



RESEARCH ARTICLE

INVESTIGATION OF STABILITY LOSSES OF THE PACKER WITH A GIRDLE-CYLINDRICAL-CONICAL CONSTRUCTION

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ABSTRACT

In the paper stability losses of the packer with a girdle construction have been investigated by taking into account compactness mechanism. Expanding and constricting cases of conical surface of the packer with a girdle construction have been studied, stability problem of the packer has been solved via numerical estimation by using finite differences method. With this purpose, continuity condition of the deformation has been written for packer construction (cylindrical and conical surface of the packer) and added to stability losses condition. In the first approach the solution has been carried out in boundary and junction nodes of the packer (where cylindrical and conical surfaces combine), however, in the second approach by finite differences method. During packing process stability losses limit has been studied for the first time for a girdle construction.

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INTRODUCTION

Let's study packing mechanism (Mammadov *et al.*, 2016; Mammadov and Gurbanov, 2015; Babanly *et al.*, 2016) by regulating stability losses with the participation of friction forces of packers placed between the tube attached to the pump receiver and operating pipeline in order to regulate dynamic level in the well being operated by rod well pumping unit. It has been accepted that, the packer lapping proposed increases the efficiency of the work in using the packer without tap (lapping replacing the anchor for creating packer stop). Thus, being in contact with operating pipeline thanks to friction force, its packing lapping switches to packing mode (without the help of tap lapping). Because, the construction of the packer newly created consists of two parts (whole) with cylindrical and conical surface.

Experimental Section

Let's consider the problem of stability losses during the work of the packer with ΔP girdle (cylindrical-conical surface) by external pressure distributed equally. The packer with cylindrical-conical construction is generally considered to have different thicknesses, and being connected with cylindrical-

conical parts between them. This can also be called a packer with girdle construction.

Objectives of the problem

The following two cases of the connection are possible:

- with its small diametric edge conical part is connected to the cylindrical part (fig.1) (expanding of conical part)

Initial data: Let's note the following markings:

- $\ell_{cyl}$  - the length forming the cylindrical part of the packer;
- $h_{cyl}$  - the thickness of cylindrical part wall of the packer;
- $a_{cyl}$  - the radius of the middle surface of cylindrical part of the packer;
- $E_{cyl}$  - elasticity module of packer material of cylindrical part;
- $\mu_{cyl}$  - Poisson coefficient of packer material of cylindrical part;
- $D_k = \frac{E_{cyl} h_k^3}{12(1 - \mu_{cyl}^2)}$  - cylindrical hardness of the cylindrical part;

$\ell_0$  - the distance measured along the generatrix from top surface of the cone to the edge of small diameter;

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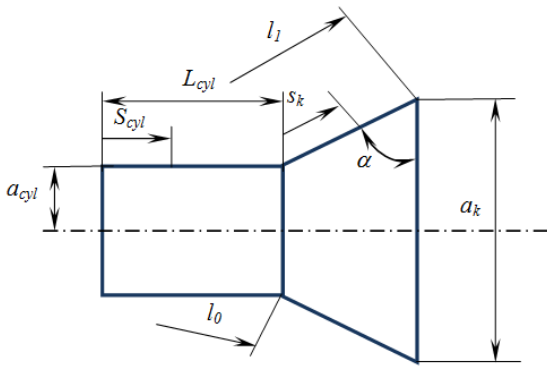
$h_k$  - the thickness of conical part wall of the packer;  
 $\alpha$  - the angle between the congeneratrix and its base;  
 $E_k$  - elasticity module of packer material of conical part;  
( $E_{cyl} = E_k$ );

$D_k = \frac{E_k h_k^3}{12(1-\mu_k^2)}$  - cylindrical hardness of the packer conical part.

Differential equation, solving stability problem of cylindrical part due to  $\Delta P = q$  external pressure impact, can be written as

follows within conditions  $\frac{h_{cyl}}{a_{cyl}} < \left(\frac{\ell_{cyl}}{a_{cyl}}\right)^2 < \frac{a_{cyl}}{h_{cyl}}$  :

$$\frac{d^4 \psi_{cyl}}{dx_{cyl}^4} k^4 \cdot \psi_{cyl} = 0 \quad (1)$$



**Fig. 1. Expanding direction parameters of conical surface of the packer with girdle construction**

here  $\psi_{cyl}(x)$  is deformation – flexion function of the packer and is connected with  $W_{cyl}(S)$  flexion as follows:

$$W_{cyl} = \psi_{cyl} \cos n_{cyl} \theta \quad (2)$$

where  $x_{cyl} = \frac{S_{cyl}}{a_{cyl}}$  is a measureless coordinate;

$n_{cyl}$  - is the number of the waves occurred in circular direction during stability losses;

$\theta$  - is the angle coordinate formed by the middle surface;

$$K^4 = P_{cyl}^3 (v_{cyl} - P_{cyl}); \quad P_{cyl} = \varepsilon_{cyl} n_{cyl}^2 \text{ and } v_{cyl} = \frac{q a_{cyl}}{E_{cyl} \cdot h_{cyl} \cdot \varepsilon_{cyl}^3}$$

- are measureless parameters;

$$\varepsilon_{cyl} = \sqrt[4]{\frac{h_{cyl}^2}{12(1-\mu_{cyl}^2) \alpha_{cyl}^2}} \text{ - are measureless parameters of the packer.}$$

1. If  $\alpha$  isn't equal to zero or  $\pi/2$ , then the system of differential equations, solving the stability problem in accordance with (4) work for conical part of the packer, can be replaced by a single equation.

