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RESEARCH ARTICLE

PROPERTIES OF ELECTRON-ION ACOUSTIC SOLITARY WAVES IN A MAGNETIZED DEGENERATE QUANTUM PLASMA

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ABSTRACT

This research focuses on the " Quality of service The nonlinear propagation of electron-ion (El) acoustic solitary waves in a degenerate quantum plasma (containing relativistic magnetized quantum electrons and light ions in presence of stationary heavy ions) have been theoretically investigated. The Korteweg-de Vries (K-dV) and modified K-dV (mK-dV) equations are derived by adopting the reductive perturbation method. Their stationary solutions are derived and analyzed analytically as well as numerically to study some new basic features of the El acoustic solitary structures that are commonly found to exist in degenerate quantum plasma. It is found that the basic properties (viz. amplitude, width, and phase speed, etc.) of the El acoustic waves are significantly modified by the effects of relativistic ally degenerate electrons and light ions, quantum pressure, number densities of plasma particles, and external magnetic field, etc. The results of this theoretical investigation may be useful for understanding the formation and features of the solitary structures in astrophysical compact objects like white dwarfs, neutron stars, etc.

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INTRODUCTION

The physics of dense quantum plasmas has received an immense interest not only because of their omnipresence in many astrophysical compact objects (like white dwarfs, neutron stars, active galactic nuclei, etc. (Woolsey *et al.*, 2004; Shapiro, 1983; Garcia-Berro, 2010; Mamun, 2010)), but also for their available application in the laboratory i.e., intense laser-solid matter interaction experiment (Berezhiani, 1992; Murkhmd, 2006). The degenerate compact objects e.g. white dwarfs, neutron stars, etc. are usually consist of extremely dense iron/oxygen/carbon and helium nuclei (Shiikla, 2011; Massey, 1976) are ideal examples of degenerate plasma systems where the quantum mechanical effects (i.e., when the de Broglie thermal wavelength of the charged particles is equal or larger than the average inter-particle distance) play an important role in the plasma dynamics (Abdikian, 2016). The plasma in the interior of white dwarfs and in the crust of neutron stars are extremely dense and highly degenerate (Mamun, 2010; Chabrier, 2006; Lai, 2001; Harding, 2006) (plasma particles number densities can be of the order of 10^{30}cm$\sim^{3}$, or even more and the magnetic field strength can also be very large, i.e., B » $W^{g}G$). For such plasmas, the Fermi temperature Tp is high and the quantum-mechanical effects associated with the quantum statistical pressure and the quantum force involving the Rohm potential (causing the plasma particles tunneling) are expected to play an important role (Manfredi, 2005). Therefore, by adding the quantum statistical pressure term (the Fermi-Dirac distribution) and the quantum diffraction term (the Bohm potential) to the fluid model generalized QHD model (Manfredi, 2001; Manfredi, 2005) is obtained. A number of works (Rouhani, 2014; Hossain, 2011; Haas, 2003; Bhowmik, 2007; Saeed-ur-Rehman, 2012; Hossen, 2015) have been done by considering the quantum effects of plasma particles in different plasma medium. The behaviour of high densities and low temperatures quantum plasmas was first studied by Pines (Pines, 1961). Haas *et al.* (Haas *et al.*, 2003) found in their investigation that the Bohm potentials associated with the plasma particles significantly modify the basic features of the nonlinear IA waves. Bhowmik *et al.* (2007) investigated the effects of quantum diffraction parameter H, and the equilibrium density ratio of the plasma species in modifying the electron-acoustic (EA) waves in a quantum ET plasma.

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On the other hand, Ali *et al.* (2007) analyzed the IA waves in an EPI plasma, and found that the nonlinear properties of the IA waves are significantly affected by the inclusion of the quantum terms in the momentum equations of electrons and positrons. But none of them considered the effects of external magnetic field and the presence of the heavy ions. It is well known that (Miller, 1987; Plastino, 1993; Gervino, 2012) the presence of external magnetic field (which causes the obliqueness of the wave propagation) plays a vital role in modifying the basic features of the linear and nonlinear waves in space and astrophysical plasmas (Mahmood, 2008; Sultana, 2010; El-Tantawy, 2012; Shahmansouri, 2013; Alinejad, 2013; Ashraf *et al.*, 2014), and that the most of astrophysical degenerate quantum plasma systems like white dwarfs and neutron stars usually contain degenerate electrons and light ions along with heavy ions (Koester, 1990). This means that the effects of heavy ions and magnetic field must be considered, specially for the study of the nonlinear phenomena in the degenerate astrophysical objects (Woolsey, 2004; Shapiro, 1983; Torres *et al*., 2010; Mamun, 2010). Obliquely propagating electron-acoustic (EA) solitary waves (SWs) in a two electron population quantum rnag-netoplasma was theoretically investigated by Masood and Mushtaq (2008). They found that propagation characteristics of the EA SWs are significantly affected by the presence of quantum corrections and the ratio of hot to cold electron concentration. Recently, Hossen and Mamun (Hossen, 2011c) have theoretically investigated the nonlinear positron-acoustic (PA) waves propagating in the fully relativistic electron-positron-ion plasma and found that the effects of relativistic degeneracy of electrons and positrons, static heavy ions, plasma particles velocity, and enthalpy, etc. have significantly modified the ba-sic properties of the PA SWs. Using a fully relativistic set of two fluids plasma equations, Lee and Choi (Lee, 2007), Tribeche and Boukhalfa (2011), Saberian *et al,* (2011), and Akbari-Moghanjoughi (2011) have studied the characteristics of the nonlinear IA waves in different fully relativistic plasmas. Therefore, in our present work, we have examined the basic properties of the El acoustic waves propagating in a degenerate quantum plasma composed of relativistically magnetized quantum electrons and light ions in the presence of stationary heavy ions. We have considered the quantum mechanical (such as tunneling associated with the Bohm potential) effects for both electrons and light ions which abundantly occurs in different astrophysical situations (viz. white dwarfs, neutron stars, active galactic nuclei, etc. (Woolsey, 2004; Shapiro, 1983; Garcia-Berro, 2010; Mamun, 2010), and laboratory plasmas like intense laser-solid matter interaction experiment (5. 6). The manuscript is organized as in Sec. II, the basic equations governing our plasma model are presented; in Sec. Ill and IV, the K-dV and mK-dV equations along with their solutions are derived; in Sec. V, a brief discussion is given.

