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RESEARCH ARTICLE

GALILEO'S TOES AND THE FALL OF BODIES: EXPERIMENT OF THE TOWER

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ABSTRACT

For Aristotle, the free-falling body tends to occupy your place natural (soil), moving in a straight line with a speed proportional to its mass. Galileo discusses this phenomenon describing an experiment supposedly held in a tower. For some commentators to your work, this experiment had a bias more rational than empirical. In the development of your reasoning Galileo predicted that a largest iron-ball 100 pounds of mass would hit the ground with a difference "two fingers" in relation to another lighter iron ball of mass 1 pound when both fall from a height 100 fathoms. To check the veracity of this statement, the experiment course reissued based on the history given in the book Discorsi, using a mathematical model implemented by software Mathematica 11.0. The results showed that both balls reach the ground with a difference much larger than two fingers provided by Galileo, more exactly 9.12 m away.

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INTRODUCTION

The study of falling bodies is a phenomenon that has always fascinated man since the beginning of the story. Many thinkers and philosophers of nature tried to explain the phenomenon, but it was Aristotle who first tried explained it more systematically. The theory of the four elements - earth, water, fire and air - dates back to the Persian prophet Zarathustra (600-583 B.C.), or Zoroaster, as called by the Greeks (Habashi, 2000). The idea was first defended in Greece by the philosopher Empedocles (...) and later amplified by Aristoteles (384-322 B.C.) and his followers. To Aristotle all objects or bodies found in nature are composed of four elements: water, earth, fire and air (Burt, 1984; Butterfield H, 1984; Drake Stillman 1981). Aristotle observed that some objects on Earth are light and others heavy. He attributed the property to be light or heavy to the intrinsic percentage of each of the four fundamental elements. In this system, the earth element is associated with higher weight (or density), the water and the air occupy an intermediate position, while the fire element is the lightest (or least dense) of all. A question always came to the discussion: which should be the natural movement of a given object? Aristotle thought that if it were heavy, his natural movement would be down, and if it were light, his natural movement would be upward. The light smoke rises straight, unless it is blown by the wind, while a stone, a block or a piece of iron falls straight when abandoned from a certain height. (Chalmers, 1990; Cohen Bernard, 1985; Drake Stillman, 1981; Popper KR, 1968). For Aristotle, the natural motion of an object was a straight line, with the ascending or descending direction determined by the vertical line passing through the center of the Earth and by the observer. A heavy body would fall in a straight line tending to reach its natural place, which is the ground, speed being proportional to its mass: "the heavier the body the greater its speed" (Geymonat L, 1983; Redondi P, 1991). Galileo addresses this subject in the book *Discorsi and Dimostrazioni Matematiche intorno a due Nuove Scienze atteneti alla Mecanica ed ai Movimenti Localli*, launched in 1638, concentrating on the movement of bodies falling in opposition to the Aristotelian theory, stating that the rate of fall is not proportional to the body mass, but dependent on external factors, namely air resistance. In this sense, tells the legend that he would have done an experiment to confirm his theory, the famous Tower experiment, in which two iron balls with different masses were abandoned from the top of a tower of 100 fathoms high. A fathom is an old unit of length equal to 1.8288 m. According to the book, the larger ball hits the ground with a difference of two fingers compared to the smaller ball, as shown in an excerpt from the book *Discorsi* written in original Italian in a dialogue between Simplicio and Salviati:

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[...] Dice Salviati. Dovevi voi dire, un grano di rena comer uma macina da Guado. Io non vorrei, Sig. Simplicio, che voi faceste comer fanno molt'altri, che il divertendo Discorso principale dal tentativa vi attaccaste mio che detto um mancasse dal vero quant'è um capello, e che questo sotto capello voleste nasconder um difetto d'un altro, grande gomona quant'una da nave. Aristotele diz: "una palla di ferro di Cento Libbre, cadendo dall'altezza braccia di Cento, Arriva na terra che sia Prime One deu um Libbra scesa um sol braccio"; io médico ch'ell'arrivano nell'istesso tempo; voi trovate, l'esperienza nel farne, che la Maggiore antecipado devido Dite Minore, cioè che when grande percuote in terra ne l'altra è lontana devido dita: ora dopo queste devido dita vorreste appiattare braccia di Aristotele Novantanove ele, e proferindo apenas o mio mínimo errore, Metter silenzio sotto l'altro Massimo. Aristotele pronunzia mobili che nel medesimo di mezzo gravità diferente se muovono (por Quanto depende Dalla gravità) com proporzionate velocitadi ai papagaio pesi, e l'esemplifica com Contem ne i quali se p scorgere ed puro il assoluto peso effetto, lasciando l'altre Considerazioni figura delle come sim eu Minimi momenti, le quali cose grande ricevono alterazione dal mezzo, che semplice effetto della altera il gravità único: Perciò che si vede l'ouro gravíssimo tutte sopra l'altre materie I Ridotto de uma só vez Foglia sottilissima vagando por aria; l'istesso fanno i sassi pestati em sottilissima polvere. Ma voi volete mantenere o proposizione universale, bisogna che voi mostriate, o delle proporzione velocità osservarsi em tutti i gravi, e che uma Sasso di venti Libbre più volte se muova dieci che di devido veloce um; il che vi médico esser falsa, e che, cadendo dall'altezza di cinquanta ou cento braccia, na terra nell'istesso tempo Arrivano [...] (Discorsi, p.27 1638)

