quadratic part of  $f$ , respectively. Denote by  $r(f_2)$  the range of the quadratic part  $f_2$  of  $f$ . If  $n = 1$ , i.e., in a space  $E_1$ , the canonical form of  $f = a_1 x^2 + 2a_1 x + a$  is well known and absolutely trivial.\n\nIf  $n = 2$  and  $n = 3$  the canonical form of  $f$  is also well known. We shall consider  $n \geq 2$ , in order to do an analogy. We put 
$$
X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}
$$
, and  $Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ , where  $x_1, ..., x_n$  and  $y_1, ..., y_n$  are systems of unknowns.

where  $a_{ij}, a_i, a \in \text{and at least } a_j \neq 0 \text{ and } x_1, \dots, x_n$  are unknowns. The forms

$$
a_n z_n = 0, \text{ i.e. } z_n = 0.
$$
  
The theorem is proved.□  
**4. Canonical forms of the hypersurfaces of the second degree**  
In the many-dimensional real affine Euclidean space  $h_n$  the surfaces of the  
 $f(x_1, ..., x_n) = \sum_{i,j=1}^n a_{ij} x_i x_j + 2 \sum_{i=1}^n a_i x_i + a = 0, a_{ji} = a_{ij}, i, j = 1, 2, ...$   
where  $a_{ij}, a_i, a \in \text{ and at least } a_j \neq 0 \text{ and } x_1, ..., x_n \text{ are unknowns.}$   
The forms  
 $f_1 = 2 \sum_{i=1}^n a_i x_i + a, f_2 = \sum_{i,j=1}^n a_{ij} x_i x_j, \quad a_{ji} = a_{ij}, i, j = 1, 2, ..., n,$   
are called a linear and a quadratic part of  $f$ , respectively. Denote by  $r(f_n)$   
in a space  $E_1$ , the canonical form of  $f = a_1 x^2 + 2a_1 x + a$  is well known as  
If  $n = 2$  and  $n = 3$  the canonical form of  $f$  is also well known. We shall c  

$$
X = \begin{pmatrix} x_1 \\ \dots \\ x_n \end{pmatrix}
$$
 and 
$$
Y = \begin{pmatrix} y_1 \\ \dots \\ y_n \end{pmatrix},
$$
  
where  $x_1, ..., x_n$  and  $y_1, ..., y_n$  are systems of unknowns.

are called a linear and a quadratic part of f, respectively. Denote by  $r(f_2)$  the rang of the quadratic part  $f_2$  of f. If  $n = 1$ , i.e. in a space  $E_1$ , the canonical form of  $f = a_1x^2 + 2a_1x + a$  is well known and absolutely trivial.

If  $n = 2$  and  $n = 3$  the canonical form of  $f$  is also well known. We shall consider  $n \ge 2$ , in order to do an analogy. We put

$$
X = \begin{pmatrix} x_1 \\ \dots \\ x_n \end{pmatrix} \text{ and } Y = \begin{pmatrix} y_1 \\ \dots \\ y_n \end{pmatrix},
$$

where  $x_1, ..., x_n$  and  $y_1, ..., y_n$  are systems of unknowns.

At first we shall obtain the canonical form of  $f$  reducing the quadratic part  $f_2$  in a canonical form by an orthogonal transformation

, , , . (4) *X QY* 11 1 1 ... ... ... ... ... *<sup>n</sup> n nn q q Q q q ij ji q q ij q i ij j x q y j j j i ij f b y a b a q j n* 1,..., <sup>0</sup> *ij <sup>a</sup> r r f* <sup>2</sup> <sup>0</sup> ,...,  *r* 2 ,..., 0  *r n r b b c a*  

We replace every  $x_i$  by  $x_i = \sum q_{ij} y_j$ . Then the linear part  $f_1$  of  $f$  obtain the form  $1$  and  $1$  and  $1$  and  $1$  and  $1$  and  $1$ *n*  $=\sum_{j=1} q_{ij} y_j$ . Then the linear part  $f_1$  of  $f$  obtain the form

$$
f_1 = 2\sum_{j=1}^n b_j y_j + a, \ b_j = \sum_{i=1}^n a_i q_{ij}, \ j = 1, ..., n. \tag{5}
$$

In view of Theorem2, we shall suppose, that at least one coefficient  $a_{ii} \neq 0$ , i.e.  $r = r(f_2) > 0$ . Let the characteristic roots  $f_1$ ,...,  $f_r$  of the quadratic part  $f_2$  of  $f$  aredifferent from zero and  $f_{r+1}$ ,...,  $f_n = 0$ . Then we set

We replace every 
$$
x_i
$$
 by  $x_i = \sum_{j=1}^{n} q_{ij} y_j$ . Then the linear part  $f_1$  of  $f$  obtain the form  
\n
$$
f_1 = 2 \sum_{j=1}^{n} b_j y_j + a, b_j = \sum_{i=1}^{n} a_i q_{ij}, j = 1,...,n.
$$
\n(5)  
\nIn view of Theorem2, we shall suppose, that at least one coefficient  $a_{ij} \neq 0$ , i.e.  $r = r(f_2) > 0$ . Let the characteristic roots  
\n $\}_{1},..., \}_{r}$  of the quadratic part  $f_2$  of  $f$  are different from zero and  $\}_{r+1},..., \}_{n} = 0$ . Then we set  
\n
$$
c = a - \frac{b_1^2}{b_1} - ... - \frac{b_r^2}{b_r}.
$$
\n(6)  
\nIn the following result we shall obtain the canonical form of the surface  $f$  of the second degree in  $\frac{1}{n}$  by orthogonal

In the following result we shall obtain the canonical form of the surface  $f$  of the second degree in  $\binom{n}{r}$  by orthogonal transformations of the unknowns and by translations. We shall suppose that  $f$  is given by equation (3).