MODEL EQUATIONS

We consider a collision less plasma system with an ambient magnetic field directed along the *z* axis, i.e., (Bo = $\hat{z}B_o$). where $\hat{z}$ is a unit vector in the *z* direction. The obliquely propagation of the nonlinear acoustic SWs through our considered plasma system is assumed such that the wave vector lies in the *x-z* plane. At equilibrium, the quasi-neutrality condition can be expressed as $Z_h\eta_{ho} + \eta_{io} - \eta_{eo} = 0$, where η_{ho}, η_{io}, and η_{eo} are the equilibrium number densities of immobile heavy ions, light ions, and electrons, respectively, and Z_h is the immobile heavy ions charge state. The unnormalized dynamic equations for the considered magnetized quantum El plasmas are as follows:

$$\frac{\partial N_s}{\partial T} + \nabla^*.(N_s U_s) = 0, \tag{1}$$

$$\frac{\partial U_s}{\partial T} + (U_s.\nabla^*)\ U_s = \frac{q_s}{m_e}\left[-\nabla^* + \frac{1}{c}U_s \times \vec{B}_0\right] - \frac{\nabla^* P_s^*}{m_e N_s} + \frac{\hbar^2}{2m_e}\nabla^*\left(\frac{1}{\sqrt{n_s}}\nabla^{*2}\sqrt{N_s}\right), \tag{2}$$

$$\nabla^{*2}\Phi = -4\pi e(N_i - N_e + Z_h N_h), \tag{3}$$

is P_s^* and the full expression for it is (Mon, 1935)

$$P_s^* = \frac{\pi m_e^4 c^5}{3h^3}\left[R_s(R_s^2+1)^{1/2}(2R_s^2-3) + 3\sinh^{-1}(R_s)\right] \tag{4}$$

in which the relativity parameter $R_s = pF_s / m_e c$ and $pF_s = (3h^3 n_s / 8\pi)^{1/3}$ is the Fermi relativistic momentum of s'th species. Taking $N_s = N_{s0} + N_{s1}, N_{s1}$ is the perturbation of the s'th species with $|N_{s1}|$ << *Nso* and the Taylor expansion up to second order, Eq. (4) turns to

$$P_s^* \approx P_{s0} + \left[\frac{\partial P_s^*}{\partial N_s}\right]_{Ns=Ns0} N_{s1} + \frac{1}{2}\left[\frac{\partial^2 P_s^*}{\partial N_s^2}\right]_{Ns=Ns0} N_{s1}^3$$

$$= P_{s0} + \frac{2 \in F_s}{3\gamma_{s0}} N_{s1} + \frac{\left(3R_0^2 + 4\right)\in F_s}{9\gamma_{s0}^3 N_{s0}} N^2{}_{s1}' \tag{5}$$

where $\gamma_{s0} = \sqrt{1+R_{s0'}^2}\ R_{s0} = P_{s0}/m_e c,\ P_{s0} = (3h^3 n_{s0}/8\pi)^{1/3}$, and $\in F_s = \hbar^2(3\pi^2 n_s)^{2/3}/2m_e$. In order to normalize the basic Eqs. (l)-(3), we use the following normalized parameters

$$T \to t\omega_{pe,}\nabla^* \to \frac{\omega_{pe}}{uF_e}\nabla, \Phi \to \frac{e\phi}{2\in F_e}, U_s \to \frac{u_s}{uF_e}$$

$$N_s \to \frac{n_s}{n_{s0}}, \omega_{ps} \to \sqrt{\frac{4\pi e^2 n_{s0}}{m_e}}, H_s \to \frac{\hbar\omega_{pe}}{2\in F_e}$$

$$uF_s \to \sqrt{\frac{2\in F_s}{m_e}}, \omega c \to \frac{eB_0}{m_e c}, \Omega c \to \frac{\omega_c}{\omega_{pe}}$$

$$P_s^* \to \frac{P_s}{2\in_{Fe} n_{s0}}, \eta \to \frac{\in Fe}{\in Fi}, \sigma \to \frac{n_{h0}}{n_{e0}}. \tag{6}$$

The normalized equations for the ion quantum fluid can be written as

$$\frac{\partial n_i}{\partial t} + \nabla.\left(n_i u_i\right) = 0, \tag{7}$$

$$\frac{\partial u_i}{\partial t} + \left(u_i.\nabla\right)u_i = -\nabla\phi + \Omega_c(u_i \times \hat{z}) - \frac{\nabla P_i}{n_i}$$

$$+ \frac{H^2}{2(1-\sigma Z_h)^{2/3}}\nabla\left(\frac{1}{\sqrt{n_i}}\nabla^2\sqrt{n_i}\right), \tag{8}$$

and the normalized equations for the electron quantum fluid can be written as

$$\frac{\partial n_e}{\partial t} + \nabla.\left(n_e u_e\right) = 0, \tag{9}$$

$$\frac{\partial u_e}{\partial t} + \left(u_e.\nabla\right)u_e = \nabla\phi - \Omega_c(u_e \times \hat{z}) - \frac{\eta\nabla P_e}{n_e}$$

$$+ \frac{H^2}{2}\nabla\left(\frac{1}{\sqrt{n_e}}\nabla^2\sqrt{n_p}\right), \tag{10}$$

and the normalized form of the Poisson equation is

$$\nabla^2\phi = n_e - n_i(1-\sigma Z_h) - Z_h\sigma. \tag{11}$$

K-dV EQUATION

To study the nonlinear propagation of the electrostatic perturbation modes in the relativistically magne-tized quantum plasmas (under consideration), we will derive the K-dV equation employing the reductive perturbation method (47). So, we first introduce the stretched coordinates (Washimi, 1966) as

$$\xi = \in^{1/2} (l_z + l_y + l_z - V_p t), \tag{12}$$

$$\tau = \in^{3/2} t, \tag{13}$$

where $V_p [= \omega / \kappa(\omega$ is angular frequency and *k* is the wave vector)) is the wave phase speed, $\in (0 < \in < 1)$ is a measure of the solitary wave amplitude i.e., a measure of the weakness of the dispersion or of the nonlinear effect, $l_x, l_y,$ and l_z are the direction cosines of the wave vector **k** along the *x, y,* and *z* axes, respectively, so that $l_x^{\ 2} + l_x^{\ 2} = 1.$ We then expand the perturbed quantities $n_i, n_e, u_{sx,y,} u_{sz'}$ and ϕ about their equilibrium values in power series of $\in$ as

$$n_i = 1 + \in n_i^{(1)} + \in^2 n_i^{(2)} + \in^3 n_4^{(3)} + ..., \tag{14}$$

$$n_e = 1 + \in n_e^{(i)} + \in^2 n_e^{(2)} + \in^3 n_e^{(3)} + ..., \tag{15}$$

$$u_{sx,y} = \in^{3/2} u_{sx.y}^{(1)} + \in^2 u_{sx.y}^{(2)} + \in^{5/2} u_{sx.y}^{(3)} + ..., \tag{16}$$

$$u_{sz} = \in u_{sz}^{(1)} + \in^2 u_{sz}^{(2)} + \in^3 u_{sz}^{(3)} + ..., \tag{17}$$

$$\phi = \in \phi^{(1)} + \in^2 \phi^{(2)} + \in^3 \phi^{(3)} + ... \tag{18}$$

Now, substituting Eqs. (12)-(18) in Eqs. (7)-(ll), and equating the coefficients for the lowest order of $\in$, we obtain the first order continuity equations, the *z* -component of the momentum equations, and Poisson's equation, which after simplification, we can write as

$$n_i^{(1)} = \frac{3\gamma_{i0} l_z^2}{3\gamma_{i0} V_p^2 - l_z^2} \phi^{(1)}, \tag{19}$$

$$u_{iz}^{(1)} = \frac{3\gamma_{i0} l_z V_p}{3\gamma_{i0} V_p^2 - l_z^2} \phi^{(1)}, \tag{20}$$

$$n_e^{(1)} = \frac{3\gamma_{e0} l_z^2}{\eta l_z^2 - 3_{\gamma e0}^2 V_p^2} \phi^{(1)}, \tag{21}$$

$$u_{ez}^{(1)} = \frac{3\gamma_{e0} l_z^2 V_p}{\eta l_z^2 - 3\gamma_{e0} V_p^2} \phi^{(1)}, \tag{22}$$

$$v_p = \sqrt{\frac{\gamma_{e0} l_z^2 + \eta \gamma_{i0} l_z^2 (l - Z_h \sigma)}{3\gamma_{e0} \gamma_{i0} (2 - Z_h \sigma)}}, \tag{23}$$

