ISSN: 0975-833X

Available online at http://www.journalcra.com

International Journal of Current Research
Vol. 12, Issue, 06, pp.11991-11999, June, 2020

DOI: https://doi.org/10.24941/ijcr.39032.06.2020

INTERNATIONAL JOURNAL OF CURRENT RESEARCH

RESEARCH ARTICLE

EVALUATING HOW DATA OF "RETENTION LIMITS FOR SAUDI INSURANCE MARKET" FITS A PROGRESSIVE TYPE-II CENSORED SAMPLE FOR WEIBULL GENERALIZED EXPONENTIAL DISTRIBUTION

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ARTICLE INFO

Article History:
Received 20th March, 2020
Received in revised form
09th April, 2020
Accepted 17th May, 2020
Published online 29th June, 2020

Key Words:

Retention Limits, Saudi Insurance Market, Reinsurance, Risk Management, Weibull Generalized Exponential Distribution, censored sample, Maximum Likelihood Estimation, MCMC.

ABSTRACT

Aims: The main aim is to determine the optimal orientation of retention rates and then determine reinsurance shares for the various insurance branches in the Saudi insurance market by building statistical modelling with model parameters by which risk factors can be monitored according to the insurance branch, as well as forecasting growth or possible contraction in retention rates by Insurance branch in the Saudi insurance market. **Place and Duration of Study:** Retention rates by insurance activity in the Saudi insurance market for the year 2018 and up. **Methodology:** Weibull Generalized Exponential distribution parameters based on censored samples have been discussed by using Maximum Likelihood, Bayesian Estimation based on the Markov Chain Monte Carlo method. **Results:** An increase in the risk factors for instability in the retention rates with the most extreme values, whether the rate is more extreme, increased, or decreased. Represented in branches; Health insurance, Energy insurance, Vehicle insurance, Protection and savings, an increase in the rate of retention for branches; Energy insurance and Aviation insurance, insurance branches whose retention rate is expected to decrease are Health insurance followed by Vehicle insurance, Bayesian Estimation is better and more efficient than the MLE and MPS estimation. **Conclusion:** In approximately most of the situations, we notice that the measures of Bayesian estimates are preferable than the measures of MLE estimates. As the data of retention limits by activity in the Saudi insurance market are fitting to the model and how the schemes work in practice, effectiveness in determining retention limits "Reinsurance" ensures balanced financial performance and stable profitability level

Citation: ***Hany A Saleh, Ehab M. Almetwally and Hisham. M. Almongy, 2020.*** "Evaluating How Data of "Retention Limits for Saudi Insurance Market" fits a Progressive Type-II Censored Sample for Weibull Generalized Exponential Distribution", *International Journal of Current Research*, 12, (06), 11991-11999.

INTRODUCTION

The retention limit is one of the important factors that helps the insurance institution achieve its goals. This is because the retention limit aims to increase the outputs with the stability of the inputs, which leads to an increase in insurance operations and thus an increase in profits. Determining the "retention threshold" is one of the most difficult issues facing a direct insurance company when building a reinsurance strategy. Although there are multiple methods and models that can be relied upon to determine the extent of retention, most companies fear due to the direct impact on their profitability.

Optimum retention limit criteria:

The insurance institution is supposed to observe the following when determining the retention limit:

- The extent of the insurance company's ability to cope with losses.
- Determine the maximum possible total losses that the company can bear.
- Build a probability distribution of losses.
- Build a probability distribution of expected losses in the future.
- Determine all reliable sources to face losses.
- Study the types of optimum reinsurance agreements.
- The most important determinants that affecting in determining the retention limit are:
- Portfolio size.

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- Probability and severity of loss.
- Emergency loads.
- Investment policy.
- Reinsurance price.
- Probability of specific destruction.
- Capital.
- Provisions.
- Return rate.

The importance of this study is demonstrated in an attempt to build a statistical modeling commensurate with the nature of retention limits data according to the insurance branch on the one hand, and this modeling also achieves statistically significant parameters on the other hand. The aim of this study is to determine the optimal orientation of retention rates and then determine reinsurance shares for the various insurance branches in the Saudi insurance market by building statistical modeling with model parameters by which risk factors can be monitored according to the insurance branch, as well as forecasting growth or possible contraction in retention rates by Insurance branch in the Saudi insurance market. According to (Iqbal &Rehman, 2014a) Reinsurance could be considered as the transfer of risks from primary insurer to another insurer (named Reinsurer) by agreement under which the reinsurer agrees to indemnify the primary insurer for some or all of the financial consequences of certain loss. Reinsurance contributes to the growth of the insurance sector and then helps in development of gross economy. (Swiss Re, 2004; Iqbal &Rehman, 2014b). Reinsurance aids primary insurers to manage risks of underwriting and actuarial risks that expose to (Swiss Re, 2004; Curak, Utrobicic, &Kovac, 2014). According to (Veprauskaite and Sherris, 2012) Reinsurance appears in earnings, solvency and economic value of direct insurance institutions. Bourguignon et al. (2014) proposed Weibull generalized family of distributions using Weibull Generator. Mustafa et al. (2016) used the Weibull generalized family to generate the new distribution by assuming exponential distribution as a baseline distribution, which is denoted Weibull Generalized Exponential Distribution (WGED). Almetwally et al. (2018) discussed estimation of the WGED parameters with progressive type-II censoring schemes by using the maximum likelihood and Bayesian estimation methods. Gupta and Jamal (2019) discussed estimation of WGED parameters based on generalized order statistics and they derived the sub models of generalized order statistics such as order statistics and record values.