$$\frac{d^2}{dx_k^2} \left( x_k^3 \frac{d^2 \psi_k}{dx_k^2} \right) + \left( \frac{P_k^4}{x_k^2} - P_k^2 v_k \right) \psi_k = 0 \quad (3)$$

where  $\psi_k(x)$  is flexion function and is connected with  $W_k(S)$  flexion by the following relation.

$$W_k = \psi_k \operatorname{tg} \alpha \cos n_k \theta \quad (4)$$

where  $x_k = \frac{S_k}{\ell_1}$  - is a measureless coordinate;

$n_k$  - is the number of the waves occurred in circular direction during stability losses;

$P_k = \varepsilon_k \frac{n_k^2}{\cos^2 \alpha}$  and  $v_k = \frac{q \ell_1}{E_k h_k} \left( \frac{\operatorname{ctg} \alpha}{\varepsilon_k} \right)^3$  are measureless parameters;

$\varepsilon_k = \sqrt[4]{\frac{h_k^2 \operatorname{ctg}^2 \alpha}{12(1-P_k^2) \ell_1^2}}$  - is a measureless parameter of the packer construction.

After transformations, (1) and (3) equations can be as follows:

$$\frac{1}{P_{cyl}} \cdot \frac{d^4 \varphi_{cyl}}{dx_{cyl}^4} + P_{cyl} \psi_{cyl} = v_{cyl} \psi_{cyl} \quad (5)$$

$$\frac{x_k^3}{P_k^3} \cdot \frac{d^4 \psi_k}{dx_k^4} + \frac{6x_k^2}{P_k^2} \cdot \frac{d^3 \psi_k}{dx_k^3} + \frac{6x_k}{P_k} \cdot \frac{d^2 \psi_k}{dx_k^2} + \frac{P_k}{x_k^3} \cdot \psi_k = v_k \psi_k \quad (6)$$

Thus, the same pressure  $q$  impacts on the construction formed by cylindrical and conical surfaces of the packer, then  $v_{cyl}$  parameter can be indicated by  $v_k$ , that is the following equation is obtained:

$$v_{cyl} = v_k \frac{E_k h_k a_{cyl}}{E_{cyl} h_{cyl} \ell_1} \left( \frac{\varepsilon_k \operatorname{tg} \alpha}{\varepsilon_{cyl}} \right)^3 \quad (7)$$

The approximate differential equations (5) and (6) do not allow for precise formation of elements attachment conditions (cylindrical and conical parts) of the packer with girdle construction.

Let's carry out only the following four of the eight attachment conditions.

- 1) Equality of  $W$  displacement projections perpendicularly to the rotation axis of a girdle construction of the packer to displacement projections  $U$  along generatrix of cylinder and cone, as they are very small,  $W$  can not be taken into account;
- 2) The equality of rotational angles  $\nu$  forming cylinders and cones,
- 3) Equality of the spindle angles  $M_1$ ;
- 4) Equality of sliding forces  $T_{12}$ ;

During the connection of conical part with its large diameter edge to the cylindrical part (Fig. 2) (cone limitation), the positive direction of the coordinates is presented in Fig.2. Positive directions of  $W$  displacement, rotational angle, bending moment  $M_1$ , and sliding force  $T_{12}$  is presented in Fig. 3.

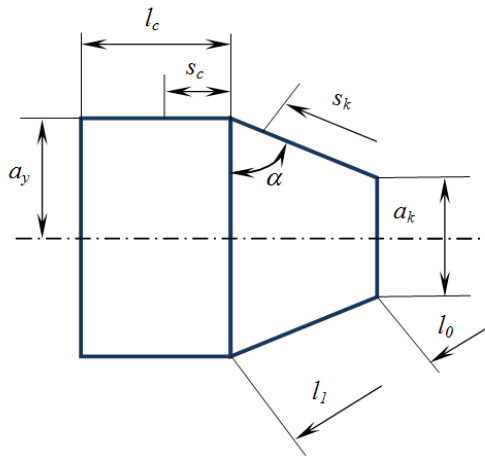


Fig. 2. Direction of coordinates in the constricting of the conical surface of the packer with a girdle construction

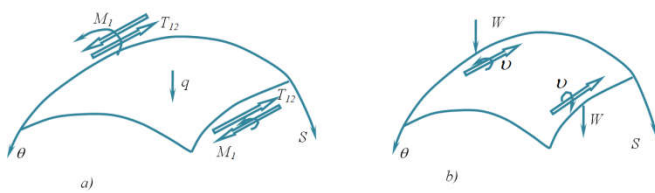


Fig.3. Positive directions:

- a - bending moment  $M_1$  and sliding force  $T_{12}$  vectors;
- b - W displacement and rotation angle vectors  $\nu \leftarrow \vartheta$ .

