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International Journal of Current Research
Vol. 9, Issue, 12, pp.63136-63141, December, 2017

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

STUDY ON EINSTEIN- KAEHLERIAN DECOMPOSABLE RECURRENT SPACE OF FIRST ORDER

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ARTICLE INFO

Article History:
Received 20th September, 2017
Received in revised form
23rd October, 2017
Accepted 10th November, 2017
Published online 31st December, 2017

Key words:
Kaehlerian space,
Einstein space,
Einstein-Kaehlerian space,
recurrent space,
Curvature tensor,
Projective curvature tensor.

ABSTRACT

Takano [1967] have studied decomposition of curvature tensor in a recurrent space. Sinha and Singh [1970] have been studied and defined decomposition of recurrent curvature tensor field in a Finsler space. Singh and Negi studied decomposition of recurrent curvature tensor field in a Kaehlerian space. Negi and Rawat [1995] have studied decomposition of recurrent curvature tensor field in Kaehlerian space. Rawat and Silswal [2007] studied and defined decomposition of recurrent curvature tensor fields in a Tachibana space. Further, Rawat and Kunwar Singh [2008] studied the decomposition of curvature tensor field in Kaehlerian recurrent space of first order. In the present paper, we have studied the decomposition of curvature tensor fields R_{ijk}^h in terms of two non-zero vectors and a tensor field in Einstein-Kaehlerian recurrent space of first order and several theorem have been established and proved.

Citation: Rawat, K.S. and Sandeep Chauhan, 2017. "Study on einstein- kaehlerian decomposable recurrent space of first order", *International Journal of Current Research*, 9, (12), 63136-63141.

INTRODUCTION

An n (=2m) dimensional Kaehlerian space K_n is a Riemannian space, which admits a tensor field F_i^h satisfying the conditions

$$F_i^h F_h^j = - \delta_i^j \tag{1.1}$$

$$F_{ij} = - F_{ji} , (F_{ij} = F_i^a g_{aj}) \tag{1.2}$$

and $F_{i,j}^h = 0$, (1.3)

Where the (,) followed by an index denotes the operation of covariant differentiation with respect to the metric tensor g_{ij} of the Riemannian space.

The Riemannian curvature tensor R_{ijk}^h is given by

$$R_{ijk}^h = \partial_i \begin{Bmatrix} h \\ j\ k \end{Bmatrix} - \partial_j \begin{Bmatrix} h \\ i\ k \end{Bmatrix} + \begin{Bmatrix} h \\ i\ \alpha \end{Bmatrix} \begin{Bmatrix} \alpha \\ j\ k \end{Bmatrix} - \begin{Bmatrix} h \\ j\ \alpha \end{Bmatrix} \begin{Bmatrix} \alpha \\ i\ k \end{Bmatrix},$$

The Ricci tensor and the scalar Curvature tensor are respectively given by

$$R_{ij} = R_{\alpha ij}^{\alpha} \text{ and } R = g_{ij} R_{ij} \tag{1.4}$$

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It is well known that these tensors satisfies the following identities

$$R^{a}_{ijk} = R_{jk,i} - R_{ik,j} \quad (1.5)$$

$$R_{,i} = 2R^{a}_{i,a} \quad (1.6)$$

$$F^{a}_{i} R_{aj} = - R_{ia} F^{a}_{j} \quad (1.7)$$

and

$$F^{a}_{i} R^{i}_{a} = R^{a}_{i} F^{i}_{a} \quad (1.8)$$

The holomorphically projective curvature tensor P^{h}_{ijk} is defined by

$$P^{h}_{ijk} = R^{h}_{ijk} + \frac{1}{(n+2)} (R_{ik}\, \delta^{h}_{j} - R_{jk} \delta^{h}_{i} + S_{ik} F^{h}_{j} - S_{jk}\, F^{h}_{i} + 2S_{ij}\, F^{h}_{k}) \quad (1.9)$$

where $S_{ij} = F^{a}_{i}\, R_{aj}$

Let us suppose that a Kaehlerian space is Einstein one, and then the Ricci tensor satisfies

$R_{ij} = \frac{R}{n} g_{ij}\ ,\ R_{,a} = 0$

from which, we obtain

$R_{ij}, a = 0\ ,\ S_{ij,a} = 0$

and $S_{ij} = \frac{R}{n}\, F_{ij}$

The Bianchi identity for Einstein-Kaehlerian space are given by

$$R^{h}_{ijk} + R^{h}_{jki} + R^{h}_{kij} = 0 \quad (1.10)$$

and

$$R^{h}_{ijk,a} + R^{h}_{ika,j} + R^{h}_{iaj,k} = 0 \quad (1.11)$$

The Commutative formulae for the Curvature tensor fields are given as follows

$$T^{i}_{,jk} - T^{i}_{,kj} = T^{a}\, R^{i}_{ajk} \quad (1.12)$$

$$T^{h}_{i,ml} - T^{h}_{i,lm} = T^{a}_{i}\, R^{h}_{aml} - T^{h}_{a}\, R^{a}_{iml} \quad (1.13)$$

