Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 10, Issue, 07, pp.71792-71796, July, 2018

INTERNATIONAL JOURNAL
OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

CHARACTERIZATIONS OF C-A-CONTINUOUS FUNCTIONS

*Govindappa Navalagi

Department of Mathematics, KIT Tiptur-572202, Karnataka, India

ARTICLE INFO

Article History:
Received 08th April, 2018
Received in revised form
19th May, 2018
Accepted 21st June, 2018
Published online 31st July, 2018

Key Words:
Preopen sets, α-Open Sets, Semiopen Sets, Compact Subsets, Precontinuouity, C-Continuity, pre-α-Openness, pre-α-Closedness.

ABSTRACT

In 1970, Gentry and Hoyle have defined and studied the notion of c-conti-nuity in topological spaces. Later, Long et al and Gauld have studied some more properties of c-continuity in the literature. In 1965, O. Njastad, had defined the concept of α-sets, latter these sets were called as α-open sets. 1983,Mashhour et al have defined and studied the concepts of α-closed sets, α-continuity, α-openness and α-closedness in topological spaces. In this paper, we define and study the concepts of c-α-continuity, αc-continuity, c*-α-continuity and almost cα-continuity in topological spaces. Also, we characterize their basic properties.

Citation: ***Govindappa Navalagi*, 2018.** "Characterizations of c-α-continuous functions", *International Journal of Current Research*, 10, (07), 71792-71796.

INTRODUCTION

In 1970, Gentry and Hoyle (Gentry, 1970) have defined and studied the new class of functions called c-continuous functions. Latter, in 1974 & 1975, Long et al (Long, 1974; Long, 1975) have studied further properties of c-continuous functions and defined a new class of functions called c*-continuous functions in topological spaces. Again, in 1978 Gauld (1978) has defined and studied some more properties of c-continuous functions via cocompact topologies. In 1965, O. Njastad (1965), had defined the concept of α-sets, latter these sets were called as α-open sets. 1983,Mashhour *et al.* (1983) have defined and studied the concepts of α-closed sets, α-continuity, α-openness and α-closedness in topological spaces. In this paper, we define and study the concept of c-α-continuity, αc-continuity, c*-α-continuity and almost -cα-continuity. Also, we characterize their basic properties.

2.Preliminaries

Throughout the present paper, spaces (X,τ) and (Y, σ) (or simply, X and Y) always mean topological space s on which no separation axioms are assumed unless explicitly stated. Moreover, in this paper wherever compactness is taken to mean every open cover has a finite subcover and subsets of a space are compact provided they are compact considered as subspace (cf.10).Let A be a subset of a space X. The closure and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A of a space X is called regular open (in brief, r-open) if A = Int Cl(A) and regular closed (in brief, r-closed) if A = Cl Int(A).

The following definitions and results are useful in the sequel:

Definition 2.1: A subset A of a space X is said to be:

(i) α -open (23) if A $\subset$ Int(Cl(Int(A)))
(ii) semi-open (9) if A $\subset$ Cl(Int(A))
(iii) pre-open (16) if A $\subset$ Int(Cl(A)
(iv) β-open (1) if A $\subset$ Cl Int Cl(A).

The family of all α –open (resp. semi-open, pre-open) sets in a space X is denoted by α O(X) (resp. SO(X) PO(X.) The complement of an α -open (resp. pre-open) set is said to α -closed (18) (resp. pre-closed (5)).

Definition 2.2: The intersection of all α-closed sets containing A is called the α-closure of A and is denoted by **α**ClA (Mashhour *et al.,* 1983).

The union of all pre-open sets contained in A is called pre-interior of A and is denoted by pInt(A) (Mashhour *et al.,* 1984).

***Corresponding author: Govindappa Navalagi,**
Department of Mathematics, KIT Tiptur-572202, Karnataka, India.
DOI: https://doi.org/10.24941/ijcr.31751.07.2018

Definition 2.3: A function f:X →Y is said to be:

- precontinous (16), if the inverse image of each open subset of Y is preopen subset in X.
- semicontinuous(9), if the inverse image of each open subset of Y is semiopen subset in X.
- α-continuous (18), if the inverse image of each open subset of Y is α-open subset in X.

Definition 2.4 (3): A function f:X →Y is said to be pre-α-open(resp. pre-α-closed) if the image of each α-open (resp. α-closed) subset of X is α-open (resp. α-closed) subset in Y.