We define $l_z = cos\,\delta$,. where δ is the angle between the directions of the wave propagation vector k and the external magnetic field B_0- The Eq. (23) represents the dispersion relation for the acoustic type electrostatic waves in the degenerate quantum plasma under consideration. We can write the first order *x*— and *y*—components of the momentum equations as

$$u_{iy}^{(1)} = \frac{3 l_x \gamma_{i0} V_p^2}{\Omega_e (3\gamma_{i0} V_p^2 - l_z^2)} \frac{\partial \phi^{(1)}}{\partial \xi}, \tag{24}$$

$$u_{ix}^{(1)} = -\frac{3l_y\gamma_{i0}V_p^2}{\Omega_c(3\gamma_{i0}V_p^2 - l_z^2)}\frac{\partial\phi^{(1)}}{\partial\xi}, \tag{25}$$

$$u_{ey}^{(1)} = -\frac{3l_y\gamma_{e0}V_p^2}{\Omega_c(\eta l_z^2 - 3\gamma_{e0}V_p^2)}\frac{\partial\phi^{(1)}}{\partial\xi}, \tag{26}$$

$$u_{ez}^{(1)} = \frac{3l_y\gamma_{e0}V_p^2}{\Omega_c(\eta l_z^2 - 3\gamma_{e0}V_p^2)}\frac{\partial\phi^{(1)}}{\partial\xi}, \tag{27}$$

Eqs. (24)-(27), represent the *x*— and *y*—components of the ion-electron electric field drift, respectively. Again, substituting Eqs. (12)-(18) in Eq. (8) and Eq. (10) and using Eqs. (19)-(27), we obtain the next higher order *x*-and *y*—components of the momentum equations as

$$u_{iy}^{(2)} = \frac{3V_p^3\gamma_{i0}l_y}{\Omega_c^2(3\gamma_{i0}V_p^2 - l_z^2)}\frac{\partial^2\phi^{(1)}}{\partial\xi^2}, \tag{28}$$

$$u_{iy}^{(2)} = \frac{3V_p^3\gamma_{i0}l_x}{\Omega_c^2(3\gamma_{i0}V_p^2 - l_z^2)}\frac{\partial^2\phi^{(1)}}{\partial\xi^2}, \tag{29}$$

$$u_{ey}^{(2)} = -\frac{3V_p^3\gamma_{e0}l_y}{\Omega_c^2(\eta l_z^2 - 3\gamma_{e0}V_p^2)}\frac{\partial^2\phi^{(1)}}{\partial\xi^2}, \tag{30}$$

$$u_{ex}^{(2)} = -\frac{3V_p^3\gamma_{e0}l_x}{\Omega_c^2(\eta l_z^2 - 3\gamma_{e0}V_p^2)}\frac{\partial^2\phi^{(1)}}{\partial\xi^2}, \tag{31}$$

Eqs. (28)-(31), denote the *y*— and x—components of the ion-electron polarization drift, respectively. Further, substituting Eqs. (12)-(18) in Eqs. (7)-(ll), we obtain the next higher order continuity equations, the *z*- component of the momentum equations, and Poisson's equation, which can be given as

$$\frac{\partial n_i^{(1)}}{\partial\tau} - V_p\frac{\partial n_i^{(2)}}{\partial\xi} + l_z\frac{\partial u_{ix}^{(2)}}{\partial\xi} + 1_y\frac{\partial u_{iy}^{(2)}}{\partial\xi} + l_z\frac{\partial u_{iz}^{(2)}}{\partial\xi} + l_z\frac{\partial}{\partial\xi}\left(n_i^{(l)}u_{iz}^{(l)}\right) = 0, \tag{32}$$

$$\frac{\partial u_{iz}^{(1)}}{\partial\tau} - V_p\frac{\partial u_{iz}^{(2)}}{\partial\xi} + l_z u_{iz}^{(1)}\frac{\partial u_{iz}^{(1)}}{\partial\xi} + 1_z\frac{\partial\phi^{(2)}}{\partial\xi} + \frac{l_z}{3\gamma_{i0}}\frac{\partial n_i^{(2)}}{\partial\xi} + \frac{l_z\left(3R_{i0}^2+4\right)}{9\gamma_{i0}^3}n_i^{(l)}\frac{\partial n_i^{(l)}}{\partial\xi} - \frac{l_z}{3\gamma_{i0}}n_i^{(l)}\frac{\partial n_i^{(l)}}{\partial\xi} - \frac{H^2l_z}{4}\frac{\partial^3 n_i^{(1)}}{\partial\xi^3} = 0, \tag{33}$$

$$\frac{\partial n_e^{(1)}}{\partial\tau} - V_p\frac{\partial n_e^{(2)}}{\partial\xi} + l_x\frac{\partial u_{ex}^{(2)}}{\partial\xi} + l_y\frac{\partial u_{ey}^{(2)}}{\partial\xi} + l_z\frac{\partial u_{ez}^{(2)}}{\partial\xi} + l_z\frac{\partial}{\partial\xi}\left(n_e^{(1)}u_{ez}^{(1)}\right) = 0, \tag{34}$$

$$\frac{\partial u_{ez}^{(1)}}{\partial \tau} - V_p \frac{\partial u_{ez}^{(2)}}{\partial \xi} + l_x u_{ez}^{(1)} \frac{\partial u_{ez}^{(1)}}{\partial \xi} - l_z \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{\eta l_z}{3\gamma_{e0}} \frac{\partial n_e^{(2)}}{\partial \xi}$$
$$+ \frac{\eta l_z (3R^2{}_{e0} + 4)}{9\gamma_{e0}^3} n_e^{(1)} \frac{\partial n_e^{(1)}}{\partial \xi} - \frac{\eta l_z}{3\gamma_{e0}^3} n_e^{(1)} \frac{\partial n_e^{(1)}}{\partial \xi}$$
$$- \frac{H^2 l_z}{4(1 - Z_h \sigma)^{2/3}} \frac{\partial^3 n_e^{(1)}}{\partial \xi^3} = 0, \tag{35}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = n_e^{(2)} - (1 - Z_h \sigma) n_i^{(2)} \tag{36}$$

Now, simplifying Eqs. (32)-(36) by using Eqs. (19)-(31), and combining each other, we finally obtain our desired equation in the form

$$\frac{\partial^2 \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \tag{37}$$

Eq. (37) is the well known K-dV equation describing the dynamics of the SWs propagating in the degenerate quantum plasma. The nonlinear coefficient, *A* and the dispersion coefficient, *B* are given as

$$A = \frac{L_1 + L_2 + L_3 + L_4}{6\gamma_{e0}^2 V_p a^3 b + 6\gamma_{p0}^2 V_p a b^3 (1 - Z_h \sigma)}, \tag{38}$$

$$B = \frac{K_1 - K_2 - K_3 + K_4 - K_5}{72\gamma_{e0}^2 V_p l_z^2 a^2 \Omega_c^2 + 72\gamma_{i0}^2 V_p l_z^2 b^2 \Omega_c^2 (1 - Z_h \sigma)}, \tag{39}$$

