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RESEARCH ARTICLE

INTERACTION OF LUMP WAVE WITH KINK SOLITON SOLUTIONS OF A (3+1)-DIMENSIONAL NONLINEAR EVOLUTION EQUATION VIA HIROTA BILINEAR METHOD

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ABSTRACT

In this paper, we reveal a (3+1)-dimensional nonlinear evolution equation to determine interaction between lump waves and kink soliton. In this regard, we cast the model into it Hirota bilinear form firstly. We offer periodic lump wave through a test function in-terms of exponential and periodic cosine functions. We also consider test function as a combination of a general quadratic polynomial with exponential function to reveal interaction of lump wave and kink soliton. Finally, the interactions of solitary waves and lump waves are presented with an entire analytic derivation. Some graphs are incorporated to visualize the dynamics of the obtained wave solutions.

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INTRODUCTION

Generally, all the basic equations of physics are nonlinear and such types of nonlinear evolution equations (NLEEs) are often very difficult to get exact solution (Gu *et al*, 1999; Fan, 2000; Zhang, 2004; Wang *et al*, 1996; Liu *et al*, 2001; Wang *et al*, 2005; Manafian *et al*, 2016; González-Gaxiola, 2017; Zaid Odibat, 2017; Bira, 2015; Shakeel *et al*, 2016; Hossen *et al*, 2017; Roshid *et al*, 2017; Wazwaz, 2013; Roshid, 2017; Roshid *et al*, 2017; Muller *et al*, 2005; Roshid , 2017; Kharif *et al.*, 2009; Akhmediev *et al.*, 2009; Guo *et al*, 2012; Li *et al*, 2013). So, to seek the exact solutions of NLEEs still have very attraction to diverse group of researcher. The Darboux transformation (Gu *et al.,* 1999), the tanh-function method (Fan, 2000), the extended tanh-function method (Zhang, 2004), the homogeneous balance method (Wang *et al*, 1996), the Jacobi elliptic function expansion method (Liu *et al*, 2001), the F-expansion method (Wang *et al*, 2005), Hirota bilinear method (Roshid *et al*, 2017; Roshid, 2017) and so on which many powerful and systematic approaches to obtain the exact solutions of NLEEs. Among those methods, the Hirota's bilinear method is rather heuristic and possesses significant features that make it practical for the determination of multiple soliton solutions, and for multiple singular soliton solutions (Roshid, 2017) for an extensive class of NLEEs in a direct method. Recently, we have seen two types of phenomena such as soliton fission and soliton fusion respectively (Roshid, 2017) in many nonlinear science and engineering field such as the gas dynamics, laser, plasma physics, electromagnetic, and passive random walker dynamics (Wazwaz, 2013; Roshid, 2017; Roshid *et al*, 2017). Also, rogue wave solutions have drawn a big attention of mathematicians and physicists globally for amusing class of lump-type solutions. Such types of phenomena are found in different fields in physics such as plasmas, the deep ocean, nonlinear optic and even finance (Muller *et al*, 2005; Roshid, 2017; Kharif *et al*, 2009). On the basis of Hirota bilinear forms, it is natural and interesting to hunt for rogue type solutions of NLEEs (Guo *et al*, 2012; Li *et al*, 2013). In this paper, we consider a (3+1)-dimensional nonlinear evolution equation to determine interaction between lump waves and kink soliton. That's why we cast the equation into Hirota bilinear form firstly. Then we offer periodic lump wave through a test function in-terms of exponential and periodic cosine functions. Finally, the interactions of solitary waves and lump waves are presented with an entire analytic derivation. Some graphs are incorporated to visualize the dynamics of the obtained wave solutions.

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2. Lump and solitary wave solutions to the Breaking Soliton equation

2.1 The bilinear form of (3+1) D nonlinear evolution equation

Consider the (3+1)-dimensional nonlinear evolution equation as

$$u_{yt} - u_{xxxy} - 3(u_x u_y)_x - 3u_{xx} + 3u_{zz} = 0. \tag{1}$$

Through the dependent variable transformation as

$$u = 2(\ln f)_x, \tag{2}$$

Eq.(1) can be reduce to bilinear D operator form.