In this dialogue, Simplicius is an enthusiast of Aristotelian ideas and Salviati personifies the new conception of Galileo's mechanics. Salviati claims that a 100-pound iron ball, loosened from a height of 100 fathoms**,** anticipates two fingers to a 1-pound iron ball, position diametrically opposed to the Aristotelian theory of proportionality between velocity and mass. This experiment, supposedly carried out by Galileo, fuels much epistemological discussion among commentators of his work. Empirists like Stillman Drake advocate experimental practice in developing their scientific legacy.

According to Drake, the experience must be associated with the measurements and calculations: "the new foundation for the science of Galileo was careful measurement by which sought to replace the old search for causes for a modern pursuit of physical laws (Drake-Stillman, 1981). For him, the physics of Galileo was based on their own actual calculations, which by talent and accuracy of their measurements, led him to develop the law of falling bodies. To legitimize your opinion, Drake refers to the text in which Galileo, on the third day of the book *Discorsi* describes in Italian the famous experiment of the inclined plane.

[…] In un regolo, o vogliàn dir corrente, di legno, lungo circa 12 braccia, e largo per un verso mezo bracio e per l'altro 3 dita, si era in questa minor larghezza incavato un canaletto, poco più largo d'un dito; tiratolo drittissimo, e, per averlo ben pulito e liscio, incollatovi dentro una carta pecora zannata e lustrata al possibile, si faceva in esso scendere una palla di bronzo durissimo, ben rotondata e pulita; costituito che si era il detto regolo pendente, elevando sopra il piano orizontale una delle sue estremità un braccio o due ad arbitrio, si lasciava (come dico) scendere per il detto canale la palla, notando, nel modo che appresso dirò, il tiempo che consumava nello scorrerlo tutto, replicando il medesimo atto molte volte per assicurarsi bene della quantità del tiempo, nel quale non si trovava mai differenza né anco della decima[…].(Discorsi, giornata terza del moto locali, 1638)

In this experiment, Galileo makes an association between the vertical drop movement of a ball and the rolling motion of the same ball in a plane with almost no friction. The plan acts as a dilute of the speed of the ball, facilitating greatly the measures of time. According to Drake, Galileo's experience was the basis of the reality that was being investigated and mathematics a restricted role in the formulation of physical laws that explained the phenomenon.

[...] The key to Galileo's mathematical physics was its application of a theory of proportionality and real measurements, which should be done as accurately as possible by the means available at the time (Drake Stillman, 1981, page 9)

According to the above quotes, Drake argues the Galilean legacy of the inductive character, where experimental observations of the phenomenon were carefully measured and then studied in the light of reason, leading to the mathematical formulation of physical laws. Rationalists argue the hypothetical-deductive (for example, assumption) character of scientific incursions in an attempt to explain nature. Alexander Koyrè represents one of the exponents of this point of view. For him, Galileo's conception of the scientific method involves a predominance of reason over the simple experiment, replacing a reality empirically known as idealized models by taking mathematics as an anchor, prioritizing the theory of facts (Koyrè A, 1961; Koyrè A, 1986; Koyrè A, 1991). Only then, the limitations of Aristotelian empiricism can be overcome, leading to the establishment of the essence of the true experimental method (Burt, 1984; J Henry, 1987; Kneller G, 1978) The main characteristic of Koyrè's thought is the preponderance of the role of reason in Galileo's legacy.

[...]. We must not forget that the spontaneous experience of common sense did not play an important role in its science, if it did it was a negative role, an obstacle in the foundation of modern science. Aristotle's physics was much closer to the experience of common sense than the analysis of Galileo or Descartes [...] (Koyrè A, 1991, p.15) In the development of his thought, he underestimates the confrontation with the empirical. It eliminates common-sense experiences and reinforces experiences with a high level of rationality. As himself says:

[...] Galileo's conception of the correct scientific method involves the predominance of reason to pure and simple experience, the replacement of an empirically known reality with idealized mathematical models, and the prevalence of theory over facts. Only in this way, the limitations of Aristotle's empiricism can be overcome, leading to the establishment of a true experimental method [...] (Koyrè A, 1991, p.154).