**Theorem3.** Let  $r > 0$  be the rang of the quadratic part  $f_2$  of the surface  $f$  of the second degree in the real affine Euclidean space  $n, n \geq 2$ . Then the following cases hold, in which the numbers  $b_i$  and  $c, j = 1,...,n$ , are defined by (5)and (6), respectively. 1)Let  $r = n$ . Then the surface f is with a canonical equation  $\mathcal{L} = QY, \quad Q = \begin{vmatrix} 1 & \cdots & \cdots & 1 \\ q_1 & \cdots & q_m & \cdots & q_n \end{vmatrix}, \quad q_{ij} = q_{ji}, \quad q_{ij} \in \mathcal{L}$ <br> *n* are phase every  $x_i$  by  $x_i = \sum_{j=1}^{n} q_{ij} y_j$ . Then<br>  $\mathcal{L} = 2 \sum_{j=1}^{n} b_j y_j + a, \quad b_j = \sum_{i=1}^{n} a_i q_{ij}, \quad j = 1, \ldots, n$ .<br>  $\mathcal{L} = 2 \sum_{j$ place every  $x_i$  by  $x_i = \sum_{j=1}^n q_{ij} y_j$ . Then the linear part  $f$ <br>  $2\sum_{j=1}^n b_j y_j + a$ ,  $b_j = \sum_{i=1}^n a_i q_{ij}$ ,  $j = 1,...,n$ .<br>
w of Theorem2, we shall suppose, that at least one<br>  $\sum_{j=1}^n b_j^2 + a_j^2$ ,  $\sum_{j=1}^n b_j^2 + a_j^2$ ,  $\sum$  $a - \frac{b_1^2}{b_1^2} - ... - \frac{b_r^2}{b_r^2}$ .<br>
he following result we shall obtain therefore shall obtain therefore shall obtain thereformations of the unknowns and by trancherent or **orem3.** Let  $r > 0$  be the rang of the quade, We replace every  $x_i$  by  $x_j = \sum_{j=1}^{n} q_{ij} y_j$ . Then<br>the linear part  $f_1$  of  $f$  obtain the form<br>  $f_1 = 2 \sum_{j=1}^{n} b_j y_j + a$ ,  $b_j = \sum_{j=1}^{n} a_j q_{ij}$ ,  $j = 1,...,n$ .<br>
In view of Theorem2,<br>
we shall suppose, that at least one coe  $\sum_{j=1}^{n} b_j y_j + a$ ,  $b_j = \sum_{i=1}^{n} a_i q_{ij}$ ,  $j = 1,...,n$ .<br> *c* of Theorem2, we shall suppose, that at least one coefficient  $a_{ij} \neq 0$ , i.e<br> *i*, of the quadratic part  $f_2$  of  $f$  are<br>different from zero and  $\}_{r+1},...$ ,  $\}_{n$  $2\sum_{j=1}^{n} b_j y_j + a$ ,  $b_j = \sum_{i=1}^{n} a_i a_{ij}$ ,  $j = 1,...,n$ .<br>
(5)<br>
w of Theorem2, we shall suppose, that at least one coefficient  $a_{ij} \neq 0$ , i.e.  $r = r(f_2) > 0$ . Let the characteristic roots<br>  $\sum_{i=1}^{n} b_i$ ,  $\sum_{j=1}^{n} \frac{b_j^2}{$ (5)<br>
uppose, that at least one coefficient  $a_{ij} \neq 0$ , i.e.  $r = r(f_2) > 0$ . Let the characteristic roots<br>
of  $f$  are<br>
different from zero and  $\}_{r=1},...$ ,  $\}_{n} = 0$ . Then we set<br>
(6)<br>
obtain the canonical form of the surface that at least one coefficient  $a_{ij} \neq 0$ <br>different from zero and  $\}_{r+1},...$ ,  $\}_{n}$ :<br>the canonical form of the surface<br>anslations. We shall suppose that  $f$ <br>aadratic part  $f_2$  of the surface  $f$  of<br>d, in which the number *f*,  $\int_{r}^{1}$  of the quadratic part  $f_2$  of  $f$  are<br>different from zero and  $\int_{r+1},...$ ,  $\int_{n}^{1} = 0$ . Then we set<br> $-\frac{b_1^2}{1} - ... - \frac{b_r^2}{1}$ .<br>Tollowing result we shall obtain the canonical form of the surface  $f$  of t  $c = a - \frac{1}{1} \sum_{1}^{n} \cdots - \sum_{j=1}^{n} \sum_{k=1}^{n}$ <br>
In the following result we shall obtain the canonical form of the sarface f of the second degree in  $\pi$ , by<br>
runsformations of the unknowns and by translations. We shall sup owing result we shall obtain the cantions of the unknowns and by translatio<br>
Let  $r > 0$  be the rang of the quadratic<br>
2. Then the following cases hold, in wh<br>
n. Then the surface  $f$  is with a canonic:<br>  $+\int_n z_n^2 = -c$ ,<br>
def al form of the surface  $f$  of the second degree in  $n$  by orthogonal<br> *i* e shall suppose that  $f$  is given by equation (3).<br>  $f_2$  of the surface  $f$  of the second degree in the real affine Euclidean space<br>
ne numbers  $b$ 

$$
\} _{1}z_{1}^{2} + ... + \, _{n}z_{n}^{2} = -c \,, \tag{7}
$$

where c is defined by (6) for  $r = n$ . Besides the following subcases hold.

**1.1**)If  $\}$ <sub>1</sub>,...,  $\}$ <sub>n</sub> have the same sign and  $c \neq 0$ , then f is an *ellipsoid*(an *ellipse*,if  $n = 2$ ).

**1.2**)Letat least two of the signs of  $\}$ <sub>1</sub>,...,  $\}$ <sub>n</sub> are different and  $c \neq 0$ . Then  $f$  is *ahyperboloid* (*a hyperbola*, if  $n = 2$ ).

**1.3**)If  $c = 0$ , then f is acone. Besides, if  $\}$ <sub>1</sub>,...,  $\}$ <sub>n</sub> have the same signs, then f is an imaginary cone with only one real point  $(0,0,...,0)$ . (If  $n = 2$  and the signs of  $\}$ <sub>1</sub> and  $\}$ <sub>2</sub> are different, then f is *a pair intersecting straight lines*.) 2)Let  $r = n - 1$ . We have two subcases.  $2^{2} + ... + 3_{n}z_{n}^{2} = -c$ ,<br>
re *c* is defined by (6) for  $r = n$ . Besides t<br>
If  $3_{1},...,3_{n}$  have the same sign and  $c \neq 0$ <br>
Letat least two of the signs of  $3_{1},...,3_{n}$  are<br>
If  $c = 0$ , then *f* is acone. Besides, if  $3_{1}$ ,<br> 1)). Let  $r = n$ . Then the surface  $f$  is with a canonical equation<br>  $\int_{1}^{n} z^{2} + ... + \int_{n} z_{n}^{2} = -c$ ,<br>
where  $c$  is defined by (6) for  $r = n$ . Resides the following subcases hold.<br>
1.1)If  $\frac{1}{2}, ..., \frac{1}{n}$ , have the same si Besides the following subcases hold.<br> *i*  $d \, \mathcal{C} \neq 0$ , then  $f$  is an *ellipsoid*(an *ellipse*, if  $n = 2$ ).<br>  $,..., \, f_n$  are different and  $\, \mathcal{C} \neq 0$ . Then  $f$  is *ahyperboloid (a hyperboloi,* if  $n = 2$ ).<br> *i*,..., 2) Leta least two of the signs of  $\int_1, ..., \int_n$  are different and  $c \neq 0$ . Then  $f$ <br>3) If  $c = 0$ , then  $f$  is acone. Besides, if  $\int_1, ..., \int_n$  have the same signs, then  $f$ .<br>3) If  $c = 0$ , then  $f$  is acone. Besides, if  $\int_1$ *i.1)I*  $f_1,..., f_s$ , have the same sign and  $c \neq 0$ , then  $f$  is an ellipsoid(an ellipsoid In hyperbola if  $n = 2$ ).<br> *r n z n f n <i>c n n f s n <i>n c e <i>n f n n* 

**2.1**)Let  $b_n \neq 0$ . Then the surface  $f$  is *aparaboloid*(a *parabola*, if  $n = 2$ ) with a canonical equation

$$
\Sigma_1 z_1^2 + \dots + \Sigma_{n-1} z_{n-1}^2 = -2b_n z_n \,. \tag{8}
$$

Besides if  $\{1, ..., \}_{n=1}$  are with the same signs, then f is *an elliptical paraboloid*andif at least two of signs of  $\{1, ..., \}_{n=1}$  are different, then  $f$  is *ahyperbolic paraboloid*. ....,0). (If  $n = 2$  and the signs of  $\}_{1}$  and<br>
st  $r = n - 1$ . We have two subcases.<br>
Let  $b_n \neq 0$ . Then the<br>
surface  $f$  is a parabonally<br>  $2^2 + ... + \}_{n=1} z_{n-1}^2 = -2b_n z_n$ .<br>
des if  $\}_{1}$ ,...,  $\}_{n=1}$  are with the same sig

**2.2**) Let  $b_n = 0$ . Thenthesurface f is *acylinder* with a canonical equation

$$
\Sigma_1 z_1^2 + \dots + \Sigma_{n-1} z_{n-1}^2 = -c \,. \tag{9}
$$

**3**)Let  $1 \le r \le n-2$ . Then the surface f is acylinder. Besides the following subcases hold.

**3.1**)If at least one of the coefficients b<sub>i</sub> is non-zero,  $i = r+1,...,n$ , then f is a *parabolic cylinder* and f has a canonical equation .  $(10)$ des if  $\}_{1}$ ,...,  $\}_{n=1}$  are with the same signs, then  $f$  is an elliptical paraboloida<br>rent, then  $f$  is ahyperbolic paraboloid.<br>Let  $b_n = 0$ . Then the surface  $f$  is acylinder with a canonical equation<br> $x^2 + ... + \}_{n=1} z_{$ 

**3.2**)If  $b_{r+1} = ... = b_n = 0$ , then the canonical form of f is

58525	International Journal of Current Research, Vol. 9, Issue, 10, pp.58521-58529, October, 2017
<b>3.2</b> ) If $b_{r+1} = ... = b_n = 0$ , then the canonical form of $f$ is	
$\frac{1}{2}z_1^2 + ... + \frac{1}{2}z_r^2 = -c$ .	(11)
<b>Proof.</b> After the reduction of $f_2$ inacanonicalformby orthogonal transformation (4), equation (2), in view of (5), obtains the form	
$\frac{1}{2}x_1^2 + ... + \frac{1}{2}y_1^2 + 2\sum_{r=1}^{n}b_ry_r + a = 0$ .	