Substitute the equation (2) with $f = f(x, y, z, t)$ into equation (1) we obtain

$$(D_t D_y - D_x^3 D_y - 3D_x^2 + 3D_z^2) f.f = 0 \tag{3}$$

where $D_t D_y$, $D_x^3 D_y$, D_x^2 and D_z^2 are all the bilinear derivative operators (20, paper) defined by

$$D_x^\alpha D_y^\beta D_t^\gamma (\rho.Q) = (\frac{\partial}{\partial x} - \frac{\partial}{\partial x'})^\alpha (\frac{\partial}{\partial y} - \frac{\partial}{\partial y'})^\beta$$
$$\times (\frac{\partial}{\partial t} - \frac{\partial}{\partial t'})^\gamma \rho(x, y, t) Q(x' y' t') \Big|_{x'=x, y'=y, t'=t} \tag{4}$$

Using formula Eq.(4), Eq.(3)reduces to

$$ff_{xxxy} - f_{xxx} f_y - 3 f_x f_{xxy} + 3 f_{xx} f_{xy} = 0. \tag{5}$$

2.2 Lump wave solutions

Let us adopt that Eq. (5) has a ansatz in the following form:

$$f = l_1 \cos(a_1 x + a_2 y + a_3 z + a_4 t) + \exp(a_5 x + a_6 y + a_7 z + a_8 t) + l_2 \exp\{-(a_5 x + a_6 y + a_7 z + a_8 t) \tag{6}$$

where a_i, $(1 \le i \le 8)$ are arbitrary constants to be determined later. Setting Eq. (6) into bilinear form Eq. (5), we obtain some polynomials which are functions of the variables x, y, z and t. Equating all the coefficient of $\cos$, $\sin$ and $\exp$ to be zero, we can obtain the set of algebraic equations for a_i, $(1 \le i \le 8)$. Solving the system with the aid of symbolic computation system Maple, gives the following relations between the parameters a_i:

Set-1:

$$a_1 = 0,\ a_4 = -\frac{3a_3^2}{a_2}, a_5 = \frac{\sqrt[3]{P}}{2a_2} - 4a_3{}^{-\sqrt[3]{}}\sqrt[3]{P}, a_6 = 0, a_7 = 4a_3{}^{-\sqrt[3]{}}\sqrt[3]{P} - \frac{\sqrt[3]{P}}{2a_2}, \tag{7}$$

Where $P = (4a_2 a_8 + 4\sqrt{\frac{a_2^3 a_8^2 + 32a_3^3}{a_2}})a_2^2$ and a_2, a_3, a_8 are arbitrary constants.

Therefore, substituting Eq. (7) and Eq. (6) along with Eq.(2) yields the following periodic lump wave solution,

$$u = 2(\ln f)_x, \tag{8}$$

where

$$f = l_1\cos(a_2 y + a_3 z - \frac{3a_3^2}{a_2}t) + \exp\{(\frac{\sqrt[3]{P}}{2a_2} - 4a_3\sqrt[-3]{P})x - (\frac{\sqrt[3]{P}}{2a_2} - 4a_3\sqrt[-3]{P})z + a_8 t\}$$
$$+ l_2\exp\{-(\frac{\sqrt[3]{P}}{2a_2} - 4a_3\sqrt[-3]{P})x + (\frac{\sqrt[3]{P}}{2a_2} - 4a_3\sqrt[-3]{P})z - a_8 t\}$$

and $P = (4a_2a_8 + 4\sqrt{\frac{a_2^3a_8^2 + 32a_3^3}{a_2}})a_2^2$.

Fig.1 shows the sketch of lump waves occurs periodically for the values $a_2 = 2,\ a_3 = a_8 = 1, l_1 = l_2 = 1$, (a) gives 3D views from which one can reveal the lump wave or one dimensional rogue wave feathers in the $xt-$plane at $y = z = 0$. It is also clear that the Fig.1 of Eq. (8) is the familiar eye-shaped lump wave solution which has a local deep whole and a height peak (clears from the views (b)) in each lump wave. Besides this, we discover that lump wave has the uppermost peak in its surrounding waves. The figures in the other plane exhibits similar characteristics but periodicity of lump may differ.