Definition 2.5 (7): A function f: X → Y is said to be c-continuous if for each x ∈ X and each open set V ⊂ Y containing f (x) and having compact complement, there exists an open set U containing x such that f(U) ⊂ V.

Theorem 2.6 (7, Th.1): Let f: X→Y be a function. Then the following statements are equivalent:

- f is c-continuous.
- If V is an open subset of Y with compact complement, then $f^{1}(V)$ is open subset of X. These statements are implied by:
- If F is a compact subset of Y, then $f^{1}(F)$ is closed subset of X and, moreover, if Y is Hausdorff, then all the above statements: (i)-(iii) are equivalent.

Theorem 2.7: Let f: X → Y be a function. Then, f is c-continuous if and only if:

- The inverse image of each open subset of Y having compact complement is open in X (Long, 1974).
- The inverse image of each closed compact subset of Y is closed in X (Singh, 1986).

Definition 2.8 (Govindappa Navalagi, 2014**):** A function f: X → Y is said to be c-precontinuous if for each x ∈ X and each open set V ⊂ Y containing f (x) and having compact complement, there exists an preopen set U containing x such that f(U) ⊂ V.

Definition 2.9 (Caldas *et al.,* 2005; Govindappa Navalagi, 2014**):** A function f: X → Y is said to be c-semicontinuous if for each x ∈ X and each open set V ⊂ Y containing f (x) and having compact complement, there exists an semiopen set U containing x such that f(U) ⊂ V.

Definition 2.10 (Govindappa Navalagi, 1965**):** A function f: X → Y is said to be c-β-continuous if for each x ∈ X and each open set V ⊂ Y containing f (x) and having compact complement, there exists an β-open set U containing x such that f(U) ⊂ V.

Definition 2.10 (Aho, 1994**):** A space X is a PS-space iff each preopen subset of X is semiopen. It means that, a space X is PS-space if PO(X) ⊂ SO(X).

3. Properties of c-α-continuous functions

We, define the following.

Definition 3.1: A function f: X → Y is said to be c-α-continuous if for each x ∈ X and each open set V ⊂ Y containing f (x) and having compact complement, there exists an α-open set U containing x such that f(U) ⊂ V. As, we know that every α-open set is preopen and semiopen, so the following imply:

- Every c-continuous function is c-α-continuous.
- C-α-continuous function is c-precontinuous.
- C-α-continuous function is c-semicontinuous.

We, have the following:

Lemma 3.2: In a PS-space, if f:X→Y is c-precontinuous then it is c-semicontinuous.

We, prove the following.

Theorem 3.3: Let f: X→Y be a function. Then the following statements are equivalent:

- f is c-α-continuous.
- If V is an open subset of Y with compact complement, then $f^{1}(V)$ is α-open subset of X.

These statements are implied by:

- If F is a compact subset of Y, then $f^{1}(F)$ is α-closed subset of X and, moreover, if Y is Hausdorff, then all the above statements: (i)-(iii) are equivalent.

Proof follows by Theorem 2.7 and 2.8 above. Easy proof of the following is omitted.

Lemma 3.4: A function f: X → Y is said to be c-α-continuous if the inverse image of each open subset of Y having compact complement is α-open in X.

Lemma 3.5: A function f: X → Y is said to be c-α-continuous if the inverse image of each closed compact subset of Y is α-closed in X.

We, recall the following.

Lemma 3.6(Mashhour, 1119**):** If A is either preopen or semiopen subset of X and V is a α-open subset of X, then A ∩ V is a α-open subset in the subspace (A, τ/A). Next, we prove the following.

Theorem 3.7: If f: X → Y is c-α-continuous function and A be an either preopen or semiopen subset of X, then f/A: A → Y is also c-α-precontinuous. Easy proof of the Theorem follows by Lemma – 3.5 above. We, define the following.

Definition 3.8: A function f: X →Y is said to be M-α-continuous, if the inverse image of each α-open subset of Y is α-open subset in X., equivalently, if the inverse image of each α-closed subset of Y is α-closed subset in X.

Theorem 3.9: If f: X → Y is M-α-continuous and g: Y→ Z is c-α-continuous, then gof is c-α-continuous.

Proof. Let U be an open subset of Z having compact complement. Then, $g^{-1}(U)$ is α-open set in Y, since g is c-α-

continuous. Again, as f is M-α-continuous and $g^{-1}(U)$ is α-open subset of Y, $(gof)^{-1}(U) = f^{-1}(g^{-1}(U))$ is α-open subset in X. This shows that gof is c-α-continuous function.
We, define the following.