**Proof.**Afterthereductionof  $f_2$  inacanonicalformby orthogonal transformation (4), equation (2), in view of (5), obtains the form

58525 International Journal of Current Research, Vol. 9, Issue, 10, pp.58521-58529, October, 2017  
\n3.2) If 
$$
b_{r+1} = ... = b_n = 0
$$
, then the canonical form of  $f$  is  
\n
$$
\begin{aligned}\n\frac{1}{2} \zeta_1^2 + ... + \frac{1}{2} \zeta_r^2 &= -c. \qquad (11) \\
\text{Proof.} \text{Afterthe reduction of } f_2 \text{ inacanonicalform by orthogonal transformation (4), equation (2), in view of (5), obtains the form} \\
\frac{1}{2} \gamma_1^2 + ... + \frac{1}{2} \gamma_r^2 + 2 \sum_{j=1}^n b_j y_j + a &= 0. \n\end{aligned}
$$
\nIn this equation we make the following transformation  
\n
$$
\begin{aligned}\n\frac{b_1^2}{2} + 2b_i y_i &= \frac{b_i^2}{2} - \frac{b_i^2}{2} \,, \quad z_i = y_i + \frac{b_i}{2} \,, \quad i = 1, ..., r.\n\end{aligned}
$$
\n(11)

In this equation we make the following transformation

58525 *International Journal of Current Research, Vol. 9, Issue, 10, pp.58521-58529, October, 2017*  
\n3.2) If 
$$
b_{r+1} = ... = b_n = 0
$$
, then the canonical form of  $f$  is  
\n
$$
\frac{1}{2}z_1^2 + ... + \frac{1}{2}z_r^2 = -c.
$$
\n(11)  
\nProof. Afterthereduction of  $f_2$  inacanonicalformby orthogonal transformation (4), equation (2), in view of (5), obtains the form  
\n
$$
\frac{1}{2}y_1^2 + ... + \frac{1}{2}y_r^2 + 2\sum_{j=1}^n b_j y_j + a = 0.
$$
\nIn this equation we make the following transformation  
\n
$$
\frac{1}{2}y_i^2 + 2b_i y_i = \frac{1}{2}z_i^2 - \frac{b_i^2}{2i}, \quad z_i = y_i + \frac{b_i}{2i}, \quad i = 1,...,r.
$$
\n(12)  
\nWe add the last equalities with  $z_{r+1} = y_{r+1}, ..., z_n = y_n$  and we obtain a translation of the surface (12). In this way (12) obtains the form  
\n
$$
\frac{1}{2}z_1^2 + ... + \frac{1}{2}z_r^2 + 2\sum_{i=r+1}^n b_i z_i + c = 0.
$$
\n(13)

We add the last equalities with  $z_{r+1} = y_{r+1},..., z_n = y_n$  and we obtain a translation of the surface (12). In this way (12) obtains the form

58235 *International Journal of Current Research, Vol. 9, Isour, 10, pp.58521-58529, October, 2017*  
\n3.2) If 
$$
b_{r+1} = ... = b_n = 0
$$
, then the canonical form of  $f$  is  
\n $\frac{1}{2}z_1^2 + ... + \frac{1}{r}z_r^2 = -c$ .  
\n6.21  $z_1^2 + ... + \frac{1}{r}z_r^3 = \frac{b_1y_1}{r^2} + a = 0$ .  
\n6.22  $\int_0^{\infty} \int_0^{\infty} \int_0^{\infty}$ 

We shall consider the different cases of the theorem.

1)Let  $r = n$ . Then (13) obtains the form (7) andcase 1 of the theorem is fulfilled together with the consider subcases. 2)Let  $r = n - 1$ . Equation (13) obtains the form

$$
\Sigma_1 z_1^2 + \dots + \Sigma_{n-1} z_{n-1}^2 = -2b_n z_n - c \,. \tag{14}
$$

We consider the following subcases.

2.1) Let 
$$
b_n \neq 0
$$
. Since  
\n
$$
2b_n z_n + c = 2b_n \left( z_n + \frac{c}{2b_n} \right),
$$

then by making the translation  $t_n = z_n + \frac{1}{n-1}$ ,  $t_1 = z_1, \ldots, t_{n-1} = z_{n-1}$ , (14) obtains form (8). Besides the subcaseshold for elliptical and hyperbolic paraboloids, i.e. statement 2.1 of the theorem is fulfilled. shall consider the different cases of the theorem.<br> *n* and the theorem is fulfilled together vect  $r = n$ . Then (13) obtains the form (7) and case 1 of the theorem is fulfilled together vect  $r = n - 1$ . Equation (13) obtains Example 1 of the theore<br>
tains the form<br> *c*.<br>
S.<br>
S.<br>
S.<br>
S.<br>
S.<br>  $\frac{1}{n} = z_n + \frac{c}{2b_n}$ ,  $t_1 = z_1, ..., t_{n-1} =$ <br>
ds, i.e. statement 2.1 of the theore<br>
S.<br>
S. S. S.<br>
S.<br>
S. S. S. S.<br>
S  $\frac{c}{b_n}$ ,  $t_1 = z_1, ..., t_{n-1} = z_{n-1}$ , (14) obtains form

2.2) Let  $b_n = 0$ . Then (14) obtains form (9), i.e. f is a cylinder and condition 2.2of the theorem is fulfilled.

3) Let  $1 \le r \le n - 2$ . We consider the following subcases.

3.1) Let at leastone coefficient  $b_i$  in (13) is non-zero,  $i = r+1,...,n$ . Then, byTheorem1, wetransformthelinearhomogenouspartof(13) inalinearhomogenousform of one unknown  $t_n$ , applying an orthogonal transformation of the unknowns  $z_{r+1},..., z_n$ , expressed by  $t_{r+1},..., t_n$  and supplemented with  $z_1 = t_1,..., z_r = t_r$ . In this way we obtain the equation  $\therefore J_{1}x_{r} + \sum_{i=1}^{n} \frac{b_{i}}{r_{i}} = \sum_{i=1}^{n} \frac{b_{i}}{r_{i}}$ <br>
all consider the different cases of the theorem.<br>
all consider the different cases of the theorem.<br>  $r = n$ . Then (13) obtains the form<br>  $r = n - 1$ . Equation (13) o *i* **i** *f* **i i** *i r n z z* 1  $t_{n-1} = z_{n-1}$ , (14) obtains form (8)<br>theorem is fulfilled.<br>der and condition 2.2of the theorem<br>(13) is non-zero,  $i = r$ <br>genousform of one unknown<br> $r_{r+1},..., t_n$  and supplemented with  $z_1$ . is theorem is fulfilled together with the consider subcases.<br>
(14)<br>  $t_{n-1} = z_{n-1}$ , (14) obtains form (8). Besides the subcaseshold for<br>
theorem is fulfilled.<br>
Inder and condition 2.2of the theorem is fulfilled.<br>
(13) is the consider subcases.<br>
(14)<br>
(8). Besides the subcaseshold for<br> *rm* is fulfilled.<br> *r* + 1,..., *n*. Then, by Theorem 1,<br>  $t_n$ , applying an orthogonal<br>  $z_1 = t_1, ..., z_r = t_r$ . In this way we<br>
tain form (10), i.e.  $f$  is a cyl by making the translation  $t_n = z_n + \frac{c}{2b_n}$ ,  $t_1 = z_1, ..., t_{n-1} = z_{n-1}$ , (14) obtical and hyperbolic paraboloids, i.e. statement 2.1 of the theorem is fulfilled.<br>
Let  $b_n = 0$ . Then (14) obtains form (9), i.e.  $f$  is a cylin 2)  $\left| z \right|$  or  $t = n - 1$ . Equation (1.3) obtains the form<br>
Vs consider the following subcases.<br>
2),  $z_n^2 + ... + y_{n-1} z_{n-1}^2 = -2b_n z_n - c$ .<br>
(1.4)<br>
We consider the following subcases.<br>
2.1)  $1 \text{ et } b_n \neq 0$ . Since<br>
2.b,  $z_n + c = 2$  $\frac{c}{2h_n}$ ,  $t_1 = z_1,...,t_{n-1} = z_{n-1}$ , (14) obtains form (8). Besides the subcaseshold for<br>aternent 2.1 of the theorem is fulfilled.<br>D), i.e.  $f$  is a cylinder and condition 2.2of the theorem is fulfilled.<br>wing subcases.<br> *z*<sub>n-1</sub>, (14) obtains form (8).<br>
um is fulfilled.<br>
d condition 2.2of the theorem is<br>
is non-zero,  $i = r + 1$ <br>
form of one unknown  $t_n$ <br>  $t_n$  and supplemented with  $z_1 =$ <br>  $u_n = t_n + \frac{c}{s}$ . Then (13) Obtain is <sup>-1</sup>, (14) obtains form (8). Besides the subcaseshold for<br> *f* indifiled.<br> **f**  $P_n \neq 0$ . Since<br>  $c = 2b_n \left(z_n + \frac{c}{2b_n}\right)$ ,<br>
making the translation  $t_n = z_n + \frac{c}{2b_n}$ ,  $t_1 = z_1, ..., t_{n-1} = z_{n-1}$ , (14) obtains form (8). Besides the subcas<br>
and hyperbolic paraboloids, i.e. statement 2.1 of the theorem is