If a sample is drawn from a complete population, but either the last observation or the first are unknown, this case is called the single censored observation. This type of data is called censored or incomplete data. The most common used censoring schemes are Type-I censored (or time censored) and Type-II censored (or failure-censored). These two censoring schemes do not allow for units to be removed from the experiments while they are still alive. Progressive censoring is a more general censoring scheme which allows the units to be removed from the test (see Balakrishnan and Aggarwala, 2000). Progressive censoring is useful in a life-testing experiment because of it has ability to remove live units from the experiment, which saves time and money. Applications under PTIIC using different lifetime distributions have been discussed by many authors. For examples, see Dey and Dey (2014), Almetwally and Almongy (2018, 2019), Aslam et al. (2020) and El-Sherpieny et al. (2020). The methods of estimation under censored sample, it is divided into two categories: The maximum likelihood estimation (MLE) methods and the Bayesian estimation method. An important algorithm of the Bayesian method based on MCMC techniques and Gibbs sampling which are more general Metropolis within Gibbs samplers these are introduced by Metropolis et al. (1953). To more examples and application of Bayesian estimation using different lifetime distributions see Madi and Raqab (2009), Almetwaly and Almongy ($2018_{a,b}$), Almetwally et al. ($2019_{a,b}$) and Ahmad and Almetwally (2020). In view of the importance of the Weibull Generator distributions and progressive type-II censoring sample (PTIICS) in reliability studies, we consider the retention limits for the Saudi insurance market under design condition is assumed to censored sample. The Kolmogorov–Smirnov test is the standard goodness-of-fit tests see Massey (1951) and Marsaglia et al. (2003). So a statistical analysis based on the K–S goodness-of-fit test was applied to the data obtained from for retention limits Saudi insurance market to determine the probability distributions that best fit the Saudi insurance market. The paper is organized as follows: section 2 is devoted for model and method description. Section 3 is devoted for the estimation of the WGED parameters using the MLE method and Bayesian estimation method under PTIICS. A simulation study is performed to illustrate the statistical properties of the parameters in Section 4. Data of retention limits were analyzed in Section 5. Eventually, the concluded remarks are given in Section 6.

2. MODEL AND METHOD DESCRIPTION

Let x has WGED with vector of parameter $\Theta = (\alpha, \gamma, \theta)$,

Assume that its cumulative function (CF) is given by

$$F(x;\Theta) = 1 - e^{-\alpha(e^{\gamma x}-1)^{\theta}} \tag{2.1}$$

And the corresponding probability Density function is:

$$f(x;\Theta) = \alpha\gamma\theta e^{\gamma x}(e^{\gamma x}-1)^{\theta-1}e^{-\alpha(e^{\gamma x}-1)^{\theta}} \tag{2.2}$$

The quantile function of the WGE distribution is:

$$x=\frac{1}{\gamma}\ln\left(1+\left[\frac{-1}{\alpha}\ln(1-u)\right]^{1/\theta}\right).\quad 0<u<1 \tag{2.3}$$

The procedure of the Kolmogorov–Smirnov (KS) test is as follows: Given a sample of n observations,

$$KS=max|Ecd(x)-F(x;\Theta)|,$$

where $Ecd(x)=\frac{1}{n}\sum_{i=1}^{n}I_{[0,x]}$, $I_{[0,x]}$ is the indicator function and $F(x;\Theta)$ is CDF of WGED. The P-value can be calculated by $\text{P}-\text{value}=\frac{\sqrt{2\pi}}{x}\sum_{k=1}^{\infty}e^{-\pi^2\frac{(2k-1)^2}{8x^2}}$, for more information see Marsaglia et al. (2003). If the value of the P-value exceeds the critical value as 0.05, the hypothesis that the observations are from the WGED is accepted. In PTIICS, let's set that n is independent sorted observations placed on a life testing where $X_1<X_2<\cdots<X_n$ and the progressive censoring scheme $R_i, i=1,2,\ldots,m$. The number of failures m, and removal R are fixed given by experimenter. At the time of the first failure, x_1, R_1 units are randomly removed from the remaining $(n-1)$ surviving items, in the time of the second failure, x_2, R_2 units of the remaining $n-2-R_1$ units are randomly removed and so on the test continues until the m^{th} failure at which time, all the remaining $n-m-R_1-R_2-\cdots-R_{m-1}$ units are removed. The data from PTIICS is as follows $X_{1:m:n}<X_{2:m:n}<\cdots<X_{m:m:n}$.