Koyrè also States that:

[...] The experience for Galileo is associated with the Latin word *Experimentum* which represents the opposition to common experience. *Experimentum* is a question posed to nature, using special language, geometric and mathematical language [...] (Koyrè 1991, page 151).

It is precisely the development of experimental method, understood as a rational reflection method, formulated from a language of Mathematics (geometry), which Koyrè identifies the time of discontinuity of the work of Galileo, when compared with those of their predecessors. The position taken by Koyrè emphasizes two important elements of the historiography by him built and the epistemology of Bachelard: opposition between common sense and scientific knowledge and innovative vision in the development of scientific thought. Ludovico Geymonat in his works (Geymonat L, 1993; Geymonat L and Giorello, 1986; Geymonat L, 1997) adopts a less asymmetric aspect and defends the genial character of Galileo in the development of his physics. According to him, the experimental incursions of Galileo combined with hypothetical deductive incursions. This transition was made naturally, and there is no exclusive demarcation line in the development of his thinking. The invitation to experience according to Geymonat was a gradually articulated process where Galileo involved all his creativity and technical knowledge. That is, scientific research is not a moment of pure passivity, as some think, but of a kind of diversified empirical activity that is purely theoretical. Therefore, the observation time for Galileo is not contrary to the moment of mathematical elaboration: it represents distinct but not opposed phases of scientific research.

The famous Tower experiment seems to have been more of an idealization, a Gedankenexperiment, a term used by Ernst Mach (thought experiment) to denote an imaginary behaviour analogous to the search procedures that should be used by scientific experimenters. Galileo already knew the result; in this specific case, it seems probable that the experimental work had a secondary character in the elaboration of his thesis (Geymonat L, 1997; Kneller G, 1978). To prove Galileo's thesis on the difference of space travelled by the two balls, a re-reading of the Tower's legendary experience was carried out using a computational resource to mathematically model the phenomenon, Mathematica 11.0 software. The data used in the development of the program are in the official notes contained in the original edition of the book *Discorsi*.

Methods and Procedures

Figure 1 shows the forces present in a solid ball that drops falls under the action of the force of gravity in the presence of the air resistance and buoyancy.

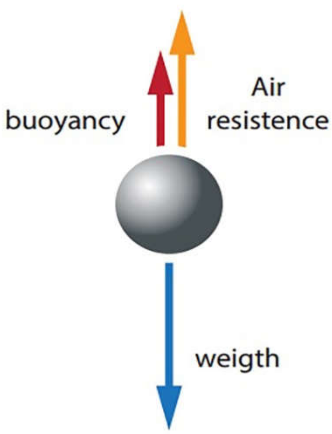

Figure 1. Forces present in the fall of a body in a viscous medium in the presence of the force of gravity, air resistance to movement and buoyancy

The dynamic description of a body in fall is given by the equation:

$$F = P - E - R \tag{1}$$

F, *P*, *E* and *R* being the net force, the body weight, the buoyancy caused by the medium and the air resistance force, respectively.

For subsonic speeds between 86 km/h and 1,200 km/h, the air resistance force (*R*) is proportional to the square of speed (Timoshenko, 1951):

$$R = \beta v^2 \tag{2}$$

In the particular case of the iron ball whose density is much greater than that of the medium (air), the buoyancy is insignificant and can be eliminated from the equation.
Equation (1) can be rewritten:

$$ma = mg - \beta v^2 \tag{3}$$

or,

$$a = g - \frac{\beta}{m} v^2 \tag{4}$$

Resulting differential equation:

$$\frac{dv}{dt} = g - \frac{\beta}{m} v^2 \tag{5}$$

whose solution to the initial condition $v(0) = 0$ is as follows:

$$v(t) = \sqrt{\frac{mg}{\beta}} \tanh\left(\sqrt{\frac{\beta g}{m}} \cdot t\right) \tag{6}$$

The height as a function of time, *H*(*t*), can be obtained through the direct integration of (6) respecting the condition $H(0) = 0$:

$$H(t) = \frac{m}{\beta} \ln\left[\cosh\left(\sqrt{\frac{\beta g}{m}} t\right)\right] \tag{7}$$

where is the aerodynamic coefficient β is given by:

$$\beta = \frac{1}{2} C \rho A \tag{8}$$

The constant *C* in (8) depends on the shape of the body, ρ represents the mean density of the material and *A* is the cross-sectional area of the body. The constants used in the development of the calculations were: g (acceleration due to gravity), 9.81 m/s^2; air density, 1.22 kg/m^3; *C* (aerodynamic drag for the ball), 0.47; and iron density, 7.87×10^3 kg/m^3. The bodies used in the simulation consisted of two iron balls, the largest being 100 pounds, or 45.4 kg, and the smallest one pound, or 0.454 kg. Using equation (8), the aerodynamic coefficients were calculated from the two balls, as shown in Table 1.