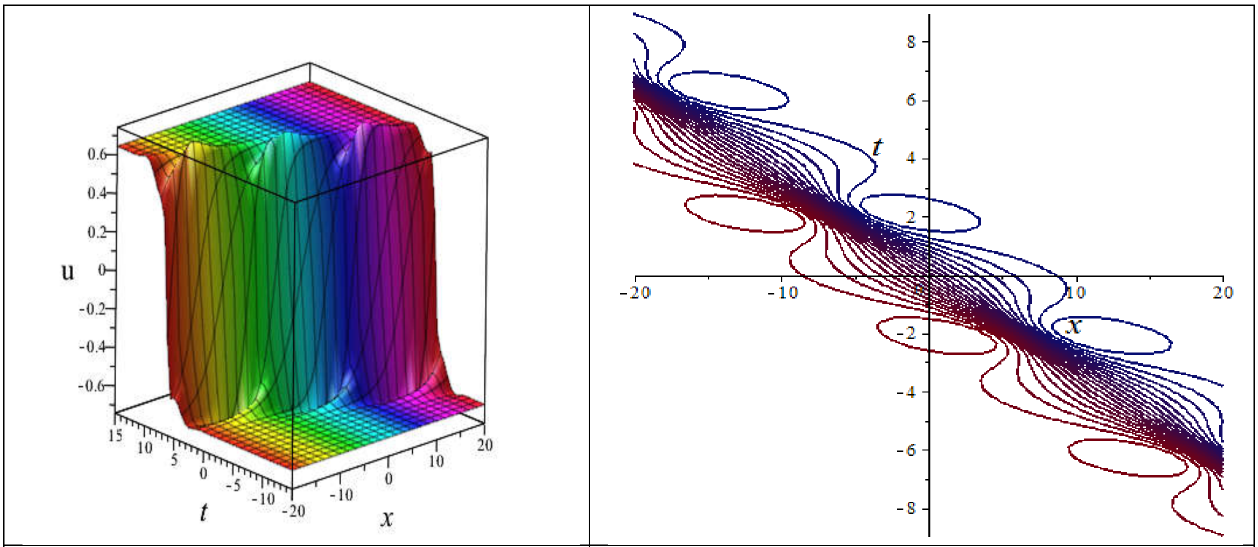

Fig 1. Lump wave solution (8) for Eq. (1) by choosing suitable parameters: $a_2 = 2,\ a_3 = a_8 = 1, l_1 = l_2 = 1$.**(a) Perspective view of the wave at** $y = z == 0$. **(b) Corresponding contour plot of the wave.**

Set-2: $a_1 = 0,\ a_4 = -\frac{3a_3^2}{a_2}, a_5 = \frac{\sqrt[3]{P}}{2a_2} + 4a_3\sqrt[-3]{P}, a_6 = 0, a_7 = \frac{\sqrt[3]{P}}{2a_2} + 4a_3\sqrt[-3]{P}$, (9)

where $P = (4a_2a_8 + 4\sqrt{\frac{a_2^3a_8^2 - 32a_3^3}{a_2}})a_2^2$ and a_2, a_3, a_8 are arbitrary constants.

Therefore, substituting Eq. (9) and Eq. (6) along with Eq.(2) yields the following periodic lump wave solution,
$u = 2(\ln f)_x$, (10)

Where

$$f = l_1\cos(a_2 y + a_3 z - \frac{3a_3^2}{a_2}t) + \exp\{(\frac{\sqrt[3]{P}}{2a_2} - 4a_3\sqrt[-3]{P})x + (\frac{\sqrt[3]{P}}{2a_2} - 4a_3\sqrt[-3]{P})z + a_8 t\}$$
$$+ l_2\exp\{-(\frac{\sqrt[3]{P}}{2a_2} - 4a_3\sqrt[-3]{P})x - (\frac{\sqrt[3]{P}}{2a_2} - 4a_3\sqrt[-3]{P})z - a_8 t\}$$

where $P = (4a_2a_8 + 4\sqrt{\dfrac{a_2^3a_8^2 - 32a_3^3}{a_2}})a_2^2$ and a_2, a_3, a_8 are arbitrary constants.

Fig.2 shows the sketch of lump waves occurs periodically for the values $a_2 = 2,\ a_3 = a_8 = 1, l_1 = l_2 = 1$, (a) gives 3D views from which one can reveal the lump wave or one dimensional rogue wave feathers in the $xt-$plane at $y = z = 0$. It is also clear that the Fig.2 of Eq. (10) is the familiar eye-shaped lump wave solution which has a local deep whole and a height peak (clears from the views (b)) in each lump wave. Besides this, we discover that lump wave has the uppermost peak in its surrounding waves. The figures in the other plane exhibits similar characteristics but periodicity of lump may differ (see Fig-2(c) and (d) in the $xy-$plane).