If  $S_{cyl} = \ell_{cyl}$  and  $S_k = \ell_0$  are available, for “open to outside” girdle construction, the followings can be taken:

$$\begin{cases} W_{cyl}^{(\ell_{cyl})} = W_k^{(\ell_0)} \cdot \sin \alpha; & \nu_{cyl}^{(\ell_{cyl})} = \nu_k^{(\ell_0)} \\ M_{1cyl}^{(\ell_{cyl})} = M_{1k}^{(\ell_0)}; & T_{12cyl}^{(\ell_{cyl})} = T_{12k}^{(\ell_0)} \end{cases} \quad (8)$$

when  $S_k = 0$  and  $S_k = \ell_1$  are available, then for the “constricting” girdle construction of the packer the following equation can be taken

$$\begin{cases} W_{cyl}^{(0)} = W_k^{(\ell_1)} \cdot \sin \alpha; & \nu_{cyl}^{(0)} = \nu_k^{(\ell_1)} \\ M_{1cyl}^{(0)} = M_k^{(\ell_1)}; & T_{2cyl}^{(0)} = T_{12}^{(\ell_1)} \end{cases} \quad (9)$$

the following equation can be written for  $M_{1cyl}$ ,  $M_{1k}$  bending moments and  $T_{12cyl}$ ,  $T_{12k}$  sliding forces:

$$\begin{aligned} M_{1cyl} &= -\frac{D_{cyl}}{d_{cyl}^2} \left( \frac{d^2 \psi_{cyl}}{dx_{cyl}^2} - \mu_{cyl} n_{cyl}^2 \psi_{cyl} \right) \cos n_{cyl} \theta; \\ T_{12cyl} &= -\frac{E_{cyl} h_{cyl}}{a_{cyl} n_{cyl}^3} \cdot \frac{d^3 \psi_{cyl}}{dx_{cyl}^3} \cdot \sin n_{cyl} \theta; \\ M_{1k} &= -\frac{D_k \operatorname{tg} \alpha}{\ell_1^2} \cdot \left( \frac{d^2 \psi_k}{dx_k^2} + \frac{2}{x_k} - \frac{\mu_k n_k^2}{x_k^2 \cos^2 \alpha} \cdot \psi_k \right) \\ T_{12k} &= -\frac{E_k h_k}{\ell_1 n_k^3} \cdot x_k^2 \operatorname{tg}^2 \alpha \cdot \cos^3 \alpha \left( \frac{d^3 \psi_k}{dx_k^3} + \frac{2}{x_k} \cdot \frac{d^2 \psi_k}{dx_k^2} \right) \sin n_k \theta \end{aligned} \quad (10)$$

The following equation should be carried out during stability losses in the merging part of the cylindrical and conical parts of the packer with girdle construction in accordance with the conditions of the deformation unfolding and internal force factors.

$$n_{cyl} = n_k = n$$

On the base of this attachment condition, (8) and (9) can be written as follows:

$$\begin{cases} \psi_{cyl}^{(r)} = \psi_k^{(s)} \cdot \sin \alpha \operatorname{tg} \alpha; & \frac{d \psi_{cyl}^{(r)}}{dx_{cyl}} = x_s \frac{d \psi_k^{(s)}}{dx_k} \cdot \sin \alpha; \\ \frac{d \psi_{cyl}^{(r)}}{dx_{cyl}^2} = \frac{D_k}{D_{cyl}} \cdot x_s^2 \left[ \frac{d \psi_k^{(s)}}{dx_k^2} + \frac{\mu_k}{x_s} \cdot \frac{d \psi_k^{(s)}}{dx_k} \right] \cdot \sin \alpha \cos \alpha + \left( \mu_{cyl} - \mu_k \cdot \frac{D_k}{D_{cyl}} \right) n^2 \varphi_k^{(s)} \operatorname{tg} \alpha; \\ \frac{d^3 \psi_{cyl}^{(r)}}{dx_{cyl}^3} = \frac{E_k h_k}{E_{cyl} h_{cyl}} \cdot x_s^3 \left( \frac{d^3 \psi_k^{(s)}}{dx_k^3} + \frac{2}{x_s} \cdot \frac{d^2 \psi_k^{(s)}}{dx_k^2} \right) \operatorname{tg}^2 \alpha \cos^4 \alpha \end{cases} \quad (11)$$

where  $\psi_{cyl}^{(r)}$  and  $\psi_k^{(s)}$  are flexion function of their derivatives for merging junction of cylindrical-conical parts of the packer.

The solution of the tasks

Thus, since the solution of differential equation (6) is indefinable in a closed case, let’s solve the stability problem of the packer with girdle construction by numerical evaluation using the finite difference method (Mareev and Stankova, 2012). Let’s divide the length forming cylindrical area of the packer with girdle construction into  $m_{cyl}$  equal parts and the length forming the conical area to  $m_k$  equal parts.

Then, the step for cylindrical area will be  $t_{cyl} = \frac{\ell_{cyl}}{a_{cyl} m_{cyl}}$ ,

however, for conical area it will be  $t_k = \frac{1-x_0}{m_k}$ , where

$$x_0 = \frac{\ell_0}{\ell_1}.$$

Let’s indicate the derivatives in the boundary zones (lapping) and at the junction of the girdle construction of the packer with the first convergence expressions for finite values, and for all intermediate nodes with second convergence expressions (Godunov, 1999)

Then attachment condition (11) will be as follows:

$$\begin{cases} \psi_{cyl}^{(r)} = \psi_k^{(s)} e_1; \\ \psi_{cyl}^{(r+1)} - \psi_{cyl}^{(r-1)} = \psi_k^{(s+1)} e_2 - \psi_k^{(s-1)}; \\ \psi_{cyl}^{(r+1)} - 2\psi_{cyl}^{(r)} + \psi_{cyl}^{(r-1)} = \psi_k^{(s+1)} e_3 - \psi_k^{(s)} \cdot e_1 + \psi_k^{(s-1)} e_5; \\ \psi_{cyl}^{(r+2)} - 2\psi_{cyl}^{(r+1)} + 2\psi_{cyl}^{(r-1)} - \psi_{cyl}^{(r-2)} = \psi_k^{(s+2)} e_6 - \psi_k^{(s+1)} \cdot e_7 - \psi_k^{(s)} e_8 + \psi_k^{(s-1)} \cdot e_9 - \psi_k^{(s-2)} e_{10} \end{cases} \quad (12)$$