Table 1. Aerodynamic coefficients of the two iron balls (β), cross section radius(R), and mass (M)

Iron density (ρ_{Fe}): 7.87 x 10^3 kg/m³		
	Big ball	Small ball
Radius (m)	0.11	0.024
Mass (kg)	45.4	0.454
β (kg/m)	0.011	0.00052

The inference of the aerodynamic coefficient of the larger ball in equation (7) and the values of its mass and the height of fall (183.00 m or 100 fathoms) allow the determination of the time of its arrival in the ground. The procedure can be done with the corresponding parameters of the smaller ball to calculate the distance covered by it in the same time interval. Calculations were performed using Mathematica 11.0 software. Wolfram Mathematica is a computer program, originally designed by Stephen Wolfram and continuously developed by Wolfram Research, based in Champaign, Illinois, which implements a computer algebra system based on symbolic computation. The software contains a series of ready-to-use programming libraries for various fields of engineering, biology, chemistry, image processing, finance, statistics, and math, among others and also serves as a means for rapid program development. Mathematica is very effective in obtaining equations based on physical parameters, defined by the user, in presenting the results as graphs, tables and animations in the assembly and execution of calculations tool.

RESULTS

The time of fall of the largest ball was 6.15647 s ≈ 6.16 s, found by solving equation (7) with H = 183.00 m using Mathematica software. For the smaller ball, the drop time (for the same height H), is 6.32677 s ≈ 6.33 s. The substitution of time values and ball-related parameters in equations (6) and (7) led to the construction of Table 2.

Table 2. Speed and distance travelled by the balls at the time of the fall of the ball higher and the distance between them (ΔH) at that time

	Big ball	Small ball	Distance variation traveled ΔH (m)
Speed (m/s)	58.59	53.02	
Distance (m)	183.00	173.88	9.12

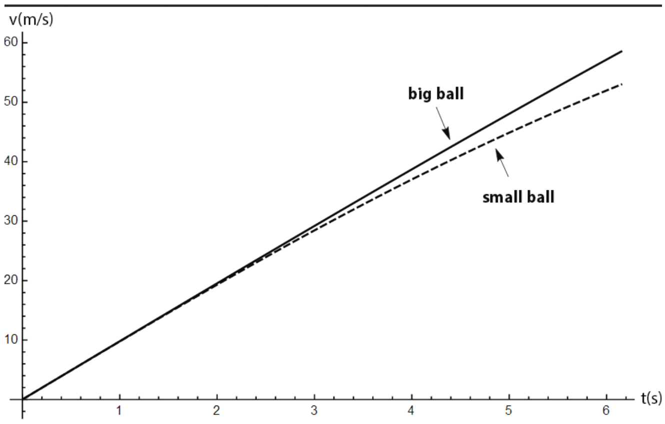

Figure 2. Graph of the speed of the two balls in the time between 0.00 and 6.16 s. The larger ball is represented by the continuous curve, while the smaller ball is represented by the dashed curve. At 6.16 s, the speed of the largest ball is 58.59 m/s and the speed of the smaller ball is 53.02 m/s

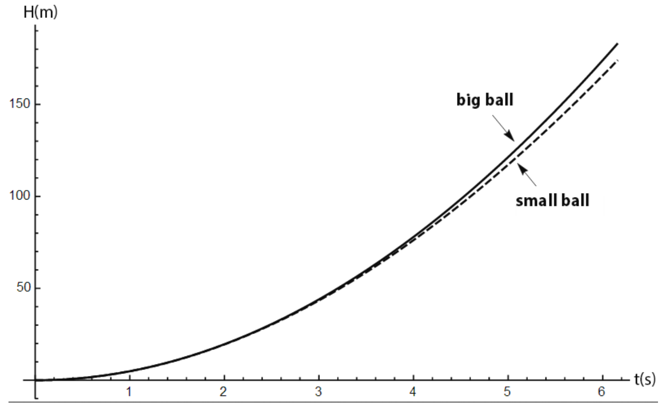

Figure 3. Graph of the distances covered by the two balls in the time interval between 0.00 and 6.16 s. The larger ball is represented by the continuous curve, while the smaller ball is represented by the dashed curve. In the time of 6.16 s, the larger ball covered 183.00 m, while the smaller ball ran 173.88 m

Equations (6) and (7) were also used to compute the graphs of the speed and distance travelled by two balls in the approximate time of 6.16 s, shown in figures 2 and 3, respectively.