Where

$$a = (3\gamma_{i0} V_p^2 - l_z^2), \quad b = (\eta l_z^2 - 3\gamma_{e0} V_p^2),$$
$$L_1 = 27\gamma_{e0}^3 V_p^2 a^3 l_z^2), L_2 = \eta a^3 l_z^4 (3R_{e0}^2 - 3\gamma_{e0}^2 + 4),$$
$$L_3 = 27\gamma_{i0}^3 V_p^2 b^3 l_z^2 (1 - Z_h \sigma),$$
$$L_4 = b^3 l_z^4 (3R_{i0}^2 - 3\gamma_{i0}^2 + 4)(1 - Z_h \sigma),$$
$$K_1 = 4a^2 b^2 \Omega_c^2, \quad K_2 = 36\gamma_{e0}^2 V_p^4 a^2 (1 - l_z^2),$$
$$K_3 = 9\gamma_{e0}^2 l_z^4 H^2 a^2 \Omega_c^2,$$
$$K_4 = 36\gamma_{i0}^2 V_p^4 b^2 \ (1 - l_z^2), \ (1 - Z_h \sigma),$$
$$K_5 = 9\gamma_{i0}^2 l_z^4 H^2 b^2 \Omega_c^2), \ (1 - Z_h \sigma)^{1/3}, \tag{40}$$

Now, to investigate the properties of the El acoustic SWs, we are interested in the SWs solution of the K-dV equation. To do so, we introduce another stretched coordinates. $\zeta = \xi - u_{0\tau}$. After the coordinate transformation, the steady state $(\partial/\partial\tau = 0)$ solution of the SWs can be written as

$$\phi^{(1)} = \phi_m \operatorname{sec}h^2\left(\frac{\varsigma}{\delta_1}\right), \tag{41}$$

where the amplitude $\phi_m = 3U_0 / A$ and the width, $\nabla_1 = \sqrt{4B/U_0}$. It is seen that the quantum parameter *H* is present in δ_1, whereas ϕ_m is totally independent of these parameter.

mK-DV EQUATION

The K-dV equation (Eq. (37)) is the result of the second order calculation in the smallness parameter $\in$, where the quadratic nature has been revealed by the nonlinear term $A\phi^{(1)}\partial\phi^{(1)}/\partial\xi$,.

For plasmas with more than two species as like our system, however, there can arise cases where A vanishes at a particular value of a certain parameter σ, and Eq. (37) fails to describe nonlinear evolution of perturbation. So, higher order calculation is needed at this critical value $\sigma=\sigma_c$. For this reason, to derive the mK-dV equation, we apply the following stretched coordinates (Washimi, 1966)

$$\xi=\in^{1/2}(l_x+l_y+l_z-V_p t), \tag{42}$$

$$\tau=\in^{3/2} t, \tag{43}$$

We then expand the variables $n_i, n_e, u_{sx,y}, u_{sz}$, and ϕ, in power series of $\in$ as

$$n_i=1+\in^{1/2} n_i^{(1)}+\in n_i^{(2)}+\in^{3/2} n_i^{(3)}+\ldots, \tag{44}$$

$$n_e=1+\in^{1/2} n_e^{(1)}+\in n_e^{(2)}+\in^{3/2} n_e^{(3)}+\ldots, \tag{45}$$

$$u_{ix.y}=\in u_{sz.y}^{(1)}+\in^{3/2} u_{sx.y}^{(2)}+\in^{2} u_{sx.y}^{(3)}+\ldots, \tag{46}$$

$$u_{sz}=\in^{1/2} u_{sz}^{(1)}+\in u_{sz}^{(2)}+\in^{3/2} u_{sz}^{(3)}+\ldots, \tag{47}$$

$$\phi=\in^{1/2}\Phi^{(1)}+\in\Phi^{(2)}+\in^{3/2}\Phi^{(3)}+\ldots, \tag{48}$$

By using Eq. (42)-(48) in Eqs. (7)-(ll), we found the values of $n_i^{(1)}, n_e^{(1)}, u_{iz}^{(1)}, u_{ez}^{(1)}, u_{ix,y}^{(1)}, u_{ex.y}^{(1)}$ and V_p as like as that of the K-dV equation. To the next higher order of $\in$, we obtain a set of equations, which, after using the values *of* $n_i^{(1)}, n_e^{(1)}, u_{iz}^{(1)}, u_{ez}^{(1)}, u_{ix,y}^{(1)}, u_{ex.y}^{(1)}$ and V_p can be simplified as

$$n_i^{(2)}=\frac{81V_p^2 l_z^4\gamma_{i0}^3+3l_z^6(3R_{i0}^2-3\gamma_{i0}^3+4)}{2(3\gamma_{i0}V_p^2-l_z^2)^3}\left[\Phi^{(1)}\right]^2$$

$$+\frac{3l_z^2\gamma_{i0}}{(3\gamma_{i0}V_p^2-l_z^2)}\Phi^{(2)} \tag{49}$$

$$n_e^{(2)}=-\frac{81V_p^2 l_z^4\gamma_{e0}^3+3\eta l_z^6(3R_{e0}^2-3\gamma_{e0}^2+4)}{2(\eta l_z^2-3\gamma_{e0}V_p^2)^3}\left[\Phi^{(1)}\right]^2$$

$$+\frac{3l_z^2\gamma_{i0}}{(\eta l_z^2-3\gamma_{e0}V_p^2)}\Phi^{(2)} \tag{50}$$

$$u_{iy}^{(2)}=\frac{l_x}{3\Omega_c\gamma_{i0}}\frac{\partial n_i^{(2)}}{\partial\xi}+\frac{l_x l_z^4(3R_{i0}^2-3\gamma_{i0}^2+4)}{\gamma_{i0}\Omega_c(3\gamma_{i0}V_p^2-l_z^2)^2}\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}$$

$$+\frac{3V_p^3 l_y\gamma_{i0}}{\Omega_c^2(3\gamma_{i0}V_p^2-l_z^2)^2}\frac{\partial^2\Phi^{(1)}}{\partial\xi^2}, \tag{51}$$

$$u_{ix}^{(2)} = \frac{l_y}{3\Omega_c\gamma_{i0}}\frac{\partial n_i^{(2)}}{\partial \xi} - \frac{1_y l_z^4(3R_{i0}^2 - 3\gamma_{i0}^2 + 4)}{\gamma_{i0}\Omega_c(3\gamma_{i0}V_p^2 - l_z^2)^2}\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial \xi}$$

$$+\frac{3V_p^3 l_y\gamma_{i0}}{\Omega_c^2(3\gamma_{i0}V_p^2 - l_z^2)^2}\frac{\Phi^{(1)}}{\partial \xi^2}, \tag{52}$$

$$u_{ex}^{(2)} = -\frac{\eta l_y}{3\Omega_c\gamma_{i0}}\frac{\partial n_i^{(2)}}{\partial \xi} - \frac{\eta 1_y l_z^4(3R_{i0}^2 - 3\gamma_{i0}^2 + 4)}{\gamma_{i0}\Omega_c(\eta l_z^2 - 3\gamma l_{i0}V_p^2)^2}\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial \xi}$$
$$-\frac{3V_p^3 l_y\gamma_{e0}}{\Omega_c^2(\eta l_z^2 - 3\gamma_{e0}V_p^2)}\frac{\partial^2\Phi^{(1)}}{\partial \xi^2}, \tag{53}$$

$$u_{ex}^{(2)} = -\frac{\eta l_y}{3\Omega_c\gamma_{e0}}\frac{\partial n_e^{(2)}}{\partial \xi} - \frac{\eta 1_x l_z^4(3R_{e0}^2 - 3\gamma_{e0}^2 + 4)}{\gamma_{e0}\Omega_c(\eta l_z^2 - 3\gamma_{e0}V_p^2)^2}\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial \xi}$$