where,  $e_1 = \sin \alpha \operatorname{tg} \alpha$ ;  $e_2 = z_s t_{cyl} \sin \alpha$ ;

$$e_3 = \frac{D_k}{D_{cyl}} \cdot z_s \left( z_s + \frac{\mu_k}{2} \right) t_{cyl}^2 \sin \alpha \cdot \cos \alpha$$

$$e_4 = \left[ \frac{D_k}{D_{cyl}} (2z_s^2 + \mu_k n_1^2) - \mu_{cyl} n_1^2 \right] t_{cyl}^2 \sin \alpha \cdot \cos \alpha$$

$$e_5 = \frac{D_k}{D_{cyl}} \cdot z_s \left( z_s - \frac{\mu_k}{2} \right) t_{cyl}^2 \sin \alpha \cdot \cos \alpha; \quad e_6 = \frac{E_k h_k}{E_{cyl} h_{cyl}} \cdot z_s^3 t_{cyl}^3 \sin^2 \alpha \cdot \cos^2 \alpha;$$

$$e_7 = 2 \frac{E_k h_k}{E_{cyl} h_{cyl}} \left( 1 - \frac{2}{z_s} \right) z_s^3 t_{cyl}^3 \sin^2 \alpha \cdot \cos \alpha; \quad e_8 = 2 \frac{E_k h_k}{E_{cyl} h_{cyl}} \cdot z_s^2 t_{cyl}^3 \sin^2 \alpha \cdot \cos^2 \alpha;$$

$$e_9 = 2 \frac{E_k h_k}{E_{cyl} h_{cyl}} \left( 1 + \frac{2}{z_s} \right) z_s^3 t_{cyl}^3 \sin^2 \alpha \cdot \cos^2 \alpha;$$

$$z_s = \frac{x_s}{t_k}; \quad n_1 = \frac{n}{\cos \alpha}$$

Replacing (5) and (6) differential equations with finite-difference equations for the junction of the boundary zones and packer (cylindrical and conical spaces) yields the following result:

$$A \psi_{cyl}^{(i+2)} - 4A \psi_{cyl}^{(i+1)} + [6 + (P_{cyl} t_{cyl})^4] A \psi_{cyl}^{(i)} - 4A \psi_{cyl}^{(i-1)} + A \psi_{cyl}^{(i-2)} = v_k \psi_{cyl}^{(i)}; \quad (13)$$

$$B_j \left( 1 + \frac{3}{z_j} \right) \psi_k^{(j+2)} - 2B_j \left( 2 + \frac{3}{z_j} - \frac{3}{z_j^2} \right) \psi_k^{(j+1)} + B_j \left( 6 + \frac{12}{z_j} + \frac{P_k^4}{x_j^2 z_j^4} \right) \psi_k^{(j)} -$$

$$2B_j \left( 2 - \frac{3}{z_j} \right) \psi_k^{(j-1)} + B_j \left( 1 - \frac{3}{z_j} \right) \psi_k^{(j-2)} = v_k \psi_k^{(j)}$$

Where

$$A = \frac{E_{cyl} h_{cyl} \ell_1}{E_k h_k \alpha_{cyl} t_{cyl}} \left( \frac{\varepsilon_{cyl} \operatorname{ctg} \alpha}{\varepsilon_k P_{cyl} t_{cyl}} \right)^3; \quad B_j = \frac{z_j^3}{P_k^3 t_k}$$

$i$  - is the boundary zone index of the cylindrical part of the packer with girdle construction and obtains 0 and  $m_{cyl}$  values;

$j$  - is the boundary zone index of the conical part of the packer with girdle construction, obtains 0 and  $m_k$  values;

Finite-difference equations for the intermediate zones of the packer with girdle construction will be as follows:

$$-\frac{A}{6} \psi_{cyl}^{(i+3)} + 2A \psi_{cyl}^{(i+2)} - 6A \psi_{cyl}^{(i+1)} + [9 + (P_{cyl} t_{cyl})^4] A \psi_{cyl}^{(i)} - 6A \psi_{cyl}^{(i-1)} + 2A \psi_{cyl}^{(i-2)} - \frac{A}{6} \psi_{cyl}^{(i-3)} = v_k \psi_{cyl}^{(i)}$$

$$B_j \left( \frac{1}{15z_j^2} - \frac{3}{4z_j} - \frac{1}{6} \right) \psi_k^{(j+3)} + 2B_j \left( 1 + \frac{3}{z_j} - \frac{9}{10z_j^2} \right) \psi_k^{(j+2)} + B_j \left( \frac{9}{z_j^2} - \frac{39}{z_j} - 6 \right) \psi_k^{(j+1)} +$$

$$+ B_j \left( 9 - \frac{16}{z_j} + \frac{P_k^4}{x_j^2 z_j^4} \right) \psi_k^{(j)} + B_j \left( \frac{9}{z_j^2} + \frac{39}{z_j} - 6 \right) \psi_k^{(j-1)} + 2B_j \left( 1 - \frac{3}{z_j} - \frac{9}{10z_j^2} \right) \psi_k^{(j-2)} +$$

$$+ B_j \left( \frac{1}{15z_j^2} + \frac{3}{4z_j} - \frac{1}{6} \right) \psi_k^{(j-3)} = v_k \psi_k^{(j)} \quad (14)$$

In (14) equations  $i$  index can obtain whole numerical values from 1 to  $m_{cyl} = 1$  however,  $j$  index can obtain the values from 1 to  $m_k - 1$