A Einstein-Kaehlerian space is said to be Einstein-Kaehlerian recurrent space of first order, if its curvature tensor field satisfy the condition

$$R^{h}_{ijk,a} = \lambda_{a}\, R^{h}_{ijk} \quad (1.14)$$

where λ_{a} is a non - zero vector and is known as recurrence vector field. The following relations follow immediately from equation (1.14),

$$R_{ij,a} = \lambda_{a}\, R_{ij} \quad (1.15)$$

and

$$R_{,a} = \lambda_{a} R \quad (1.16)$$

2. Decomposition of Curvature Tensor Field R^{h}_{ijk}

We Consider the decomposition of recurrent curvature tensor field R^{h}_{ijk} in the following form

$$R^h_{ijk} = V^h_i \Phi_j \Psi_k \tag{2.1}$$

where the non - zero tensor field V^h_i and vector Φ_j , Ψ_k are such that

$$\lambda_h V^h_i = P_i \tag{2.2}$$

Theorem (2.1) : Under the decomposition (2.1), the Bianchi identities for R^h_{ijk} takes the forms

$$P_i \Phi_j \Psi_k + P_j \Phi_k \Psi_i + P_k \Phi_i \Psi_j = 0 \tag{2.3}$$

and

$$\lambda_a \Phi_j \psi_k + \lambda_j \Phi_k \psi_a + \lambda_k \Phi_a \psi_j = 0 \tag{2.4}$$

Proof:- From Equations (1.10) and (2.1), we have

$$V^h_i \Phi_j \Psi_k + V^h_j \Phi_k \psi_i + V^h_k \Phi_i \Psi_j = 0 \tag{2.5}$$

Multiplying (2.5), by λ_h , and using (2.2), we get relation (2.3)

$$P_i \Phi_j \Psi_k + P_j \Phi_k \Psi_i + P_k \Phi_i \Psi_j = 0$$

From Equations (1.11), (1.14) and (2.1), we have

$$V^h_i [\lambda_a \phi_j \psi_k + \lambda_j \phi_k \psi_a + \lambda_k \Phi_a \psi_j] = 0 \tag{2.6}$$

Multiplying (2.6) by λ_h and using (2.2), we get relation (2.4).

Theorem (2.2) : Under the decomposition (2.1), the tensor field R^h_{ijk}, R_{ij} and vectors ϕ_j , ψ_k satisfies the relations

$$\lambda_a R^a_{ijk} = \lambda_i R_{jk} - \lambda_j R_{ik} = P_i \phi_j \psi_k \tag{2.7}$$

Proof : With the help of Equations (1.5), (1.14) and (1.15), we have

$$\lambda_a R^a_{ijk} = \lambda_i R_{jk} - \lambda_j R_{ik} \tag{2.8}$$

Multiplying (2.1) by λ_h , and using relation (2.2), we get

$$\lambda_h R^h_{ijk} = P_i \phi_j \psi_k \tag{2.9}$$

in view of (2.8) and (2.9), we get the required relation (2.7).

Theorem (2.3) : Under the decomposition (2.1), the quantities λ_a and V^h_i behave like the recurrent vector and tensor field. The recurrent form of these quantities are given by

$$\lambda_{a,m} = \mu_m \lambda_a \tag{2.10}$$

$$V^h_{i,m} = \mu_m V^h_i \tag{2.11}$$

Proof : Differentiating (2.7), covariantly with respect to x^m, and using (2.1) and (2.7) , we obtain

$$\lambda_{a,m} V^a_i \phi_j \psi_k = \lambda_{i,m} R_{jk} - \lambda_{j,m} R_{ik} \tag{2.12}$$

Multiplying (2.12) by λ_a and using (2.1) and (2.8), we get

$$\lambda_{a,m}(\lambda_i R_{jk} - \lambda_j R_{ik}) = \lambda_a(\lambda_{i,m} R_{jk} - \lambda_{j,m} R_{ik}) \tag{2.13}$$

Now, multiplying (2.13) by λ_h , we have

$$\lambda_{a,m} (\lambda_i R_{jk} - \lambda_j R_{ik}) \lambda_h = \lambda_a \lambda_h(\lambda_{i,m} R_{jk} - \lambda_{j,m} R_{ik}) \tag{2.14}$$

Since the expression of right hand side of the above equation is symmetric in a and h, therefore