$$-\frac{3V_p^3 l_x\gamma_{e0}}{\Omega_c^2(\eta l_z^2 - 3\gamma_{e0}V_p^2)}\frac{\partial^2\Phi^{(1)}}{\partial \xi^2}, \tag{54}$$

$$u_{iz}^{(2)} = \frac{9V_p l_z^3\gamma_{i0}^3 + l_z^5(3R_{i0}^2 - 3\gamma_{i0}^2 + 4)}{\gamma_{i0}(3\gamma_{i0}V_p^2 - l_z^2)^2}\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial \xi}$$

$$+\frac{l_z}{V_p}\Phi^{(2)}\frac{l_z}{3V_p\gamma_{i0}}n_i^{(2)}, \tag{55}$$

$$u_{ez}^{(2)} = \frac{9V_p l_z^3\gamma_{e0}^3 + \eta l_z^5(3R_{e0}^2 - 3\gamma_{e0}^2 + 4)}{\gamma_{e0}(\eta l_z^2 - 3\gamma_{e0}V_p^2)^2}\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial \xi}$$

$$-\frac{l_z}{V_p}\Phi^{(2)} + \frac{\eta l_z}{3V_p\gamma_{e0}}n_e^{(2)}, \tag{56}$$

$$\rho^{(2)} = -\frac{1}{2}A_1\left[\Phi^{(1)}\right]^2 = 0, \tag{57}$$

where

$$A_1 = \left[\frac{T_1 + T_2}{\left(\eta l_z^2 - 3\gamma_{e0}V_p^2\right)^3} + \frac{(1 - Z_h\sigma)(P_1 + P_2)}{\left(3\gamma_{i0}V_p^2 - l_z^2\right)^3}\right], \tag{58}$$

Where

$$T_1 = 81V_p^2 l_z^4\gamma_{e0}^3{}' \qquad T_2 = 3\eta l_z^6(3R_{e0}^2 - 3\gamma_{e0}^2 + 4)'$$
$$P_1 = 81V_p^2 l_z^4\gamma_{i0}^3{}' \qquad P_2 = 3\eta l_z^6(3R_i^2 - 3\gamma_{i0}^3 + 4)' \tag{59}$$

To the next higher order of $\in$, we obtain a set of equations which after simplification as follows

$$\frac{\partial n_i^{(3)}}{\partial \xi} = \frac{18V_p l_z^2\gamma_{i0}^3}{(3\gamma_{i0}V_{i0}^2 - l_z^2)^2}\frac{\partial \Phi^{(1)}}{\partial \tau} + \frac{3l_z^{\ 2}\gamma_{i0}}{(3\gamma_{i0}V_p^2 - l_z^2)}\frac{\partial \Phi^{(3)}}{\partial \xi}$$

$$+\left[\frac{9V_p^4\gamma_{i0}^2\left(1-l_z^2\right)}{\Omega_c^2(3\gamma_{i0}V_p^2-l_z^2)^2}-\frac{9l_z^4H^2\gamma_{i0}^2}{4(1-Z_h\sigma)^{2/3}(3\gamma_{i0}V_p^2-l_z^2)^2}\right]$$
$$\frac{\partial^3\Phi^{(1)}}{\partial\xi^3}+\left[\frac{81V_p^2l_z^4\gamma_{i0}^3-3l_z^6(3R_{i0}^2-3\gamma_{i0}^3+4}{(3\gamma_{i0}V_p^2-l_z^2)^3}\right]$$
$$\frac{\partial[\Phi^{(1)}\Phi^{(2)}]}{\partial\xi}+\left[\frac{F_1+F_2+F_3+F_4}{2(3\gamma_{i0}V_p^2-l_z^2)^5\gamma_{i0}}+\frac{F_5+F_6-F_7}{(3\gamma_{i0}V_p^2-l_z^2)^4}\right.$$
$$\left.\frac{F_8+F_9}{2(3\gamma_{i0}V_p^2-l_z^2)^5}\right]\left[\Phi^{(1)}\right]^2\frac{\partial\Phi^{(1)}}{\partial\xi}, \qquad (60)$$

$$\frac{\partial n_e^{(3)}}{\partial\xi}=-\frac{18V_pl_z^2\gamma_{e0}^2}{(\eta l_z^2-3\gamma_{e0}V_p^2)^2}\frac{\partial\Phi^{(1)}}{\partial\tau}+\frac{3l_z^2\gamma_{e0}}{(\eta l_z^2-3\gamma_{e0}V_p^2)}\frac{\partial\Phi^{(3)}}{\partial\xi}$$
$$+\left[\frac{9V_p^4\gamma_{e0}^2(1-l_z^2)}{\Omega_c^2(\eta l_z^2-3\gamma_{e0}V_p^2)^2}+\frac{9l_2^4H^2\gamma_{e0}^2}{4(\eta l_z^2-3\gamma_{e0}V_p^2)^2}\right]^2\frac{\partial^3\Phi^{(1)}}{\partial\xi^3}$$
$$-\left[\frac{81V_p^2l_z^4\gamma_{e0}^3+3\eta l_z^6(3R_{e0}^2-3\gamma_{e0}^3+4}{(\eta l_z^2-3\gamma_{e0}V_p^2)^3}\right]\frac{\partial\left[\Phi^{(1)}\Phi^{(2)}\right]}{\partial\xi}$$
$$+\left[\frac{G_1+G_2+G_3+G_4}{2(\eta l_z^2-3\gamma_{e0}V_p^2)^5\gamma_{e0}}-\frac{G_5+G_6-G_7}{(\eta l_z^2-3\gamma_{e0}V_p^2)^4}\right.$$
$$\left.-\frac{G_8+G_9}{2(\eta l_z^2-3\gamma_{e0}V_p^2)^5}\right]\left[\Phi^{(1)}\right]^2\frac{\partial\Phi^{(1)}}{\partial\xi}, \qquad (61)$$

$$\frac{\partial\Phi^{(1)}}{\partial\xi}=n_e^{(3)}-(1-Z_h\sigma)n_i^{(3)} \qquad (62)$$

where

$$F_1=486V_p^2l_z^8\gamma_{i0}^5,\quad F_2=18l_z^{10}\gamma_{i0}^2(3R_{i0}^2-3\gamma_{i0}^2+4),$$
$$F_3=81V_p^2l_z^8\gamma_{i0}^2(3R_{i0}^2-3\gamma_{i0}^2+4),$$
$$F_4=3l_z^{10}(3R_{i0}^2-3\gamma_{i0}^2+4)^2,$$
$$F_5=81V_p^2l_z^6\gamma_{i0}^4,\quad F_6=9V_pl_z^8\gamma_{i0}(3R_{i0}^2-3\gamma_{i0}^2+4),$$
$$F_7=3l_z^8\gamma_{i0}(3R_{i0}^2-3\gamma_{i0}^2+4),\quad F_8=729V_p^4l_z^6\gamma_{i0}^5$$
$$F_9=27V_p^2l_z^8\gamma_{i0}^2(3R_{i0}^2-3\gamma_{i0}^2+4),$$
$$G_1=486\eta V_p^2l_z^8\gamma_{e0}^5,\quad G_2=18\eta^2l_z^{10}\gamma_{e0}^2(3R_{e0}^2-3\gamma_{e0}^2+4)'$$
$$G_3=81\eta V_p^2l_z^8\gamma_{e0}^3(3R_{e0}^2-3\gamma_{e0}^2+4)'$$
$$G_4=3\eta^2l_z^{10}(3R_{e0}^2-3\gamma_{e0}^2+4)^2,$$
$$G_5=81V_p^2l_z^6\gamma_{e0}^4,\ G_6=9\eta V_pl_z^8\gamma_{e0}^2(3R_{e0}^2-3\gamma_{e0}^2+4),$$
$$G_7=3l_z^8\gamma_{e0}(3R_{e0}^2-3\gamma_{e0}^2+4),\ G_8=729V_p^4l_z^6\gamma_{e0}^5,$$
$$G_9=27\eta V_p^2l_z^8\gamma_{e0}^2(3R_{e0}^2-3\gamma_{e0}^2+4). \qquad (63)$$