The following finite differences equation for the amplitude values of bending moments and sliding forces in boundary zones are written as follows:

$$M_{cyl}^{(i)} = -\frac{D_{cyl}}{2 a_{cyl}^2 t_{cyl}} \left[ \psi_{cyl}^{(i+1)} - (2 + \mu_{cyl} n^2 t_{cyl}^2) \psi_{cyl}^{(i)} + \psi_{cyl}^{(i-1)} \right]; \quad (15)$$

$$T_{12cyl}^{(i)} = -\frac{E_{cyl} t_{cyl}}{2 a_{cyl} n^2 t_{cyl}^3} \left[ \psi_{cyl}^{(i+2)} - 2\psi_{cyl}^{(i+1)} + 2\psi_k^{(i-1)} - \psi_{cyl}^{(i-1)} \right];$$

$$M_k^{(j)} = -\frac{D_k \operatorname{tg} \alpha}{\ell_k^2 t_k} \left[ \left( 1 + \frac{\mu_k}{2z_j} \right) \psi_k^{(j+1)} - \left( 2 + \frac{\mu_k n^2}{z_j} \right) \psi_k^{(j)} + \left( 1 - \frac{\mu_k}{2z_j} \right) \psi_k^{(j-1)} \right];$$

$$T_{12k}^{(j)} = -\frac{E_k h_k}{2 \ell_k t_k} \cdot z_j^2 \cdot \frac{\operatorname{tg}^2 \alpha}{n^2} \left[ \psi_k^{(j+2)} - 2 \left( 1 - \frac{2}{z_j} \right) \psi_k^{(j+1)} - \frac{8}{z_j} \psi_k^{(j)} + 2 \left( 1 + \frac{2}{z_j} \right) \psi_k^{(j-1)} - \psi_k^{(i-2)} \right]$$

For “spreading” case of the packer construction, the following equation is obtained from finite difference equation (12) condition of the junction zone of the cylindrical part of the packer with a girdle construction:

$$\psi_{cyl}^{(m_{cyl})} = e_1 \psi_k^{(0)};$$

$$\psi_{cyl}^{(m_{cyl}+1)} = f_{11} \psi_{cyl}^{(m_{cyl}-2)} + f_{12} \psi_{cyl}^{(m_{cyl}-1)} + f_{13} \psi_k^{(0)} + f_{14} \psi_k^{(1)} + f_{15} \psi_k^{(2)};$$

$$\psi_{cyl}^{(m_{cyl}+2)} = f_{21} \psi_{cyl}^{(m_{cyl}-2)} + f_{22} \psi_{cyl}^{(m_{cyl}-1)} + f_{23} \psi_k^{(0)} + f_{24} \psi_k^{(1)} + f_{24} \psi_k^{(2)};$$

$$\psi_{cyl}^{(-2)} = f_{31} \psi_{cyl}^{(m_{cyl}-2)} + f_{32} \psi_{cyl}^{(m_{cyl}-1)} + f_{33} \psi_k^{(0)} + f_{34} \psi_k^{(1)} + f_{35} \psi_k^{(2)};$$

$$\psi_k^{(-1)} = f_{41} \psi_{cyl}^{(m_{cyl}-2)} + f_{42} \psi_{cyl}^{(m_{cyl}-1)} + f_{43} \psi_k^{(0)} + f_{44} \psi_k^{(1)} + f_{45} \psi_k^{(2)} \quad (16)$$

where,

$$f_{11} = f_{12} = \frac{e_5 - e_2}{e_2 + e_5}; \quad f_{13} = e_2 \frac{2e_1 - e_4}{e_2 + e_5}; \quad f_{14} = e_2 \frac{e_3 + e_5}{e_2 + e_5}; \quad f_{15} = 0;$$

$$f_{21} = -1; \quad f_{22} = \frac{8e_5}{e_2 + e_5}; \quad f_{23} = 4e_2 \frac{2e_1 - e_4}{e_2 + e_5} - e_1 [6 + (P_{cyl} t_{cyl})^4] + v_k \frac{e_1}{A};$$

$$f_{33} = \left\{ \frac{e_4 - 2e_1}{e_2 + e_5} (2e_9 + 2e_2) - e_8 + \left[ 6 + (P_{cyl} t_{cyl})^4 - \frac{v_k}{A} \right] e_1 \right\};$$

$$f_{34} = \frac{1}{e_6} \left( e_9 \frac{e_2 - e_3}{e_2 + e_5} - 2e_2 \frac{e_3 + e_5}{e_2 + e_5} - e_7 \right);$$

$$f_{35} = 1; \quad f_{41} = 0; \quad f_{42} = \frac{2}{e_2 + e_5}; \quad f_{43} = \frac{e_4 - 2e_1}{e_2 + e_5};$$

$$f_{44} = \frac{e_2 - e_3}{e_2 + e_5}; \quad f_{45} = 0$$

In “constricting radial direction” of the packer with a girdle construction:

$$\psi_{cyl}^{(0)} = e_1 \psi_k^{(m_k)};$$

$$\psi_{cyl}^{(-1)} = \delta_{11} \psi_{cyl}^{(2)} + \delta_{12} \psi_{cyl}^{(1)} + \delta_{13} \psi_k^{(m_k)} + \delta_{14} \psi_k^{(m_k-1)} + \delta_{15} \psi_k^{(m_k-2)};$$