$$
\partial_1 t_1^2 + \ldots + \partial_r t_r^2 + st_n + c = 0, \quad s = \sqrt{b_{r+1}^2 + \ldots + b_n^2} > 0.
$$

For this equation we make the translation  $u_1 = t_1, ..., u_{n-1} = t_{n-1}$ ,  $u_n = t_n + \frac{1}{s}$ . Then (13) Obtain form (10), i.e.  $f$  is a cylinder and subcase 3.1 of the theorem holds.

3.2) Let  $b_{r+1} = ... = b_n = 0$ . Then f is also a cylinder and (13) obtain form (11), i.e. subcase 3.2 of the theorem is fulfilled. The theorem is proved. $\square$ 

We note that  $r < n$ , then the surface f belong to aparabolicclass.

## **5. Hypersurfaces of the maximal absolute and relative inaccuracies**

In this section instead of an error of a physical experiment we shall use the concept an inaccuracy ((Kolikov *et al*., 2010; Kolikov *et al*., 2010; Kolikov *et al*., 2015)). *Kiril Kolikov et al. Canonization of the hypersurfaces of the first and relative ina<br>*  $r < n$ *, then the surface*  $f$  *belong to aparabolicclass.*<br> **aces of the maximal absolute and relative inaccuracy**<br> **h** instead of an err

Let Y be an indirectly measurable variable depending on the directly measurable variables  $X_1, X_2, ..., X_n$ . Denote by f the real function of arguments  $X_i$   $(i = 1, 2, ..., n)$  such that  $Y = f(X_1, X_2, ..., X_n)$ . Let k observations of  $x_{i1}, x_{i2}, ..., x_{ik}$  of *Xiril Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and application for the maximal absolute<br>
and relative inaccuracies<br>
ypersurfaces of the maximal absolute and relative inaccuracie Xii canonization of the hypersurfaces of the first and the second degree and application for the maximal absolute<br>
surface*  $f$  *belong to aparabolicclass.<br> X<i>X*  $\cdot$  **i** *i* and *i* **i i i** *x* **i** *x* **i** *Xiril Kolikov et al. Canonization of the hypersurfaces of the fund relative<br>
<i>X*ive note that  $r < n$ , then the surface  $f$  belong to aparabolicclas<br> **Xj. Hypersurfaces of the maximal absolute and relative inaccu**<br>
in *r* < *n*, then the surfaces of the maximal<br>faces of the maximal<br>n instead of an error <br>Xolikov *et al.*, 2015))<br>indirectly measurab<br>of arguments  $X_i$  (*i*,...,*n*) are made in a<br>absolute inaccuracy<br> $\left|\Delta X_i\right|$ , *Kiril Kolikov et al. Canonization of the hypersurfaces of the first and the same relative inaccuracies***<br>
<b>***ii <i>i ii <i>i z ii z ii z ii z ii ii z <i>z ii* **<b>***z* **EXECUTE:**<br> **EXECUTE: EXECUTE: EXECUTE:**

 $X_i$   $(i = 1, 2, ..., n)$  are made in an experimental investigation.

The maximal absolute inaccuracy by the method of (Kolikov *et al*., 2010) is

$$
\Delta^1 Y = \sum_{i=1}^n A_i \left| \Delta X_i \right|,\tag{15}
$$

where

*SETS*  
\n*Kirli Kollkov et al. Canonization of the hypersurfaces of the first and the second degree and application for the maximal absolute and relative inaccuracies*  
\nWe note that 
$$
r < n
$$
, then the surface  $f$  belong to a parabolic class.  
\n**5. Hypersurfaces of the maximal absolute and relative inaccuracies**  
\nIn this section instead of an error of a physical experiment we shall use the concept an inaccuracy ((Kolikov *et al.*, 2010; Kolikov *et al.*, 2010; Kolikov *et al.*, 2010).  
\nLet  $Y$  be an indirectly measurable variable depending on the directly measurable variables  $X_1, X_2, ..., X_n$ . Denote by  $f$  the real function of arguments  $X_i$  ( $i = 1, 2, ..., n$ ) such that  $Y = f(X_1, X_2, ..., X_n)$ . Let  $k$  observations of  $x_{i1}, x_{i2}, ..., x_{i_k}$  of  $X_i$  ( $i = 1, 2, ..., n$ ) are made in an experimental investigation.  
\nThe maximal absolute inaccuracy by the method of (Kolikov *et al.*, 2010) is  
\n
$$
\Delta Y = \sum_{i=1}^{e} A_i |\Delta X_i|,
$$
\nwhere  
\n
$$
A_i = \frac{1}{k} \sum_{j=1}^{k} \left| \frac{\partial f}{\partial X_i} (x_{i_m}, ..., x_{i_m}, ..., x_{i_m)} \right|, i = 1, ..., n
$$
\nwhere  $\Delta x_{ij}$  are the maximal absolute inaccuracies of the directly measurable variables.  
\nThe maximal relative inaccuracy  $\frac{\Delta^1 Y}{Y}$  of  $Y$ , according to (Kolikov *et al.*, 2012), is

and

$$
\left|\Delta X_{i}\right| = \frac{1}{k} \sum_{j=1}^{k} \left|\Delta x_{ij}\right|, i = 1, 2, ..., n, \tag{17}
$$

where  $\Delta x_{ij}$  are the maximal absolute inaccuracies of the directly measurable variables.

The maximal relative inaccuracy  $\frac{\Delta^1 Y}{\Delta}$  of Y, according to (Kolikov *et al.*, 2012), is *Y* and *Y* an  $\frac{\Delta^1 Y}{\Delta}$  of Y, according to (Kolikov *et al.*, 2012), is

$$
\frac{\Delta^1 Y}{Y} = \sum_{i=1}^n B_i \left| \frac{\Delta X_i}{X_i} \right|,\tag{18}
$$

where

The maximal absolute macuracy by the method of (Konkov *et al.*, 2010) is  
\n
$$
A_1^1 = \sum_{i=1}^k A_i |AX_i|,
$$
\n(15)  
\nwhere  
\n
$$
A_2 = \frac{1}{k} \sum_{n=1}^k \left| \frac{\partial f}{\partial X_i} (X_{1m}, ..., X_{nn}, ..., X_{nn}) \right|, i = 1, ..., n
$$
\n(16)  
\nand  
\n
$$
|AX_i| = \frac{1}{k} \sum_{j=1}^k |Ax_{ij}|, i = 1, 2, ..., n,
$$
\n(17)  
\nwhere  $\Delta x_{ij}$  are the maximal absolute inaccuracies of the directly measurable variables.  
\nThe maximal relative inaccuracy  $\frac{\Delta^1 Y}{Y}$  of Y, according to (Kolikov *et al.*, 2012), is  
\n
$$
\frac{\Delta^1 Y}{Y} = \sum_{i=1}^n B_i \left| \frac{\Delta X_i}{X_i} \right|,
$$
\n(18)  
\nwhere  
\n
$$
B_i = \frac{1}{k} \sum_{n=1}^k \left| \frac{X_{2m}}{f(X_{1m}, ..., X_{nn})} \frac{\partial f}{\partial X_i} (X_{1m}, ..., X_{nn}) \right|, i = 1, ..., n
$$
\n(19)  
\nand  
\n
$$
\left| \frac{\Delta X_i}{X_i} \right| = \frac{1}{k} \sum_{j=1}^k \left| \frac{X_{2j}}{x_{ij}} \right|, i = 1, 2, ..., n,
$$
\n(20)  
\nwhere  $\frac{\Delta x_{ij}}{x_{ij}}$  are the maximal relative inaccuracies of the directly measurable variables.  
\nWe note, that  $\frac{\partial f}{\partial X} (X_{1m}, ..., X_{nn})$  and  $\frac{x_{2m}}{f(X_{2m}, ..., X_{2m})} \frac{\partial f}{\partial X_{1}} (X_{2m}, ..., X_{2m})$  in (16) and (19) are the values of  $\frac{\partial f}{\partial X_i} \frac{\partial f}{f \partial X_i}$   
\nrespectively, calculated in the *m*<sup>th</sup> observation and A<sub>1</sub> and B<sub>i</sub> are the arithmetic mean of these values for  $m = 1, 2, ..., k$ .  
\nThe maximal absolute inaccuracy  $\Delta^2 Y$  and the maximal relative inaccuracy  $\frac{\Delta^$ 