Now, combining Eqs. (60)-(62) after simplifying by using Eqs. (49)-(57), we obtain the well-known mK-dV equation as follows

$$\frac{\partial\Phi^{(1)}}{\partial\tau}+C\left[\Phi^{(1)}\right]^2\frac{\partial\Phi^{(1)}}{\partial\xi}+B\frac{\partial^3\Phi^{(1)}}{\partial\xi^3}=0, \tag{64}$$

where

$$C=\frac{a^2b^2}{18V_pa^2\gamma_{e0}^2l_z^2+18V_pb^2\gamma_{i0}^2l_z^2(1-Z_h\sigma)}\left[\left(\frac{K_1}{2b^5\gamma_{e0}}-\frac{K_2}{b^4}-\frac{G_8+G_9}{2b^5}\right)+\left\{(1-Z_h\sigma)\left(\frac{R_1}{2a^5\gamma_{e0}}-\frac{R_2}{a^4}-\frac{F_8+F_9}{2a^5}\right)\right\}\right], \tag{65}$$

Where

$$K_1=G_1+G_2+G_3+G_4,\; K_2=G_5+G_6-G_7,$$
$$R_1=F_1+F_2+F_3+F_4,\quad R_2=F_5+F_6-F_7. \tag{64}$$

Now, taking the same stretching as like the solution of the K-dV SWs, the stationary SWs solution of Eq. (64) can be directly given as

$$\Phi^{(1)}=\Phi_m\sec h\left(\frac{\zeta}{\delta_2}\right), \tag{67}$$

where the amplitude, $\Phi_m=\sqrt{6U_0/C}$ and the width, $\delta_2=\sqrt{B/U_0}$, where δ_2 depends H. On the other hand, Φ_m is totally independent of these parameter.

DISCUSSION

The properties of the acoustic SWs in degenerate quantum plasmas are discussed in this section numerically. The effects of quantum diffraction are related not only in degenerate astrophysical plasmas but also may be meaningful in some dense laboratory plasma containing electrons and light ions. in presence of stationary heavy ions.

We emphasize the effects of the propagation angle, quantum pressure, relativistic factors, immobile heavy ions charge state, and densities of the plasma components on the acoustic SWs in such degenerate quantum plasmas. For our purpose, we have derived the K-dV and

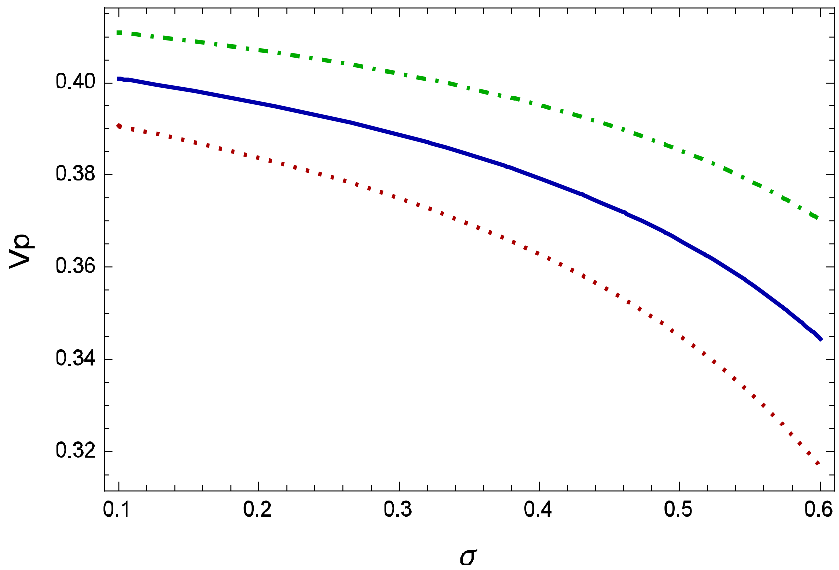


FIG. 1: Showing the variation of V_p with σ for $U_0=0.01$, δ = 40, γ_{e0} = 1.33, $\gamma_{i0}=1.29$ and Z_h = 2. The dotted line for η = 1.1, the solid line for η = 1.2, and the dotdashed line for η = 1.3.

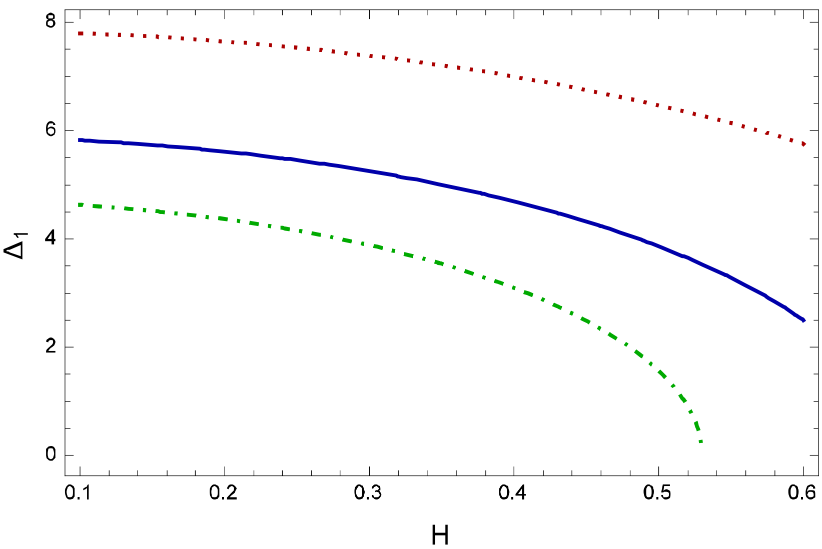
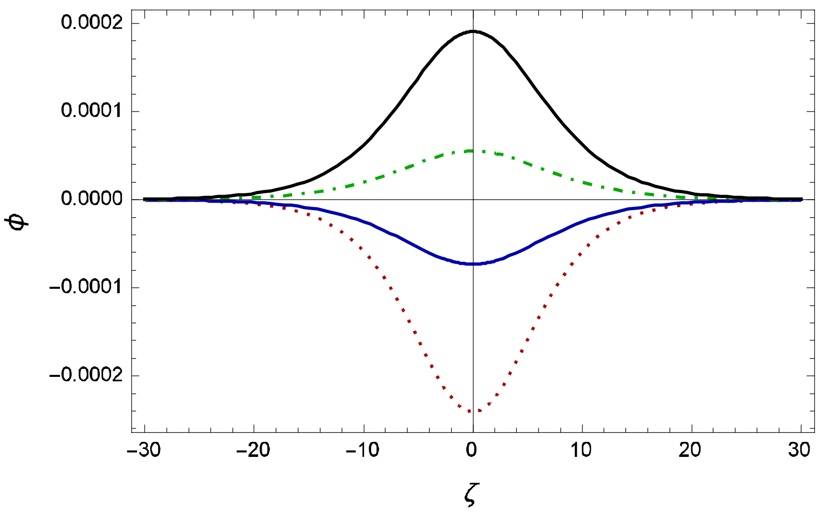


FIG. 2: Showing the variation of Δ_1 (K-dV) with H for U_0 = 0.01. γ_{e0} = 1.33, γ_{i0} = 1.29, δ = 40, $Z_h = 1$, η = 1.1. and σ = 0.1. The dotted line for $\Omega_c = 0.3$, the solid line for $\Omega_c = 0.4$, and the dotdashed line for $\Omega_c = 0.5$.