$\lambda_{a,m}\lambda_h = \lambda_{h,m}\, \lambda_a$, (2.15)

Provided that $\lambda_i R_{jk} - \lambda_j R_{ik} \neq 0$

The vector field λ_a being non-zero, we can have a proportional vector μ_m such that

$$\lambda_{a,m} = \mu_m \lambda_a \quad (2.16)$$

Further, differentiating (2,2) w.r to x^m and using (2.16), we get

$$\lambda_h V^h_{i,m} = P_{i,m} - \mu_m\, P_i \quad (2.17)$$

from the above equation, it is obvious that

$$\lambda_h\, V^h_{i,m} = \lambda_a\, V^a_{i,m} \quad (2.18)$$

Since λ_a is a non-zero recurrence vector field, we can get a proportional vector field μ_m such that

$$V^h_{i,m} = \mu_m V^h_i$$

which complete the proof.

Theorem (2.4) : Under the decomposition (2.1), the vector field P_i , ϕ_j, ψ_k behave like recurrent vectors and their recurrent form are given respectively by

$$P_{i,m} = 2\, \mu_m\, P_i \quad (2.19)$$

and

$$(\lambda_m - \mu_m)\, \phi_j\, \psi_k = \phi_{j,m}\, \psi_k + \phi_j\, \psi_{k,m} \quad (2.20)$$

Proof Differentiating (2.2) covariantly w.r. to x^m, and using equation (2.2), (2.10) and (2.11), we obtain the required result (2.19). Further, differentiating equation (2.1) covariantly w.r. to x^m, and using equation (1.14), (2.1) and (2.11), we get the required recurrent form (2.20).

Theorem (2.5) : Under the decomposition (2.1), the curvature tensor and holomorphically projective curvature tensor are equal if

$$\Phi_k\, \Psi_l\, \{(P_i\, \delta^h_j - P_j\, \delta^h_i) + P_a\, (F^a_i\, F^h_j - F^a_j\, F^h_i)\} + 2\, P_a\, \Phi_j\, \Psi_l\, F^a_i\, F^h_k = 0 \quad (2.21)$$

Proof: The equation (1.9), may be written in the form

$$P^h_{ijk} = R^h_{ijk} + D^h_{ijk} \quad (2.22)$$

where

$$D^h_{ijk} = \frac{1}{n+2}\,(R_{ik}\, \delta^h_j - R_{jk}\, \delta^h_i + S_{ik}\, F^h_j - S_{jk}\, F^h_i + 2\, S_{ij}\, F^h_k) \quad (2.23)$$

Contracting indices h and k in (2.1), we obtain

$$R_{ij} = V^l_i\, \Phi_j\, \Psi_l \quad (2.24)$$

In view of equation (2.24), we have

$$S_{ij} = F^a_i\, \Phi_j\, \Psi_l\, V^l_a \quad (2.25)$$

Making use of (2.24) and (2.25) in equation (2.22), we get

$$D^h_{ijk} = \frac{1}{n+2}\,[\Phi_k\, \Psi_l\, \{(V^l_i\, \delta^h_j - V^l_j\, \delta^h_i) + V^l_a\, (F^a_i\, F^h_j - F^a_j\, F^h_i)\} + 2\, \Phi_j\, \Psi_l\, F^a_i\, F^h_k\, V^l_a] \quad (2.26)$$

In view of (2.23), it is clear that

$P^h_{ijk} = R^h_{ijk}$ iff $D^h_{ijk} = 0$, which in view of equation (2.26) gives

$$[\Phi_k \Psi_l \{(V_i^l \delta_j^h - V_j^l \delta_i^h) + V_a^l (F_i^a F_j^h - F_j^a F_i^h)\} + 2 \Phi_j \Psi_l F_i^a F_k^h V_a^l] = 0 \quad (2.27)$$

Multiplying (2.27) by λ_l and using (2.2), we obtain the required condition (2.21).

Theorem (2.6) : Under the decomposition (2.1), the scalar curvature R, satisfy the relation.

$$\lambda_k R = g^{ij} P_i \Phi_j \Psi_l \quad (2.28)$$

Proof : Contracting indices h and k in (2.1), we get

$$R_{ij} = V_i^l \Phi_j \Psi_l \quad (2.29)$$

Multiplying (2.29) by g^{ij} both sides, we get

$$g^{ij} R_{ij} = g^{ij} V_i^l \Phi_j \Psi_l \quad (2.30)$$

in view of Equation (1.4), the above equation reduces to

$$R = g^{ij} V_i^l \Phi_j \Psi_l \quad (2.31)$$

Now multiplying (2.31) by λ_l and using (2.2), we get

$$\lambda_l R = g^{ij} P_i \Phi_j \Psi_l$$

which complete the proof of the theorem.