Definition 3.10: A function f: X →Y is said to be α*-continuous, if the inverse image of each α-open subset of Y is open subset in X.

Theorem 3.11: If f: X →Y is α*-continuous and g: Y→ Z is c-α-continuous, then gof is c-continuous function.

Proof follows from Theorem-3.6.

Theorem 3.12: Let f: X → Y be either pre-α-open or pre-α-closed surjection and let g: Y→ Z be any function such that gof is c-α-continuous.Then, g is c-α-continuous.

Proof: Suppose f is pre-α-open (resp. pre-α-closed) and V be an open subset with compact complement (resp. V be a closed compact subset) in Z. Since gof is c-α-continuous, $(gof)^{-1}(V) = f^{1}(g^{-1}(V))$ is α-open (resp. α-closed) subset in X. Since f is pre-α-open (resp. pre-α-closed) and surjective, $f(f^{1}(g^{-1}(V))) = g^{-1}(V)$ is α-open (resp. α-closed) set in Y and consequently, g is c-α-continuous function.

We, define the following.

Definition 3.13: A function f: X →Y is said to be α*-open (resp. α*-closed), if the image of each α-open (resp. α-closed) subset of X is open (resp. closed) subset in Y.

Theorem 3.14: Let f: X → Y be either α*-open or α*-closed surjection and let g: Y→ Z be any function such that gof is c-α-continuous. Then, g is c-continuous.

Proof follows by Theorem -3.8 above.

In view of the fact that an arbitrary union of preopen (resp. semiopen, α-open) sets is preopen (resp. semiopen, α-open), we have the following (Husain, 1977; Mashhour, 1982; Reilly, 1990; Mashhour, 1983).

Theorem 3.15: If X and Y are two topological spaces and X = A ∪ B, where A and B are preopen or semiopen subsets of X and f: X → Y is a function such that f|A and f|B are c-α-continuous, then f is c-α-continuous.

Proof: Assume that A and B are preopen or semiopen subsets in X. Let U be an open subset of Y with compact complement. Then, we have $f^{1}(U) = (f|A)^{-1}(U) \cup (f|B)^{-1}(U)$, each of which is α-open by Lemma-3.5 & Theorem- 3.6. Thus, $f^{-1}(U)$ is α-open in X and hence f is c-α-continuous.

Recall that a space X is called α-T_1 (3) if, for x, y ∈X such that x ≠ y, there exist preopen sets U and V such that x ∈ U, y ∉ U and y ∈ V, x∉ V. Also, it is proved that in (Caldas, 2005) a α-T_1 space every singleton set is α-closed.

In view of the above result, we give the following.

Theorem 3.16: Let f: X →Y be c-α-continuous and injective.

If Y is T_1, then X is α-T_1.

We, recall the following

Definition 3.17(16): Let f: X → Y be a function.Then, G(f) ={(x,f(x)) | x ∈X } is called the graph of f and the function g(f): X → X x Y defined as g(f)(x) = (x,f(x)) for each x∈X is called the graph function of f.

Theorem 3.18: Let f: X → Y be c-α-continuous. Then, the graph function g(f): X → X x Y is c-α-continuous.

Proof: Let U x V be any open subset in X x Y having compact complement W of X x Y. Then, we have to show that $(g(f))^{-1}(U \times V)$ is α-open set in X. Let W= X x Y \ (U x V) = (X\U) x Y ∪ X x (Y\V), in which X x (Y\V) being the closed subset of W must also be compact. Since P_Y: X x Y → Y being the projection, which is continuous, so P_Y (X x (Y \ V)) = Y\V is compact in Y. Thus, $f^{-1}(V)$ is α-open set in X. Since f is c-α-continuous, $(g(f))^{-1}(U \times V)=U\cap f^{1}(V)$, which is α-open as the intersection of an open set and an α-open set is again α-open. Therefore, g(f) is c-α-continuous.

Theorem 3.19: Let X be compact Hausdorff space. If g(f): X →X x Y is c-α-continuous, then the function f: X →Y is c-α-continuous.