FTG. 3: Showing the effects of η on the K-dV solitary profiles for U_0 = 0.01. γ_{e0} = 1.33, γ_{i0} = 1.29, σ = 0.4, δ = 40, Ω_c = 0.4, $Z_h = 1$, H = 0.3. $R_e = 2.5$ and $R_i = 2.2$. The lower red dotted line for η = 1.1. the lower blue solid line for η = 1.13, the upper green dotdashed line for η = 1.16, and the upper black solid line for η = 1.19.

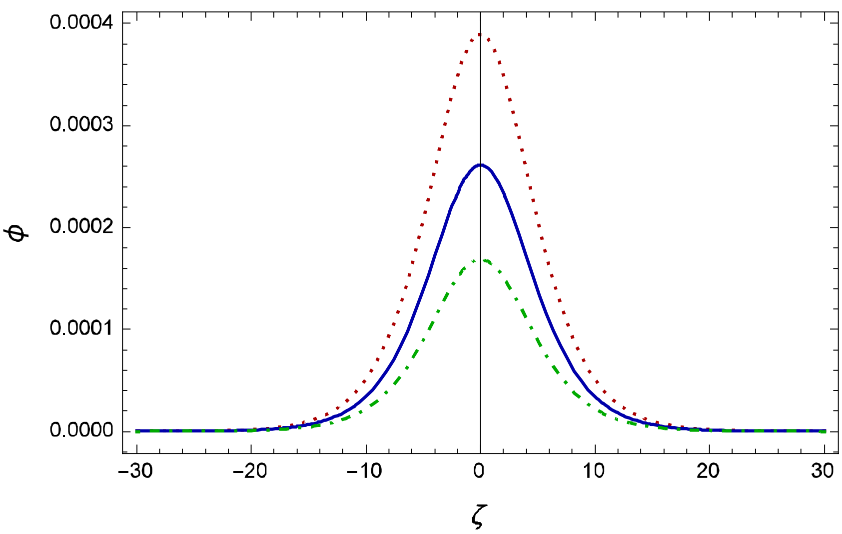


FIG. 4: Showing the effects of R_i on the (K-dV) SWs profiles for U_0 = 0.01. γ_{e0} = 1.33, γ_{i0} = 1.29, σ = 0.4, δ = 40, $\Omega_c = 0.4$, $Z_h = 1$, H = 0.3, R_e = 2.5, and η = 1.2. The dotted line for R_i = 1, the solid line for R_i = 2, and the dotdashed line for R_i = 3.

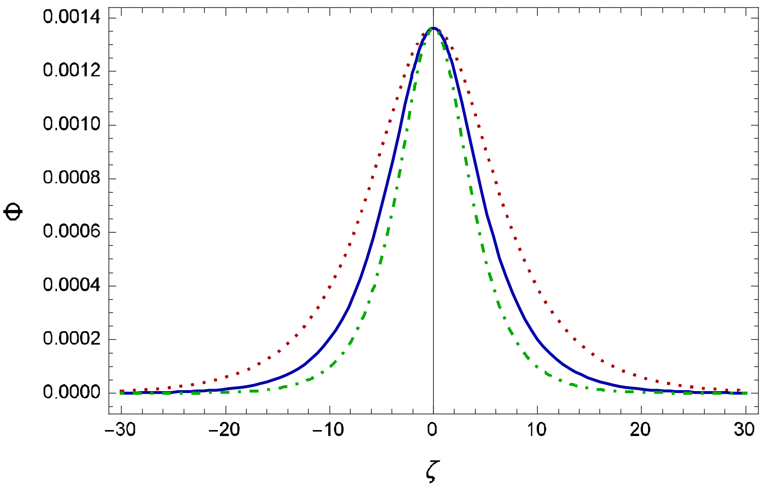


FIG. 5: Showing the effects of Ω_c on the mK-dV SWs profiles for U_0 = 0.01. γ_{e0} = 1.33, γ_{i0} = 1.29, σ = 0.2, δ = 60, $Z_h = 1$, H = 0.3, R_e = 2.5, R_i = 2.2, and η = 1.6. The dotted line for $\Omega_c = 0.3$, the solid line for Ω_c = 0.4, and the dotdashed line for $\Omega_c = 0.5$.

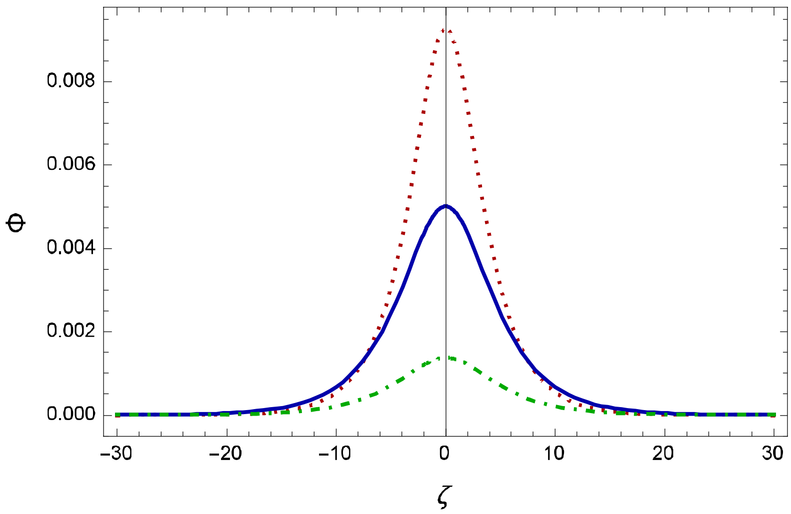


FIG. 6: Showing the effects of σ on the mK-dV SWs profiles for U_0 = 0.01. γ_{e0} = 1.33, γ_{i0} = 1.29, Ω_c = 0.4, δ = 60, $Z_h = 1$, H = 0.3. $R_e = 2.5$ and $R_i = 2.2$ and η = 1.8. The dotted line for σ = 0.1, the solid line for σ = 0.2. and the dotdashed line for σ = 0.3.