and

$$
\left| \frac{\Delta X_i}{X_i} \right| = \frac{1}{k} \sum_{j=1}^k \left| \frac{\Delta x_{ij}}{x_{ij}} \right|, \, i = 1, 2, ..., n \,, \tag{20}
$$

where  $\frac{a}{b}$  are the maximal relative inaccuracies of the directly measurable variables. *ij ij*  $x_{ii}$  $x_{ii}$  $\Delta x_{ii}$ 

We note, that  $\frac{dy}{dx}$   $(x_{1m},...,x_{nm})$  and  $x_{1m}$  of  $(x_{1m},...,x_{1m})$  in (16) and (19) are the values of  $\partial f$  and  $\frac{X_i}{x_i}$  $(x_1,...,x_{nm})$  and  $\frac{x_{nm}}{x_{nm}} = \frac{\partial f}{\partial x}(x-x)$  in (1)  $X_i^{(x_1,m},...,x_m^{(x_m,m)})$   $\frac{1}{f(x_{1,m},...,x_m)}\frac{1}{\partial X_i}(x_{1,m},...,x_{nm})$  $\partial f$  ( ) and  $\partial f$  $\partial X_i^{(x_{1m},...,x_{nm})}$  and  $\frac{C_{i_{nm}}}{f(x_{1m},...,x_{nm})}$   $\frac{C_i}{\partial X_i}(x_{1m},...,x_{nm})$  in (10) and (12) are the values of  $\frac{V_{im}}{V_{im}}$   $\frac{Q}{QX_i}(x_{lm},...,x_{nm})$  III (10) and (12)  $\frac{\partial f}{\partial X_i}(x_{1m},...,x_{nm})$  in (16) and (19) are the values of  $\frac{\partial f}{\partial X_i}$  and  $\frac{X_i}{f} \frac{\partial f}{\partial X_i}$ ,  $\partial f$  and  $X_i$   $\partial f$ ,  $\frac{\partial f}{\partial X_i}$  and  $\frac{X_i}{f} \frac{\partial f}{\partial X_i}$ ,  $\partial f$  $\partial X_i$ 

respectively, calculated in the  $m^{th}$  observation and  $A_i$  and  $B_i$  are the arithmetic mean of these values for  $m = 1, 2, ..., k$ . The maximal absolute inaccuracy  $\Delta^2 Y$  and the maximal relative inaccuracy  $\frac{\Delta^2 Y}{Z}$  of the second order of  $Y = f(X_1, X_2, ..., X_n)$ according to (Kolikov *et al*., 2015) are *i*  $\frac{\Delta^i Y}{Y}$  of *Y*, according to (Kolikov *et al.*<br>  $\left.\frac{i}{k}\right|$ ,<br>  $\left.\frac{x_{im}}{X_{im}}\right| \frac{\partial f}{\partial X_i}(x_{lm},...,x_{mn})\Big|$ ,  $i = 1,...,n$ <br>  $\left|\frac{i}{k}\right|$ ,  $i = 1, 2,...,n$ ,<br>
maximal relative inaccuracies of the directly measurabl<br>  $\frac{f}{X_i}($ cording to (Kolikov *et al.*, 2012), is<br>  $\left| \int_{\alpha} \mathbf{i} = 1,...,n \right|$ <br> *i*  $\left| \int_{\alpha} \mathbf{j} = 1,...,n \right|$ <br> *n* (16) and (19) are the values  $\left| \int_{\alpha} \frac{\partial f}{\partial X_i}(x_{i_m},...,x_{i_m}) \right|$  in (16) and (19) are the values of  $\alpha$ <br> *m*  $\mathbf{A}_$ *f*, according to (Kolikov *et al.*, 2012), is<br>  $f_{nm}$ ,  $\left| \int_i i = 1, ..., n$ <br> *f*  $\frac{x_m}{(x_m, ..., x_m)} \frac{\partial f}{\partial x_i} (x_n, ..., x_m)$  in (16) and (19) are the values of  $\frac{\partial f}{\partial x_i} (x_n, ..., x_m)$   $\frac{\partial f}{\partial x_i} (x_n, ..., x_m)$  and  $B_i$  are the arithmetic me (18)<br> *f*  $\frac{X_i}{\partial X_i}$ <br> *f*  $\frac{\partial f}{\partial X_i}$ ,<br> *f*  $\frac{X_i}{\partial X_i}$ *th*  $\frac{\partial f}{\partial X_i}(x_{1m},...,x_{nm})$ ,  $i = 1,...,n$  (19)<br> *theory*  $\left(\frac{\partial f}{\partial X_i}(x_{1m},...,x_{nm})\right)$ ,  $i = 1,...,n$  (19)<br> *theory*  $\left(\frac{\partial f}{\partial X_i}(x_{1m},...,x_{nm})\right)$  *a*<sub>*i*</sub> *f*<sub>*i*</sub> *m<sub>i</sub>*  $\frac{\partial f}{\partial X_i}(x_{1m},...,x_{nn})$  in (16) and (19) are the values o *Y*  $\left(\frac{1}{2}, \frac{1}{2}, \ldots, \frac{1}{n}\right)$  $\Delta^2 Y$  c d l s  $\sim$  c  $($  x  $)$  x  $)$ (18)<br>
(19)<br>
(20)<br>
(20)<br>
(20)<br>
(20)<br>
(20)<br>
(20)<br> *Y* =  $f(X_1, X_2,..., X_n)$ <br>
(*Y* =  $f(X_1, X_2,..., X_n)$ 

58527 *International Journal of Current Research, Vol. 9, Isou, 10, pp.5821-58529, Oecubor, 2017*  
\n
$$
\Lambda^2 Y = \sum_{i,j=1}^n A_{ij} |\Lambda X_j| |\Lambda X_j| - \frac{\Lambda^2 Y}{Y} = \sum_{i,j=1}^n A_{ij} \frac{|\Lambda X_i|}{X_i} \frac{|\Lambda X_i|}{|X_j|}.
$$
\n(21)  
\nrespectively, where  $A_{ij}$  of  $\Delta^2 Y$  and of  $\frac{\Lambda^2 Y}{Y}$  are given by the equalities:  
\n
$$
A_{ij} = \frac{1}{k} \sum_{m=1}^k \left| \frac{\partial^2 f}{\partial X_i \partial X_j} (x_{im}, ..., x_{mn}) \right|, i, j = 1, 2, ..., n
$$
\n(22)  
\nand  
\n
$$
A_{ij} = \frac{1}{k} \sum_{m=1}^k \left| \frac{x_{im} x_{jm}}{f (x_{im}, ..., x_{mn})} \frac{\partial^2 f}{\partial X_i \partial X_j} (x_{im}, ..., x_{mn}) \right|, i, j = 1, 2, ..., n
$$
\n(23)  
\nrespectively.  
\nIn (22) and (23)  $\frac{\partial^2 f}{\partial X_i \partial X_j} (x_{im}, ..., x_{mn})$  and  $\frac{x_{m} x_{jm}}{\partial X_i \partial X_k} (x_{im}, ..., x_{mn})$  are  
\nthe classical absolute in the *m*-th observation and  $A_{ij}$  is the arithmetic mean of these values of  $\frac{\partial^2 f}{\partial X_i \partial X_j} \frac{\partial \partial \partial X_j}{\partial X_i \partial X_j}$ .  
\nThe maximal absolute inaccuracy  $\Delta Y$  of *Y* of the second approximation we call the function  
\n
$$
\Delta Y = \Delta^2 Y + \frac{1}{2} \Delta^2 Y.
$$
\n(24)  
\nThe maximal relatively inaccuracy  $\frac{\Delta Y}{Y}$  of *Y* of the second approximation we call the function  
\n
$$
\frac{\Delta Y}{|Y|} = \frac{\Lambda^2 Y}{|Y|} + \frac{1}{2} \frac{\Lambda^2 Y}{|Y|}.
$$
\n(25)  
\nIn (Kolikov *et al.*, 2010; Kolkov, 2012; Kolkov, 2015) we assume that