$$\psi_{cyl}^{(-2)} = \delta_{21} \psi_{cyl}^{(2)} + \delta_{22} \psi_{cyl}^{(1)} + \delta_{23} \psi_k^{(m_k)} + \delta_{24} \psi_k^{(m_k-1)} + \delta_{25} \psi_k^{(m_k-2)};$$

$$\psi_k^{(m_k+2)} + \delta_{31} \psi_{cyl}^{(2)} + \delta_{32} \psi_{cyl}^{(1)} + \delta_{33} \psi_k^{(m_k)} + \delta_{34} \psi_k^{(m_k-1)} + \delta_{35} \psi_k^{(m_k-2)};$$

$$\psi_k^{(m_k+1)} = \delta_{41} \psi_{cyl}^{(2)} + \delta_{42} \psi_{cyl}^{(1)} + \delta_{43} \psi_k^{(m_k)} + \delta_{44} \psi_k^{(m_k-1)} + \delta_{45} \psi_k^{(m_k-2)} \quad (17)$$

Where  $\delta_{11} = 0; \delta_{12} = \frac{e_3 - e_2}{e_2 + e_3}; \delta_{13} = e_2 \frac{2e_1 - e_4}{e_2 + e_3}; \delta_{14} = e_2 \frac{e_3 + e_5}{e_2 + e_3};$   
 $\delta_{15} = 0;$

$$\delta_{21} = -1; \delta_{22} = \frac{8e_3}{e_2 + e_3}; \quad f_{23} = 4e_2 \frac{2e_1 - e_4}{e_2 + e_3} - e_1 [6 + (P_{cyl} t_{cyl})^4] + v_k \frac{e_1}{A};$$

$$\delta_{24} = 4e_2 \frac{e_3 + e_5}{e_2 + e_3}; \delta_{25} = 0; \delta_{31} = \frac{2}{e_6}; \delta_{32} = \frac{2}{e_6} \cdot \frac{e_7 - 2(e_2 + 2e_3)}{e_2 + e_5};$$

$$\delta_{33} = \frac{1}{e_6} \left\{ \frac{e_4 - 2e_1}{e_2 + e_3} (2e_2 + e_7) + e_8 + \left[ 6 + (P_{cyl} t_{cyl})^4 - \frac{v_k}{A} \right] e_1 \right\};$$

$$\delta_{34} = \frac{1}{e_6} \left( e_7 \frac{e_2 - e_5}{e_2 + e_3} - 2e_2 \frac{e_3 + e_5}{e_2 + e_3} - e_9 \right); \quad \delta_{35} = 1; \quad \delta_{41} = 0$$

$$; \delta_{42} = e_2 \frac{2}{e_2 + e_3}; \delta_{43} = \frac{e_4 - 2e_1}{e_2 - e_3}; \delta_{44} = \frac{e_2 - e_5}{e_2 + e_3}; \delta_{45} = 0$$

The value of the displacement function being outside the boundary of the packer is defined by the equaiton of boundary conditions.

Commonly, they can be presented as follows:

- for cylindrical part of the packer with a girdle construction

$$y^{(-2)} = g_{11}y^{(0)} + g_{12}y^{(1)} + g_{13}y^{(2)};$$

$$y^{(-1)} = g_{21}y^{(0)} + g_{22}y^{(1)} + g_{23}y^{(2)} \quad (18)$$

- for conical part of the packer with a girdle construction

$$y^{(p+1)} = g_{31}y^{(p-2)} + g_{32}y^{(p-1)} + g_{33}y^{(p)};$$

$$(19)$$

$$y^{(p+2)} = g_{41}y^{(p-2)} + g_{42}y^{(p-1)} + g_{43}y^{(p)}$$

where y is flexion function value in cylindrical and conical parts of the packer with a girdle construction.

Finite difference equations have been used to remove the unknowns, belonging to the zones outside the boundary of the packer with a girdle construction, from the equations, so that, it is necessary to delete the row  $m_{cyl} + 1$  and column  $m_{cyl} + 3$  from matrix A. As a result, we obtain matrix "A". Matrix "A" consists of the row  $m_{cyl} + m_k + 1$  and column  $m_{cyl} + m_k + 9$ , i.e., it is a rectangle. Let's create matrix "G" with boundary conditions, taking into account specific boundary conditions in the girdle (junction) part of the packer with a girdle construction. If we multiply matrix "A" by matrix "G" (with boundary conditions), we will have the square matrix "R" consisting of row  $m_{cyl} + m_k + 1$  and  $m_{cyl} + m_k + 1$  column and the coefficients of unknown bending functions in the nodes of the packer with a girdle construction. Thus, stability problem of the packer with a girdle construction in the effect of the same external pressure and given boundary conditions is brought to the solution of the following matrix equation.

$$(R - \nu_k E) / y = 0 \quad (20)$$

Let's consider the boundary conditions where the lower part of the cone is free, the top of the cylinder practically logged to:

Let's form the boundary conditions as follows

- in logged case of the top of the cylinder

$$W(g) = \nu(g) = 0$$

- in free case of the bottom of the cone

$$W(g) = M_1(g) = 0$$

where "g" is a boundary zone index.