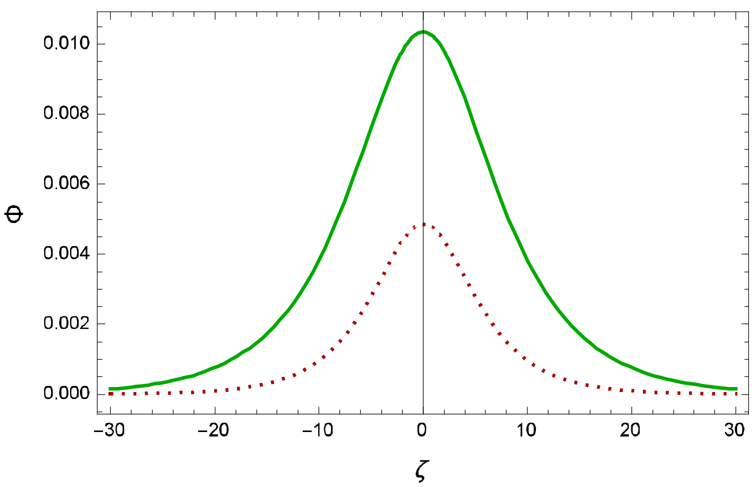


FIG. 7: Showing the mK-dV SWs profiles for U_0 = 0.01. Ω_c = 0.3, σ = 0.1, δ = 60, $Z_h = 1$, H = 0.3. and η = 1.16. The solid line for

and $R_i =$ = 2.2).

mK-dV equations, and analyzed their stationary SWs solutions based on some typical plasma parameters relevant to different astrophysical and laboratory plasma situations existed in some published works. We consider some typical plasma species density which is consistent with the relativistic degenerate astrophysical plasmas, e.g., $n_{io} = 1.1 \times 10^{29}$ cm^{-3}, $n_{e0} = 9.1 \times 10^{29}$ cm^{-3}, $n_{ho} = 0.4 \times 10^{30}$ cm^{-3}, (11, 36, 37, 48) and the other quantum parameters e.g., β = 0.1 to 0.9, H = 0.2 to 0.9 (18, 22, 49), and ambient magnetic fields ~ 10^{13}G (15, 50). The values of these parameters may change depending on different plasma situations. The results, we have found from this investigation can be summarized as follows:

- The effect σ on V_p of the El acoustic waves for different values of η are displayed in Fig. 1. The phase speed of the acoustic waves decreases with the increase of σ. This happens due to the increase of the inertia of the acoustic waves with the increase of the value of σ. It is also observed that with the increase in η, V_p of the EI acoustic waves increases.
- The variation of width, Δ_1 of the K-dV SWs with H is depicted in Fig. 2 for different values of Ω_c. It is found that as the values of both H and Ω_c increase, Δ_1 of the acoustic SWs decreases significantly.
- The variation of the K-dV SWs profiles with the η (R_i) is shown in Fig. 3 (Fig. 4). It is observed that

our plasma system supports both positive (com-pressive) and negative (rarefactive) SWs structure. In case of η both amplitude and width of the SWs increases with the increase of η. Increase of η means the increase of Fermi energy of the electrons and ions. But in case of R_i both amplitude and width of the SWs decreases with the increase of R_i (increase of relativistic effects of ions). Plasma particles can move freely in the weakly relativistic plasma more than the strong relativistic plasma.

4. The influences of the Ω_c (σ) on the mK-dV SWs profiles is shown in Fig. 5 (Fig. 6). It seems that there is no effect of Ω_c on the amplitude of the mK-dV SWs but width decreases with the increase of the Ω_c. It is also found that both width and amplitude of the mK-dV SWs decrease with the increase of σ. This occurs due to the increase of the inertia of the plasma particles.
5. Fig. 7 compares the the amplitude and width of the quantum El SWs profiles for the relativistic case and the non-relativistic case. Plasma particles can move freely in the non-relativistic plasma more than the ultra-relativistic plasma due to the less number density of plasma species in non-relativistic case than the ultra relativistic case. Therefore, the amplitude as well as width of the SWs are noticeably higher for non-relativistic case than for ultra-relativistic case. The magnitude of the external magnetic field $\mathbf{B_0}$ has no any effect on the amplitude of the SWs but it does have a direct effect on the width of SWs. We found that as the magnitude of $\mathbf{B_O}$ (i.e., Ω_p) increases, the width of SWs decreases, i.e., the magnetic field makes the solitary structures more spiky.

We hope that our lower order as well as higher order nonlinear analysis will be helpful for understanding the localized electrostatic disturbances not only in the different astrophysical degenerate compact objects (Shapiro, 1983; Garcia-Berro, 2010; Mamun, 2010)), but also in different dense laboratory plasma nonlinear experiments (Berezhiani, 1992; Murkhmd, 2006).

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REFERENCES

Abdikian, A. and Mahmood, S. 2016. Phys. Plasmas 23. 122303. "
Ali, S., Moslem, W. M., Shukla, P. K. and Schlickeiser, R. 2007. Phys. Plasmas 14, 082307.
Berezhiani, V. I., Tskhakaya, D. D. and Shukla, P. K. 1992. Phys. Rev. A 46, 6608.
Bhowmik, C., Misra, A. P. and Shukla, P. K. 2007. Phys. Plasmas 14, 122107
Chabrier, G., Saumon, D. and Potekhin, A. Y. 2006. J. Phys. A 39, 4411.
Garcia-Berro, E., Torres, S., Althaus, L. G., Renedo, I., Lorn-Aguihar, P., Crsico, A. H., Rohrmann, R. D., Salaris, M. and J. Tsern, Nature 465, 194 (2010).
Gardner, C. 1994. SIAM J. Appl. Math. 54, 409.
Gervino, G., Lavagno, A. and Pigato, D. 2012. Cent. Eur. J, Phys. 10. 594.
Haas, F., Garcia, L. G., Goedert, J. and Manfredi. G. 2003. Phys. Plasmas 10, 3858.
Harding A. K. and Lai, D. 2006. Rep. Prog. Phys. 69. 2631.
Hossain, M. M., Mamun, A. A. and Aahrafi, K. S. 2011. Phys. Plasmas 18, 103704.
Hossen M. A. and Mamun. A. A. 2015. Phys. Plasmas 22, 073505
Lai, D.2001. Rev. Mod. Phys. 73, 629.
Mamun A.A. and Shukla, P. K. 2010. Phys. Lett. A 374, 4238.

Manfredi, G. 2005. Fields Inst. Commun. 46, 263.
Manfredi, G. and Haas, F. 2001. Phys. Rev. B 64, 075316 .
Markowich, A., Ringhofer, C. and Sehmeiser, C. 1990. *Semi-conductor Equations* Springer, Vienna, Chaps. 1, 2.
Massey, H. 1976. *Negative. Ions* (Cambridge Universfly Press, Cambridge.
Miller, H. R. and Witta, P. J. 1987. *Active Galactic Nuclei* (Springer-Verlag, Berlin.
Murkhmd, M. and Shukla, P. K. 2006. Rev. Mod. Phys. 78, 591. "
Pines, D. 1961. J. Nucl. Energy Part C 2, 5.
Plastino, A. R. and Plastino, A. 1993. Phys. Lett. A 174, 384.
Rouhani, M. R., Mohammadi, Z. and Akbarian, A. 2014. Astrophys. Space Sci. 349, 265.
Saeed-ur-Rehman, 2010. Phys. Plasmas 17, 062303.
Shapiro S. L.and S. A- Teukolsky, *Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects* (John Wiley & Sons, New York, 1983).(3) E. Garcia-Berro, S. Torres, L. G. Althaus, I. Renedo, P.
Shiikla, P. K., Mamun, A. A. and Mendis, D. A. 2011. Phys. Rev. E 84, 026405.
Woolsey, N. C., Courtois, C. and Bendy, R.O. 2004. Plasma Phys. Control. Fusion 46. B397.

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