respectively, where  $A_{ij}$  of  $\Delta^2 Y$  and of  $\frac{\Delta^2 Y}{Y}$  are given by the equalities: *Y*  $\Delta^2 Y$  is a set of the set of the

$$
A_{ij} = \frac{1}{k} \sum_{m=1}^{k} \left| \frac{\partial^2 f}{\partial X_i \partial X_j} (x_{1m}, ..., x_{nm}) \right|, \ i, j = 1, 2, ..., n
$$
\n(22)

and

respectively, where 
$$
A_{ij}
$$
 of  $\Delta^2 Y$  and of  $\frac{\Delta^2 Y}{Y}$  are given by the equalities:  
\n
$$
A_{ij} = \frac{1}{k} \sum_{n=1}^{k} \left| \frac{\partial^2 f}{\partial X_i \partial X_j} (x_{1:n}, ..., x_{nn}) \right|, i, j = 1, 2, ..., n
$$
\n
$$
A_{ij} = \frac{1}{k} \sum_{m=1}^{k} \left| \frac{x_{nn} x_{j:n}}{f(x_{1:n}, ..., x_{nn})} \right|, i, j = 1, 2, ..., n
$$
\n
$$
A_{ij} = \frac{1}{k} \sum_{m=1}^{k} \left| \frac{x_{nn} x_{j:n}}{f(x_{1:n}, ..., x_{nn})} \frac{\partial^2 f}{\partial X_i \partial X_j} (x_{1:n}, ..., x_{nn}) \right|, i, j = 1, 2, ..., n
$$
\n
$$
(22)
$$
\nrespectively.  
\nIn (22) and (23)  $\frac{\partial^2 f}{\partial X_i \partial X_j} (x_{1:n}, ..., x_{nn})$  and  $\frac{x_{1:n} x_{2:n}}{f(x_{1:n}, ..., x_{nn})} \frac{\partial^2 f}{\partial X_i \partial X_j} (x_{1:n}, ..., x_{nn})$  are  
\nreducible linearly defined in the *m*-thometric form of these values for  $m = 1, 2, ..., k$ .  
\nThe maximal absolutely inaccuracy  $\Delta Y$  of  $Y$  of the second approximation we call the function  
\n
$$
\Delta Y = \Delta^1 Y + \frac{1}{2} \Delta^2 Y.
$$
\nThe maximal relatively inaccuracy  $\frac{\Delta Y}{Y}$  of  $Y$  of the second approximation we call the function  
\n
$$
\frac{\Delta Y}{|Y|} = \frac{\Delta^1 Y}{|Y|} + \frac{1}{2} \frac{\Delta^2 Y}{|Y|}.
$$
\n
$$
(24)
$$
\nIn (Kolikov *et al.*, 2010; Kolikov, 2012; Kolikov, 2015) we assume that  $\Delta X_i$  and  $\frac{\Delta X_i}{X_i}$   $(i = 1, 2, ..., n)$  in (15) and (18) are unknown values with constant coefficients. Then (15) and (21) imply, that (24) has the form  
\n
$$
\Delta Y = \
$$

respectively.

In (22) and (23)  $\frac{\partial^2 f}{\partial x \partial y} (x_{lm},...,x_{mn})$  and  $\frac{x_{lm}x_{jm}}{(x_{lm}+...,x_{mn})}$   $\frac{\partial^2 f}{\partial y^2} (x_{lm}+...,x_{mn})$  are the values of  $\frac{\partial^2 f}{\partial z^2}$  and  $\frac{x_{i}x_{j}}{x_{j}}$   $\frac{\partial^2 f}{\partial x_{l}}$ , respectively,  $\frac{1}{1m}, \ldots, \frac{1}{n}m$   $\frac{1}{1}$  $\frac{\partial^2 f}{\partial x^2}$  and x.x.  $\frac{\partial^2 f}{\partial x^2}$  $\frac{f^2 f}{2}$  ( $x_{lm},...,x_{mn}$ ) are the values of  $\frac{\partial^2 f}{\partial x^2}$  and  $x_i x_j \partial^2 f$ ,  $\frac{1}{1_m,\ldots,x_{nm}}\frac{\partial}{\partial X_i \partial X_j}(x_{1_m},...,x_{nm})$  are the variable of  $\partial^2 f$ , are the values of  $\partial^2 f$  and  $\chi_X$   $\partial^2 f$ , respect  $\partial^2 f$  and  $X_i X_j$   $\partial^2 f$ , respectively,  $\partial^2 f$ , respectively,

calculatedin them-thobservationand  $A_{ij}$  is the arithmetic mean of these values for  $m = 1, 2, ..., k$ .

The maximal absolute inaccuracy  $\Delta Y$  of Y of the second approximation we call the function

$$
\Delta Y = \Delta^1 Y + \frac{1}{2} \Delta^2 Y. \tag{24}
$$

The maximal relatively inaccuracy  $\frac{\Delta Y}{\Delta t}$  of Y of the second approximation we call the function *Y*  $\frac{\Delta Y}{\Delta x}$  of Y of the second approximation we call the function

respectively.  
\nIn (22) and (23) 
$$
\frac{\partial^2 f}{\partial X_i \partial X_j} (x_{1m}, ..., x_{nm})
$$
 and  $\frac{x_{nn}x_{jm}}{f(x_{1m}, ..., x_{nm})} \frac{\partial^2 f}{\partial X_i \partial X_j} (x_{1m}, ..., x_{nm})$  are  
\ncalculatedin them-thobservationand  $A_{ij}$  is the arithmetic mean of these values for  $m = 1, 2, ..., k$ .  
\nThe maximal absolute inaccuracy  $\Delta Y$  of Y of the second approximation we call the function  
\n
$$
\Delta Y = \Delta^1 Y + \frac{1}{2} \Delta^2 Y.
$$
\n(24)  
\nThe maximal relatively inaccuracy  $\frac{\Delta Y}{Y}$  of Y of the second approximation we call the function  
\n
$$
\frac{\Delta Y}{|Y|} = \frac{\Delta^1 Y}{|Y|} + \frac{1}{2} \frac{\Delta^2 Y}{|Y|}.
$$
\n(25)  
\nIn (Kolikov *et al.*, 2010; Kolikov, 2012; Kolikov, 2015) we assume that  $\Delta X_i$  and  $\frac{\Delta X_i}{X_i}$  ( $i = 1, 2, ..., n$ ) in (15) and (18) are unknown

In (Kolikov *et al.*, 2010; Kolikov, 2012; Kolikov, 2015) we assume that  $\Delta X_i$  and  $\frac{\Delta X_i}{X_i}$  (*i* = 1, 2, ..., *n*) in (15) and (18) are unknown values with constant coefficients. Then (15) and (21) imply, that (24) has the form