- the part of the packer with a girdle construction "spreading in radial direction" has been logged completely:

$$g_{11} = 0; \quad g_{12} = 8; \quad g_{13} = 1; \quad g_{21} = 0; \quad g_{22} = 1; \quad g_{23} = 0;$$

$$g_{31} = 0; \quad g_{32} = 1; \quad g_{33} = 0; \quad g_{41} = \frac{3 - Z_{m_k}}{3 + Z_{m_k}}; \quad g_{42} = 4 \frac{2Z_{m_k}^2 - 3}{Z_{m_k}(3 + Z_{m_k})};$$

$$g_{43} = 0 \quad (21)$$

The bottom edge of cylindrical part has been logged, however, conical edge is free:

$$g_{11} = 0; \quad g_{12} = 8; \quad g_{13} = -1; \quad g_{21} = 0; \quad g_{22} = 1;$$

$$g_{23} = 0; \quad g_{31} = 0; \quad g_{32} = -\frac{2Z_{m_k} - \mu_k}{2Z_{m_k} + \mu_k}; \quad g_{33} = 0; \quad g_{41} = \frac{3 - Z_{m_k}}{3 + Z_{m_k}};$$

$$g_{42} = -4 \frac{6Z_{m_k}^2}{Z_{m_k}(3 + Z_{m_k})}; \quad g_{43} = 0 \quad (22)$$

Both edges of the packer with a girdle construction are free:

$$g_{11} = 0; \quad g_{12} = 0; \quad g_{13} = -1; \quad g_{21} = 0; \quad g_{22} = -1; \quad g_{23} = 0;$$

$$g_{31} = 0; \quad g_{32} = -\frac{2Z_{m_k} - \mu_k}{2Z_{m_k} + \mu_k}; \quad g_{33} = 0; \quad g_{41} = \frac{3 - Z_{m_k}}{3 + Z_{m_k}};$$

$$g_{42} = -4 \frac{6Z_{m_k}^2 - \mu_k(3 - 2Z_{m_k}^2)}{Z_{m_k}(2Z_{m_k} + \mu_k)(3 + Z_{m_k})}; \quad g_{43} = 0 \quad (23)$$

"Constricting" part of the packer with a girdle construction:  
- for the logged case of the edges

$$g_{11} = 0; \quad g_{12} = 8; \quad g_{13} = -1; \quad g_{21} = 0; \quad g_{22} = 1; \quad g_{23} = 0;$$

$$g_{31} = 0; \quad g_{32} = -4 \frac{2Z_0 + \mu_k}{2Z_0 - \mu_k}; \quad g_{33} = 0; \quad g_{41} = \frac{3 + Z_0}{3 + Z_0};$$

$$g_{42} = -\frac{6Z_0^2 + \mu_k(3 - 2Z_0^2)}{Z_0(2Z_0 - \mu_k)(3 - Z_0)}; \quad g_{43} = 0 \quad (24)$$

The edge of cylindrical part is free, however, conical part edge has been logged:

$$g_{11} = 0; \quad g_{12} = 8; \quad g_{13} = -1; \quad g_{21} = 0; \quad g_{22} = 1;$$

$$g_{23} = 0; \quad g_{31} = 0; \quad g_{32} = 1; \quad g_{33} = 0; \quad g_{41} = \frac{3 + Z_0}{3 - Z_0};$$

$$g_{42} = -4 \frac{6Z_0^2 - 3}{Z_0(3 - Z_0)}; \quad g_{43} = 0 \quad (25)$$

Both edges of the packer with a girdle construction have been logged:

$$g_{11} = 0; \quad g_{12} = 0; \quad g_{13} = -1; \quad g_{21} = 0; \quad g_{22} = -1;$$

$$g_{23} = 0; \quad g_{31} = 0; \quad g_{32} = -\frac{2Z_0 + \mu_k}{2Z_0 - \mu_k}; \quad g_{33} = 0;$$

$$g_{41} = \frac{3+Z_0}{3-Z_0}; \quad g_{42} = -4 \frac{6Z_0^2 + \mu_k(3+2Z_0^2)}{Z_0(2Z_0 - \mu_k)(3-Z_0)}; \quad g_{43} = 0 \quad (26)$$

## RESULTS AND DISCUSSION

Stability problem of the packer with a girdle construction can be solved when the smallest special value  $\nu_k$  of “n” wave number, “R” matrix and according to this  $\nu_k$  value, special vector  $y$  is determined. Special vector  $y$  and “n” wave number characterize the form of the packer with a girdle construction during its stability losses, according special vector  $\nu_k$ , the following equation is used:

$$q_{cr} = \nu_k \frac{E_k h_k}{\ell_1} (\varepsilon_k \operatorname{tg} \alpha)^3 \quad (27)$$

As “n” wave number (20) isn’t determined directly by matrix equation solution, values of critic pressure and  $\nu_k$  parameter are determined. By determining “n” values and (20) solving matrix equation, a number of values of  $\nu_k$  parameter is found.

Among the values of  $\nu_k(n)$  the smallest value of  $\nu_k$  providing the following equality is chosen.

$$\nu_k(n-1) > \nu_k(n) < \nu_k(n+1)$$

Iteration method of opposite matrix has been used for solving (20) matrix equation. (Fadeev and Fadeeva, 1963).

Iteration process finishes in case the following inequality is ensured.

$$\left| \frac{\nu_k^{(m+1)} - \nu_k^{(m)}}{\nu_k^{(m+1)}} \right| \leq 10^{-3}$$

For estimating precision of the solution on the following equation:

$$\delta = \sqrt{\frac{\sum_{i=1}^N (S_i - \nu_k \gamma_i)^2}{N-1}}$$

Middle quadratic error of “R” matrix equation is calculated,

where  $S_i = \sum_{j=1}^N r_{ij} \gamma_j$ ,  $N$  is the number of matrix.

Initial calculations showed that, increasing division number of cylindrical and conical parts into lengths of generatrix from 10 to 20 specifies the solution 1%. Based on this  $m_{cyl} = m_k = 10$  has been accepted for the solution of the problem.