Proof: Let V any open set containing f(x) having compact complement. Then, we have to prove that $f^{1}(V)$ is α-open in X: Consider X x V which is open in X x Y where X x Y \(X x V) = X x (Y\V) is compact, and g(f) is c-α-continuous and hence $(g(f))^{-1}(X \times V) = f^{1}(V)$ which is α-open in X.This shows that f is c-α-continuous.

4. Properties of αc-continuous functions

We, recall the folloing.

Definition 4.1(15): A space X is said to be α-compact if every α-open cover of X has a finite subcover.

Clearly, every α-compact space is compact.

Lemma 4.2 (25): If a space X is α-compact and A is an α-closed set of X, then A is α-compact.

Now, we define the following.

Definition 4.3: A function f: X→Y is said to be αc-continuous if the inverse image of each closed α-compact set of Y is α-closed in X. It is well-known that a space X is said to be extremally disconnected (e.d), if the closure of each open subset of X is open.

We, give the following.

Lemma 4.4: The following statements hold for a function f: X→Y:

- f is αc-continuous.
- if G is an open subset of Y with compact complement, then $f^{1}(G)$ is an open subset of X, when X is an e.d.
- f is c-α-continuous function.

Next, we recall the following.

Definition 4.5 (Noiri, 1988): A function f: X→Y is said to be almost - α -continuous if the inverse image of each r-open set of Y is α-open in X.

Definition 4.6 (Singh, 1986): A function f: X→Y is said to be almost -c-continuous if the inverse image of each r-open set of Y with compact complement is open in X.

We, define the following.

Definition 4.6: A function f: X→Y is said to be almost -cα-continuous if the inverse image of each r-open set of Y with α-compact complement is α-open in X.

Next, we prove the following

Lemma 4.7: Let f:X→Y is an α-irresolute function and g:Y→Z be an almost -cα-continuous function, then gof is an almost-cα-continuous function.

Proof: Let G ⊂ Z be regular open set with compact complement, then $g^{-1}(G)$ is α-open in Y.Again, f is α-irresolute and $g^{-1}(G)$ is α-open in Y, then $f^{1}(g^{-1}(G)) = (gof)^{-1}(G)$ is α-open in X. This shows that gof is almost -cα-continuous function.

It is well-known that a space X is said to be countably compact if every countable open cover of X has a finite subcover.

We, recall the following.

Definition 4.8 (Maheshwari, 1981)**:** A space X is said to be countably α-compact if every α-open cover of X has a finite subcover. Clearly, every countably α-compact space is countably compact.

Lemma 4.9 (Maheshwari, 1981)**:** A space X is countably α-compact if every countable α-closed cover of X has a nonempty f.i.p.

Theorem 4.10 (Maheshwari, 1981)**:** Every α-closed open subspace Y of a countably α-compact space is countably α-compact.

Clearly, we have the following.

Lemma 4.11: If a space X is countably α-compact and A is an α-closed set of X,then A is countably α-compact.

Next, we recall the folloing.

Definition 4.12(Young Soo Park, 1971): A function f: X→Y is said to be c*-cintinuous if for each countably compact and closed set F of Y, $f^{1}(F)$ is closed in X.

Now, we define the following.

Definition 4.13: A function f: X→Y is said to be c*α-cintinuous if for each countably α-compact and closed set F of Y, $f^{1}(F)$ is α-closed in X.

We, prove the following.

Lemma 4.14: Let f:X→Y is an α-irresolute function and g:Y→Z be an c*-α-continuous function, then gof is an c*-α-continuous function.

Proof: Let G ⊂ Z be open set with countably α-compact complement, then $g^{-1}(G)$ is α-open in Y.A gain, f is α-irresolute and $g^{-1}(G)$ is α-open in Y, then $f^{1}(g^{-1}(G)) = (gof)^{-1}(G)$ is α-open in X. This shows that gof is c*-α-continuous function.

We,recall the following.

Lemma 4.15 (Maheshwari, 1981)**:** If f: X→Y is an open continuous function then the inverse image of every α-open set of Y is α-open in X.

Next, we give the following.

Lemma 4.16: Let f:X→Y be an open continuous function and g:Y→Z be c*-α-continuous function then gof is c*-α-continuous.

Conclusion

In the lights of e.d & PS-spaces, we have the following implication: β-open set → preopen set → semiopen set(and hence α-open set) →open set. Thus, in view of this implication, we conclude the following.

Lemma 5.1: For a function f: X→Y, then the following are equivalent:

- F is c-β-continuous,
- F is c-precontinuous,
- F is c-semicontinuous,
- F is c-α-continuous,
- F is c-continuous.