The maximal absolute inaccuracy 
$$
\Delta Y
$$
 of Y of Y of the second approximation we call the function  
\n
$$
\Delta Y = \Delta^1 Y + \frac{1}{2} \Delta^2 Y.
$$
\n(24)  
\nThe maximal relatively inaccuracy  $\frac{\Delta Y}{Y}$  of Y of the second approximation we call the function  
\n
$$
\frac{\Delta Y}{|Y|} = \frac{\Delta^1 Y}{|Y|} + \frac{1}{2} \frac{\Delta^2 Y}{|Y|}.
$$
\n(25)  
\nIn (Kolikov *et al.*, 2010; Kolikov, 2012; Kolikov, 2015) we assume that  $\Delta X_i$  and  $\frac{\Delta X_i}{X_i}$  (*i* = 1, 2,...,*n*) in (15) and (18) are unknown  
\nvalues with constant coefficients. Then (15) and (21) imply, that (24) has the form  
\n
$$
\Delta Y = \sum_{i=1}^n A_i \cdot |\Delta X_i| + \frac{1}{2} \sum_{i,j=1}^n A_{ij} \cdot |\Delta X_i| \cdot |\Delta X_j|.
$$
\n(26)  
\nFrom (18) and (21) we  
\nobtainanalogously, that (25) has the form

From  $(18)$  and  $(21)$  weobtainanalogously, that  $(25)$  has the form

respectively, where 
$$
A_{ij}
$$
 of  $A^2Y$  and of  $\frac{d^2Y}{Y}$  are given by the equalities.  
\n
$$
A_{ij} = \frac{1}{k} \sum_{k=1}^{k} \left| \frac{\partial^2 f}{\partial X_i \partial X_j} \left( X_{1m}, \dots, X_{nm} \right) \right|, i, j = 1, 2, \dots, n
$$
\n(22)  
\nand  
\n
$$
A_{ij} = \frac{1}{k} \sum_{k=1}^{k} \left| \frac{x_{1m} X_{1m}}{\int (X_{1m}, \dots, X_{nm})} \frac{\partial^2 f}{\partial X_i \partial X_j} \left( X_{1m}, \dots, X_{nm} \right) \right|, i, j = 1, 2, \dots, n
$$
\n(23)  
\nrespectively.  
\nIn (22) and (23)  $\frac{\partial^3 f}{\partial X_i \partial X_j} \left( X_{1m}, \dots, X_{nm} \right)$  and  $\frac{x_{1m} x_{2m}}{\int (X_{1m}, \dots, X_{nm})} \frac{\partial^2 f}{\partial X_i \partial X_j} \left( X_{1m}, \dots, X_{nm} \right)$  are  
\nrespectively.  
\nThe maximal absolute in every  $\Delta Y$  of *Y* of the second approximation we call the function  
\n
$$
\Delta Y = \Delta^1 Y + \frac{1}{2} \Delta^2 Y.
$$
\n(24)  
\nThe maximal relatively inaccuracy  $\Delta Y$  of *Y* of the second approximation we call the function  
\n
$$
\frac{\Delta Y}{|Y|} = \frac{\Delta^1 Y}{|Y|} + \frac{1}{2} \frac{\Delta^2 Y}{|Y|}.
$$
\n(25)  
\nIn (Kolikov *et al.*, 2010; Kolikov, 2012; Kolikov, 2015) we assume that  $\Delta X_i$  and  $\frac{\Delta X_i}{X_i}$   $(i=1, 2, \dots, n)$  in (15) and (18) are unknown  
\nvalues with constant coefficients. Then (15) and (21) imply, that (24) has the form  
\n
$$
\Delta Y = \sum_{i=1}^{k} A_i \left| \Delta X_i \right| + \frac{1}{2} \sum_{i,j=1}^{k} A_{ij} \left| \Delta X_i \right| \left| \Delta X_j \right|.
$$

Wechange  $\Delta Y$  with  $y_{n+1}$  and  $\Delta X_i$  with  $x_i$  in (26) and, analogously,  $\frac{y_{n+1}}{x_i}$  with  $y_{n+1}$  and  $\frac{y_{n+1}}{x_i}$  with  $x_i$  in (27)  $(i = 1, 2, ..., n)$ .  $\Delta Y$  with  $y_{n+1}$  and  $\Delta X_i$  with  $x_i$  in (26) and, analogously,  $\frac{\Delta Y}{Y}$  with  $y_{n+1}$  and  $\frac{\Delta X_i}{Y}$  with  $x_i$  in (27)  $(i = 1, 2, ...,$  $Y$   $X_i$   $Y_{i+1}$  $\Delta Y$   $\Delta X_i$   $\Delta X_i$  $y_{n+1}$  and  $\frac{\Delta X_i}{X_i}$  with  $x_i$  in (27)  $(i = 1, 2, ..., n)$ . 

Thenwe obtain, that the maximal absolute and relative inaccuracies of Y of the second approximation in  $(n+1)$ -dimensional affine Euclidean space  $E_{n+1}$  have the general kind

$$
y_{n+1} = \sum_{i,j=1}^{n} a_{ij} x_i x_j + 2 \sum_{i=1}^{n} a_i x_i , \qquad (28)
$$

where  $a_{ij}$  and  $a_i$  are non-negative constants,  $a_{ij} = a_{ji}$  and at least one of the coefficients is distinct from zero  $(i, j = 1, 2, ..., n)$ . We shall make an algebraic classification of  $(28)$ , i.e. a classification of the surfaces of the maximal inaccuracies of Y of the second approximationasa corollary from the canonization of the surfaces in the many-dimensional affine Euclidean space  $E_{n+1}$ , i.e. as acorollary of Theorems 2 and 3. *y*  $\frac{\Delta Y}{|Y|} = \frac{\Delta^1 Y}{|Y|} + \frac{1}{2} \frac{\Delta^2 Y}{|Y|}$ .<br> *n* (Kolikov *et al.*, 2010; Kolikov, 2012; Kolikov, 2015) we assume that<br>
radius with constant coefficients. Then (15) and (21) imply, that (24) has<br>
radius with consta  $\frac{1}{K} \sum_{n=1}^{K} \frac{1}{\sqrt{K_{n}} (K_{n},...,K_{n})} \frac{1}{nK_{n}} \sum_{n=1}^{K} \sum_{n=1}^{K}$ *i*<sup>2</sup>  $\int_{r=1}^{r} \int_{r}^{2} \int_{r+1}^{2} \int_{r+$ 

**Theorem 4.** Let r be the rang of the quadratic part of  $f_2$  of the surface f of the maximal absolute (relative) inaccuracy of the second approximation in the real affine Euclidean space  $E_{n+1}$ ,  $n \ge 1$ . Then  $f$  is from parabolic typeand the following cases hold. (i)Let  $r = n$ . Then the canonical equation of f is *Kiril Kolikov et al. Canonization of the hypersurfaces of the first and thand relative inaccura*<br> **em 4.** Let *r* be the rang of the quadratic part of  $f_2$  of the surfaction<br> **1** approximation in the real affine Euclide 8 Kiril Kolikov et al. Canonization of a<br> **Orem 4.** Let *r* be the rang of the quadr<br>
and approximation in the real affine Eucl<br>
et  $r = n$ . Then the canonical equation of<br>  $x^2 + ... + \frac{1}{2}x^2 = z_{n+1}$ ,<br>
f is a paraboloid. Bes **158528** *Kirll Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and application for the maximal and relative inaccuracies<br> 1686 and relative inaccuracies<br> 1796 and relative inacc* bes of the first and the second degree and application<br>
d relative inaccuracies<br>  $f_2$  of the surface  $f$  of the maximal absolute<br>  $F_{n+1}$ ,  $n \ge 1$ . Then  $f$  is from parabolic ty<br>  $\}$ <sub>1</sub>,...,  $\}$ <sub>n</sub> of  $f_2$  arewithsam **EXALUATE EXECUTE:** Kirl Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and application for the maxima and relative inaccuracies<br>
order 4.1. Let *r* be the rang of the quadratic part o **heorem 4.**Let *r* be the rang of the quadratic part of  $f_2$  of the surface  $f$  ofthe maximal absolut<br>cond approximation in the real affine Euclidean space  $E_{n+1}$ ,  $n \ge 1$ . Then  $f$  is from parabolic type<br>Let  $r = n$ . Th *Kiril Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and application*<br> **and relative inaccuracies**<br> **n 4.Let** *r* be the rang of the quadratic part of  $f_2$  of the surface  $f$  of the *Rivit Kolikov et al. Canonization of the hypersurfaces of the first and the second degree and application for the maximal absolute<br>
and relative ineccuracies<br> Choosen 41. Let <i>I* be the rang of the quadratic part of  $f$ *Rird Kulthor et al. Comunication of the hyperaryface of the first and the second degree and application for the maximal absolute<br> com 4.1*  $z$ *t <i>r* be the rang of the quadratic part of  $f_2$  of the surface  $f$  of the ma *sometion of the hypersurfaces of the first and and relative inaccural and relative inaccuration of the quadratic part of*  $f_2$  *of the surface independent on the real affine Euclidean space*  $E_{n+1}$ *,*  $n \ge 1$ *. The anomical n. Canonization of the hypersurfaces of the first and the second degree and application for the maximal absolute<br>
not calculate part of*  $f_2$  *of the surface of of the maximal absolute (relative) inaccuracy of<br>
real affin* 

$$
\big\{ \zeta_1 z_1^2 + \ldots + \zeta_n z_n^2 = z_{n+1}, \tag{29}
$$

i.e.  $f$  is a paraboloid. Besides, if the characteristic roots  $\},..., \}$ <sub>n</sub> of  $f_2$  arewithsamesigns, then  $f$  isan*elliptic paraboloid* and if at least twocharacteristicrootsof  $f_2$  are with opposite signs, then  $f$  is a *hyperbolicparaboloid*.