Figure 4 shows a snapshot of the initial and final positions of the two balls (major and minor) in the time interval of 6.16 s.

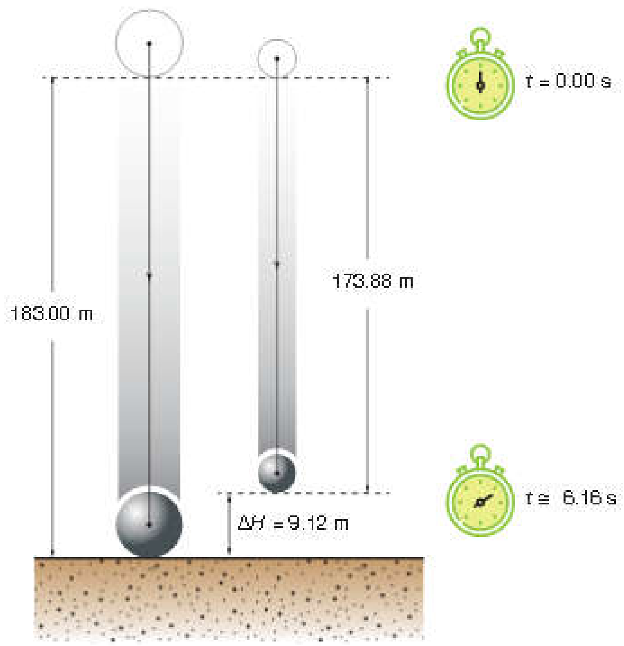

Figure 4. Positions of the balls in the time of 6.16 s. At the end of the time interval, the larger ball is 9.12 m ahead of the smaller ball

DISCUSSION

Observing the results presented in the last session, it can be stated that:

1. At the approximate time of 6.16 s, larger ball hits the ground with a speed of 58.59 m/s, or 210.92 km/h, while the ball less this is lively with a 53.02 speed m/s, or 190.87 km/h. The ratio between speeds (1.105) is not equal to ratio between masses (100). Thus, the

velocities of the two balls increase over time, but not directly proportional to the masses, as stated Aristotle.

2. The difference in speed is due to the strength of the air resistance, because although it acts simultaneously in both balls, the magnitude of its action is more significant in the smaller ball. The distances covered by the two balls in the approximate time of 6.16 s are very different.
3. Initially, the distances covered by the two balls are practically identical. As time passes, the larger ball begins to move away from the smaller one because of its increased velocity. When it reaches the ground covering 183.00 m, the smaller ball is in position 173.88 m, that is, 9.12 m above the larger ball.
4. The time elapsed between the arrival of the balls on the ground is $\Delta t = 0.17$ s. Such small measurements of time intervals were impossible with the technology that Galileo had at the time. For comparative purposes, if the heart rate (80 beats per minute) were used as a clock, the shortest time interval measured would be 0.75 s, that is, 4.4 times greater than Δt. The use of water clocks and other time measuring devices used by Galileo would involve both operational delays and reaction times, probably exceeding the Δt value.

Conclusion

The reissue of the Tower experiment reported by Galileo in the book Discorsi showed that his prediction was underestimated. The difference in distance between the two balls in the same time interval was much greater than the two fingers provided by him. This incongruence can be explained by the technological inability to monitor the very rapid fall process. An observer positioned at ground level would see the two balls arrive at almost the same time, since the average time spent for the smaller ball in accelerated motion runs through 9.12, approximates the time of human reaction. Galileo knew that resistance affected movement; it was not possible to quantify precisely the influence of the same action on the two balls, hence their error. Certainly, the tower experience is a myth. If it really happened, it was just a public demonstration of Galileo. Vicenzio Viviani (1622-1703), a great admirer and disciple of Galileo, published 60 years after the master's death, mentioned the experience of the tower for the first time in the work Racconto Istorico. It is interesting to note that there are no historical records of the supposed experiment despite its scientific and historical relevance. This reinforces the idea that the experiment was never carried out, consisting of an initiative purely in the mind (thinking experiment), reinforcing Galileo's genial character (Segre, 1989; Moreau, 2002). It should be noted that the error reported does not invalidate or depreciate the genius of his personality, since he has been able to confront, through very coherent arguments, the Aristotelian presuppositions that lasted almost 2000 years.

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