Easy proof is omitted.

REFERENCES

Abd El-Monsef, M.E., El-Deeb, S.N. and Mahmoud, R.A. 1983. β-open sets and β-continuous mappings, *Bull. Fac. Sci. Assiut.* Univ.,12,77-90.

Aho T. and Nieminen, T. 1994. Spaces in which preopen subsets are semiopen, Ricerche Mat., 43, 45-59.

Caldas, M., Jafari, S., Ganster, M. and Navalagi, G. 2005. On functions and generalized Λ_α -sets, Demonstratio Mathematica, 38(3), 729-738.

Chae, G.I., Pooniwala, R.K. and Singh, V.P. 2001. A note on c-semicontinuous functions, KORUS, 110-112.

El-Deeb, S.N., Hasanein, I.A., Mashhour, A.S. and Noiri, T. 1983. On p-regular spaces, Bull. Math.Soc.Sci., Math., R.S.Roumanie (N.S), 27(75), 311-315.

Gauld, D.B. 1978. C-continuous functions and cocampact topologies, Kyungpook Math. J., Vol.18, No.2, Dec-, 151-157.

Gentry K.R. and Hoyle, H.B. 1970. III, c-continuous functions, Yokohama Math. J., 18, 71-76.

Govindappa Navalagi, 2014. On c-precontinuous functions, *American J. of Mathematics and Sciences*, Vol.3, No. 1 41-48.

Govindappa Navalagi, 2014. Some more properties of c-semicontinuous functions, *American J. of Mathematics and Sciences,* Vol.3, No. 1, 35-40.

Govindappa Navalagi, On c-β-continuous functions (communicated).

Husain, T. 1977. Topology and maps, Plenum Press, New York.

Levine, N. 1963. Semiopen sets and semicontinuity in topological spaces, Amer. Math. Monthly, 70, 36-41.

Long P.E. and Hendrix, M.D. 1974. Properties of c-continuous functions, *Yokohama Math. J.,* 22, 117-123.

Long P.E. and Herrington, L.L. 1975. Properties of c-continuous functions and c*-continuous functions, Kyungpook Math. J., Vol.15, No.2, Dec- 213-221.

Maheshwari S.N. and Thakur, S.S. 1985. On α-compact spaces, *Bull.Inst. Math. Acad. Sinica*, 13.

Maheshwari, S.N. and Thakur, S.S. 1980. On α-irresolute mappings, Tamkang *J. Math*., 11,209-214.

Maheshwari, S.N. and Thakur, S.S. 1981. Countably α-compact spaces, *Jour. Sci. Res.,* Vol.3, No.2, 121-123.

Maheshwari, S.N. and Thakur, S.S. On α-sets, Jnanabha (to appear)

Mashhour A.S. *et al.* 1982. A note on semicontinuity and precontinuity, Indian J. pure appl. Math., 13(10), Oct-1119-1123.

Mashhour, A.S., Abd El-Monsef, M.E. and El-Monsef, S.N. 1982. On precontinuous and weak precontinuous mappings, Proc. *Math. Phys. Egypt*, 53, 47-53.

Mashhour, A.S., Abd El-Monsef, M.E. and Hasanein, I.A. 1984. On pre - topological spaces, *Bull. Math. Soc. Sci. Math., R.S. Roumanie,* 28(76), 39-45.

Mashhour, A.S., Hasanie I.A. and El-Deeb, S.N. 1983. α – continuous and α-open mappings, Acta Math. Hungarica, 41(3-4), 213-218.

Njastad, O. 1965. On some classes of nearly open sets, Pacific J.Math., 15, 961-970.

Noiri, T. 1988. Almost α-continuous functions*, Kyungpook Math.J.,*28,71-77.

Noiri, T. and Di Maio, G. 1988. Properties of α-compact spaces, Rendi conti. *Cir. Mate. Palermo Ser.,* (II),18,359-369.

Reilly, I.L. and Vamanamurthy, M.K. 1990. On some questions concerning preopen sets*, Kyungpook Math.J.,* 30, 87-93.

Singh I.J. and Prasad, R. 1986. Almost -c-continuous functions, Indian J.of Math.,Vol.27(1+3), 165-168.

Young Soo Park, C*. 1971. Continuous functions, *J. Korean Math. Soc.,* 8,69-72.

****