For the calculation, the packer the greatest and smallest radius and distance of which remain unchanged between the boundaries, has been selected. As changed value  $\alpha$  angle has been taken. At this time as a rule, the lengths of generatrix of cylindrical and conical parts were changed (Fig. 4 and 5). The calculation of “the logged part” of the packer with a girdle construction is carried according to measurements and characters of the given packer material.

$$a_{cyl} = 50 \text{ mm}; \quad a_k = 76 \text{ mm}; \quad L = 140 \text{ mm}; \quad E_{cyl} = E_k = 6 \text{ MPa}; \\ \mu_{cyl} = \mu_k = 0,48.$$

In “constricting” case of the packer with a girdle construction:

$$a_{cyl} = 50 \text{ mm}; \quad a_k = 28 \text{ mm}; \quad L = 140 \text{ mm}; \quad E_{cyl} = E_k = 6 \text{ MPa}; \\ \mu_{cyl} = \mu_k = 0,48$$

Table 1 presents the results of the comparison of various values of the packer with a girdle construction with experiment values.

Table 1

$\alpha^\circ$	$\ell_c, \text{mm}$	$\ell_k, \text{mm}$	$q_{cal}, \text{MPa}$	$q_{cr}^{\text{exp}}, \text{MPa}$	$\frac{q_{cr}^{\text{exp}}}{q_{cal}}$	The type of the packer during placement process
30	342	116	2,53	1,7	0,67	Constricting packer
60	227	200	3,75	2,5	0,68	
60	227	200	4,75	3,3	0,69	Expanding packer

As it is seen from the table, contact critic pressure of the packer is less  $\approx 0,7 q_{cal}$  than  $q_{cr}^{\text{exp}}$  calculation pressure.

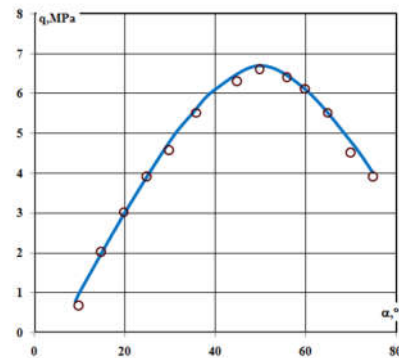


Fig. 4. The dependence of  $q_{cr}$  pressure on  $\alpha$  angle for various conditions and wall thickness of “expanding” elements of the packer with a girdle construction.

$$W_{cyl} = W_k = 0; \quad \nu_{cyl} = \nu_k = 0; \quad h_{cyl} = 95 \text{ mm}; \quad h_k = 65 \text{ mm}.$$

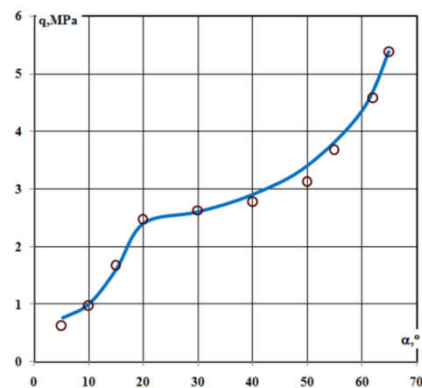


Fig. 5. The dependence of  $q_{cr}$  pressure on  $\alpha$  angle for various conditions and wall thickness of “constricting” elements of the packer with a girdle construction.  $W_{cyl} = W_k = 0$ ;

$$\nu_{cyl} = \nu_k = 0; \quad h_{cyl} = 95 \text{ mm}; \quad h_k = 65 \text{ mm}$$

## Conclusion

Initial calculations show that, increasing division number of cylindrical and conical parts into lengths of generatrix from 10 to 20 specifies the solution with 1% error. By theoretical experiments it has been grounded that, the value obtained from calculations should be multiplied  $t_0 \approx 0,7$  coefficient for determining  $q_{cr}^{exp}$  critic pressure of the packer with a girdle construction (cylindrical-conical surface), which will take into preparation technology of packer construction.

## REFERENCES

- Babanly M.B., Mamedov G.A., Mammadov V.T., Aslanov J.N. Features of the calculation while nonstationary dynamic loadings for the downhole packer sealing / Science Innovators. International Conference on European Science and Technology. Materials of the XII International Research and Practice Conference. Munich, Germany, Yuly 1st-2nd, 2016, 42-54.
- Fadeev D.K. and Fadeeva V.N. 1963. Computational methods of linear algebra: M., Fizmatgiz, p.379.
- Godunov S.K. 1999. The numerical solution of boundary value problems for systems of linear ordinary differential equations. - "Success of Mathematical Science", vol. XVI. N. 3, p. 171-174.
- Mammadov V.T., Aslanov J.N., Gajieva L.S. and Bagirova M.N. 2016. Impact of thermoelastik deformation on work of rotating preventers' sealing / Science and Education. Materials of the XII international research and practice conference. Juli 1<sup>st</sup>-2<sup>nd</sup>, Munich, Germany, p.p.95-102.
- Mammadov V.T. and Gurbanov S.R. 2015. The impact of the relatively low compression rubber to functions sealing elements of wellbore packers / ICSCCW-2015, Eighth International Conference on Soft Computing, Computing with Words and Perceptions in System Analysis, Decision and Control, Dedicated to Professor LotfiZadeh, Antalya, Turkey September 3-4, p-p. 397-399.
- Mareev V.V. and Stankova E.N. 2012. Fundamentals of finite difference methods. St. Petersburg: Publishing House of St. Petersburg University, 64 p.

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