REFERENCES

Lal, K.B. and Singh, S.S. 1971. On Kaehlerian spaces with recurrent Bochner Curvature, Academic Nazionale Dei. Lincei, Series VIII, Vol. LI, 3-4, 213-220.

Negi, D.S. and Rawat, K.S. 1995. Decomposition of recurrent Curvature tensor fields in a Kaehlerian space, Acta Cien. Ind., Vol. XXI, No. 2M, 139-142.

Negi, D.S. and Rawat, K.S. 1997. The study of decomposition in Kaehlerian space, Acta Cien. Ind., Vol. XXIII M, No. 4, 307-311.

Rawat, K. S. and Nitin Uniyal, 2011. Lie Derivative of a Linear connexion and various kinds of motions in a Kaehlerian recurrent space of first order, Journal of Inter. Aca. of Physical Sciences, Vol. 15, No.4., 467-478, ISSN - 0974-9373.

Rawat, K.S. 2005. The decomposition of recurrent Curvature tensor fields in a Kaehlerian space, Acta Cien. Ind., Vol. XXXI M, No. 1, 135-138.

Rawat, K.S. and Dobhal, Girish, 2007. Some theorems On Decomposition of Tachibana recurrent space, Jour. PAS, Vol. 13 (Ser. A), 458-464.

Rawat, K.S. and Dobhal, Girish, 2007. The study of decomposition in a Kaehlerian recurrent space, Acta Cien. Ind., Vol. XXXIII M, No. 4, 1341-1345.

Rawat, K.S. and Girish Dobhal 2008. Study of the decomposition of recurrent Curvature tensor fields in a Kaehlerian recurrent space, Jour. Pure and applied Mathematika Sciences, Vol. LXVIII, No. 1-2, ISSN 0379-3108.

Rawat, K.S. and Girish Dobhal, 2009. Decomposition of recurrent curvature tensor fields in a Tachibana space, Inter. Trans. in applied Sciences, Vol. 1, No. 4, 603-608, ISSN – 0974-7273.

Rawat, K.S. and Kunwar Singh, 2009. Decomposition of curvature tensor fields in a Tachibana first order recurrent space, Acta Cien. Ind., Vol. XXXV M, No. 3, 1081-1085.

Rawat, K.S. and Kunwar Singh, 2009. The study of decomposition of curvature tensor fields in a Kaehlerian recurrent space of first order, Jour. of the tensor Society, Vol. 3, 11-18.

Rawat, K.S. and Mukesh Kumar, 2011. On Hyper Surfaces of a Conformally flat Kaehlerian recurrent space, Jour. Pure and applied Mathematika Sciences, Vol. LXXIII, No. 1-2, 7-13, ISSN-0379-3168.

Rawat, K.S. and Mukesh Kumar, 2012. Motions in a Riemannian space and some integrability conditions, Ultra Scientist of Physical Sciences (An Inter. Jour. of Physical Sciences), Vol. 24 (1) A, 9-14, ISSN – 0970-9150.

Rawat, K.S. and Mukesh Uniyal, 2012. Study on Homothetic motions in Conformally related spaces, Jour. of Progressive Science, Vol. 03, No. 01, 54-59.

Rawat, K.S. and Nitin Uniyal, 2012. Decomposition of curvature tensor fields in a Tachibana recurrent space, Ultra Scientist of Physical Sciences (An Inter. Jour. of Physical Sciences), Vol. 24 (2) A, 424-428, ISSN – 0970-9150.

Rawat, K.S. and Nitin Uniyal, 2011. On decomposition of curvature tensor fields in a Kaehlerian Recurrent space of first order, Jnanabha (*Jour. of The Vijnana Parishad of India*), Vol. 41, 27-32, ISSN – 0304-9892.

Rawat, K.S. and Silswal, G.P. 2005. Decomposition of recurrent Curvature tensor fields in a Kaehlerian recurrent space, Acta Cien. Ind., Vol. XXXI M, No. 3, 795-799.

Rawat, K.S. and Virendra Prasad, 2010. On holomorphically Projectively flat Parabolically Kaehlerian space, Rev. Bull. Cal. Math. Soc., 18 (1), 21-26, ISSN-0008-0659.
Sinha, B. B. and Singh, S. P. 1970. On decomposition of recurrent curvature tensor fields in a Finsler space, Bull. Cal. Math. Soc., 62 91-96.
Tachibana, S. 1967. On the Bochner Curvature tensor, Nat. Sci. Report, Ochanomizu Univ. 18 (1), 15-19.
Takano, K. 1967. Decomposition of Curvature tensor in a recurrent space, Tensor (N.S), 18 (3), 343-347.
Yano, K. 1965. Differential Geometry on Complex and almost Complex spaces, Pergamon Press.

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