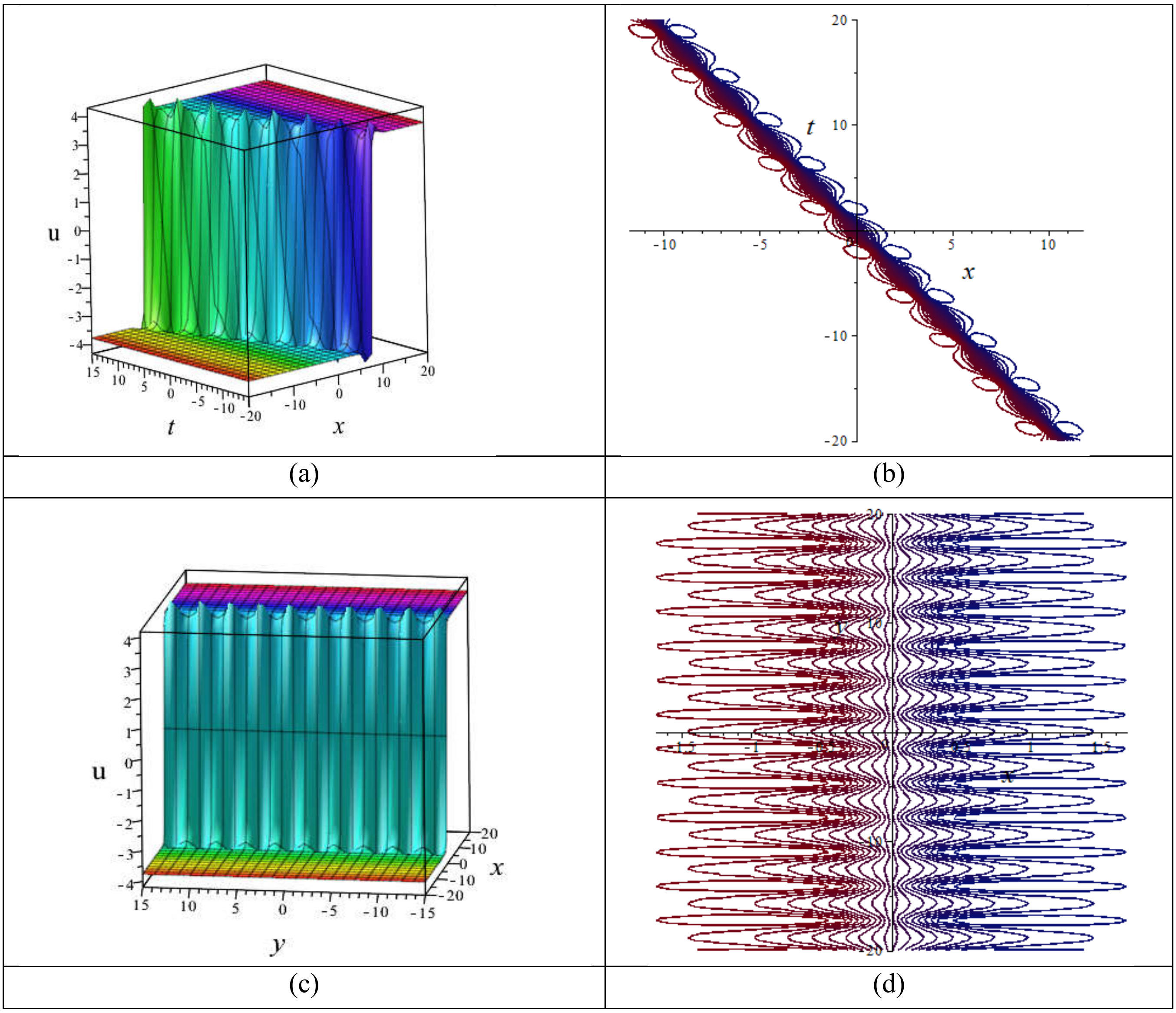

Fig. 2. Lump wave solution (10) for Eq. (1) by choosing suitable parameters: $a_2 = 2,\ a_3 = a_8 = 1, l_1 = l_2 = 1$.(a) Perspective view of the wave at $y = z == 0$. (b) Corresponding contour plot of the wave.

3.Conclusions

This paper focuses based on the Hirota bilinear process, we have adequately offered two collision phenomena between a solitary type lump wave and a periodic cosine function solution to the (3+1)-dimensional nonlinear evolution equation. The lump wave comes in term of two exponentials and periodicity comes in term of cosine function and after collision the interaction exhibits as periodic breather type periodic lump waves. Also the results have been presented graphically via 3D plot, contour plots to realize the real dynamics of the interactive waves. These outcomes are incorporated to visualize the dynamics of the obtained wave solutions in the study of water waves in mathematical physics and engineering phenomena.

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