(ii)Let  $1 \le r \le n-1$ . Then the canonical equation of f is

$$
\partial_1 z_1^2 + \dots + \partial_r z_r^2 = -Sz_n, \quad S = \sqrt{b_{r+1}^2 + \dots + b_n^2 + \frac{1}{4}},
$$
\n(30)

where  $b_i$  aredefinedby (5), i.e.  $f$  is acylinder.

(iii)If  $r = 0$ , then the canonical form of f is  $z_{n+1} = 0$ , i.e. f is a *hyperplane*, defined from the coordinate axes  $O_1z_1, ..., O_nz_n$  of same coordinate system  $O_1z_1z_2...z_{n+1}$  of  $E_{n+1}$ .

**Proof.** For the obtaining of the result we shall applyTheorem 3 and 2. Under the use of these theoremswe have to change  $E_n$  by  $E_{n+1}$ , *n* by  $n+1$  and  $b_n$  by  $\frac{b_{n+1}}{2} = \frac{1}{2}$ . The equations (28) and (29) we shall obtain directly from the cases 2.1 3.1 of the defined Theorem 3. proximation in the real affine Euclidean space  $E_{n+1}$ , *n*<br> *n*. Then the canonical equation of *f* is<br>  $+\frac{1}{2}z_n^2 = z_{n+1}$ ,<br>
a paraboloid. Besides, ifthe characteristic roots  $\}$ ,....,  $]$ <br>
haracteristic roots of  $f_$ tive) inaccuracy of the<br>
e following cases hold.<br>
(29)<br>
oticparaboloidand if at<br>
(30)<br>
e axes  $O_1z_1,..., O_1z_n$  of<br>
e have to change  $E_n$  by<br>
.1 3.1 of the defined<br>  $b_{n+1} = -\frac{1}{2} \neq 0$ . The case(i) is<br>
case (ii) is compl <sup>2</sup><br>  $\frac{2}{1} + ... + \frac{1}{2}x_n^2 = z_{n-1}$ <br>  $\int$  is a paraboloid. Besides, ifthe characteristic roots  $\}_{1}$ ,...,  $\}_n$  of  $\int_2$  are with samesigns, then  $\int$  is and liptit<br>
t two characteristic roots of  $\int_2$  are with opposit

Really, if  $r = n$ , then the canonical equation (8) (of case 2.1) of Theorem 3 obtains the form (29), sinse  $\int_{b_{n+1}=-\frac{1}{2}\neq 0}$ . The case(i) is

completed.

Let  $1 \le r \le n-1$ . It holds case 3 only with subcase 3.1 with equation (10) which obtains the form (30), i.e.case (ii) is completed. If  $r = 0$ , then case(iii) is obtained by case 1 of Theorem 2.

The proof is competed.

A significant part of the proved Theorem 4 canbeobtainedfromtheknowncanonicalformsofthehyperspacesoftheseconddegree ((Efimov, 2010), (Konstantinov, 2000) and (Shafarevich, 2013)), but indirectly, by same non-trivial additional reasonings. Wenoteexplicitly, thatcase (iii) ofTheorem 4 cannotbeobtainedfromtheknowncanonizationsofthehypersurfaces ((Efimov, 2005), (Konstantinov, 2000) (Shafarevich, 2013)), sincecase (iii)is obtained from our original Theorem 2 which gives the canonical form of an arbitraryhyperplane in  $E_n$ . Furthermore, we indicate in Theorem 4 the exact parameters of the canonical forms. have to change  $E_n$  by<br>
1 3.1 of the defined<br>
3.1 of the defined<br>
1  $e^{-\frac{1}{2}\neq 0}$ . The case(i) is<br>
1  $e^{-\frac{1}{2}\neq 0}$ . The case(i) is<br>
1  $e^{-\frac{1}{2}\neq 0}$ . The case(i) is<br>
1  $e^{-\frac{1}{2}\neq 0}$ . The completed.<br>
1  $e^{-\frac{1}{2$ directlyfrom the cases 2.1 3.1 of the defined<br>tains the form (29),sinse  $_{b_{n,n}=-\frac{1}{2}\neq0}$ . The case(i) is<br>botains the form (30), i.e.case (ii) is completed.<br>botains the form (30), i.e.case (ii) is completed.<br>nonicalfor Theorem 3.<br>
Really, if  $r = n$ , then the canonical equation (8) (of case 2.1) of<br>
completed.<br>
Let  $1 \le r \le n - 1$ . It holds case 3 only withsubcase 3.1 with equation<br>
f  $r = 0$ , then case(iii) is obtained by case 1 of Theorem 2

**Corollary 5.**If the rang r of the quadraticpart  $f_2$  of the hypersur face of the maximalabsolute (relative) inaccuracyofthesecondapproximationin  $F_2$  is 1, then the canonical equation of this surface is the parabola  $y = \int_1 x^2$ , where  $\}$  is non-zero characteristic root of  $f_2$ .  $2 \cdot$ 

**Proof.**It holds only case(i)of formulated Theorem4and the canonical equation  $y = \int_1 x^2$  is obtained from (29) by the changes  $z_1 = x$  and  $z_2 = y$ .

#### **Conclusion**

Our approach for a canonization of the hypersurfaces of the second degree in the many-dimensional space stress more to the algebraic part of the question since the geometric interpretations are well known. This approach is effective since it gives the exact coefficients of the given equation of the surface. Namely this effectiveness together with Theorem 2, gives us a possibility to obtain the canonical equation of the hypersurfaces of the maximal inaccuracies (errors).

# **REFERENCES**

Efimov, N. And Rosendorn, E. 2005. *Linear algebra and multidimensional geometry*, Fizmatlit. (in Russian)

Kolikov, K., Krastev, G., Epitropov, Y., Hristozov, D. 2010. *Analytically determining of the absolute inaccuracy (error) of indirectly measurable variable and dimensionless scale characterising the quality of the experiment*, Chemometr Intell Lab, 102, pp. 15-19 http://dx.doi.org/10.1016/j.chemolab.2010.03.001

- Kolikov, K., Krastev, G., Epitropov, Y., Hristozov, D. 2010. *Method for analytical representation of the maximum inaccuracies of indirectly measurable variable (survey)*, Proceedings of the Anniversary International Conference "Research and Education in Mathematics, Informatics and their Applications" – REMIA 2010, Bulgaria, Plovdiv, 10-12 December, pp. 159-166, 2010. http://hdl.handle.net/10525/1449
- Kolikov, K., Krastev, G., Epitropov, Y., Corlat A. 2012. *Analytically determining of the relative inaccuracy (error) of indirectly measurable variable and dimensionless scale characterising the quality of the experiment*, CSJM, vol. 20, no. 1, pp. 314- 331.http://www.math.md/en/publications/csjm/issues/v20-n1/11035/
- Kolikov, K., Epitropov, Y., Corlat, A., Krastev, G. 2015. *Maximum Inaccuracies of Second Order*, CSJM v.23, no.1 (67). http://www.math.md/en/publications/csjm/issues/v23-n1/11891/

Konstantinov, M., *Elements of analytical geometry: Curves and surfaces of the second degree*, Sofia, 2000. (in Bulgarian) Shafarevich, I. & Remizov, A., *Linear algebra and geometry*, Springer, 2013

\*\*\*\*\*\*\*