We can write the likelihood function under PTIICS as follows:

$$L\ \ (x_{i:m:n},\Theta)=A\left(\prod_{i=1}^{m}f(x_{i:m:n},\Theta)\right)\left(\prod_{i=1}^{m}\left(1-F(x_{i:m:n},\Theta)\right)^{R_i}\right) \tag{2.4}$$

Where A is a constant which does not depend on the parameters.

3. THE MLE AND BAYESIAN ESTIMATION METHOD UNDER PTIICS

This section deals with MLE and Bayesian methods of the parameters WGED based on the PTIICS data.

3.1. MLE Method

By Eq. (2.4), the likelihood function for WGED based on PTIICS can be written as

$$L(\Theta)=A(\alpha\gamma\theta)^m e^{\gamma\sum_{i=1}^{m}x_{i:m:n}}e^{-\alpha\sum_{i=1}^{m}(e^{\gamma x_{i:m:n}}-1)^{\theta}}\prod_{i=1}^{m}\left[(e^{\gamma x_{i:m:n}}-1)^{\theta-1}\left(e^{-\alpha(e^{\gamma x_{i:m:n}}-1)^{\theta}}\right)^{R_i}\right]. \tag{3.1}$$

The natural logarithm of the likelihood function equation can be obtained as follows:

$$\ln L(\Theta)=\ln A+m\ln(\alpha\gamma\theta)+\gamma\sum_{i=1}^{m}x_{i:m:n}+(\theta-1)\sum_{i=1}^{m}\ln(e^{\gamma x_{i:m:n}}-1)-\alpha\sum_{i=1}^{m}(e^{\gamma x_{i:m:n}}-1)^{\theta}-\alpha\sum_{i=1}^{m}R_i(e^{\gamma x_{i:m:n}}-1)^{\theta}. \tag{3.2}$$

Let $l(\Theta)=\ln L_1(x_{i:m:n},\Theta)$. The partial derivatives with respect to the unknown parameters of Eq. (3.2) are given as follows:

$$\frac{\partial\ln L}{\partial\alpha}=\frac{m}{\alpha}-\sum_{i=1}^{m}(e^{\gamma x_{i:m:n}}-1)^{\theta}-\sum_{i=1}^{m}R_i(e^{\gamma x_{i:m:n}}-1)^{\theta}, \tag{3.3}$$

$$\frac{\partial\ln L}{\partial\gamma}=\frac{m}{\gamma}+\sum_{i=1}^{m}x_{i:m:n}+(\theta-1)\sum_{i=1}^{m}\frac{x_i e^{\gamma x_{i:m:n}}}{(e^{\gamma x_i}-1)}-\theta\alpha\sum_{i=1}^{m}x_{i:m:n}e^{\gamma x_{i:m:n}}(e^{\gamma x_{i:m:n}}-1)^{\theta-1}-\theta\alpha\sum_{i=1}^{m}R_i x_{i:m:n}e^{\gamma x_{i:m:n}}(e^{\gamma x_{i:m:n}}-1)^{\theta-1} \tag{3.4}$$

and

$$\frac{\partial\ln L}{\partial\theta}=\frac{m}{\theta}+\sum_{i=1}^{m}\ln(e^{\gamma x_{i:m:n}}-1)-\alpha\sum_{i=1}^{m}(e^{\gamma x_{i:m:n}}-1)^{\theta}\ln(e^{\gamma x_{i:m:n}}-1)-\alpha\sum_{i=1}^{m}R_i(e^{\gamma x_{i:m:n}}-1)^{\theta}\ln(e^{\gamma x_{i:m:n}}-1). \tag{3.5}$$

The MLE of $\widehat{\Theta}$ for the WGED parameters are the solution of non-linear equations after setting them equal zero, as known. Such as, these equations are very difficult to be solved, so, we will use nonlinear optimization algorithm as Newton–Raphson method are used.

3.2. Bayesian Estimation Method

In this section, we consider the Bayesian estimation to estimate the parameters of WGED based on PTIICS under the assumption that the random variables $\Theta = (\alpha, \gamma, \theta)$ have an independent gamma prior distributions. Assumed tha $\alpha \sim Gamma(a_1, b_1)$, $\gamma \sim Gamma(a_2, b_2)$ and $\theta \sim Gamma(a_3, b_3)$ then, the joint prior density of $\alpha.\gamma$ and θ can be written as

$$\pi(\Theta) \propto \alpha^{a_1-1} e^{-\alpha b_1} \gamma^{a_2-1} e^{-\gamma b_2} \theta^{a_3-1} e^{-\theta b_3}, \tag{3.6}$$

The posterior likelihood can be represented to be proportional to the product of likelihood Eq. (3.1) and the joint prior's densities Eq. (4.1). That is,

$$\Pi(\Theta|x_{i:m:n}) \propto L(x_{i:m:n}|\Theta)\pi(\Theta)$$

Then, the joint posterior density of Θ is

$$\Pi(\Theta|x_{i:m:n}) \propto \alpha^{m+a_1-1} e^{-\alpha b_1} \gamma^{m+a_2-1} e^{-\gamma b_2} \theta^{m+a_3-1} e^{-\theta b_3} e^{\gamma \sum_{i=1}^{m} x_{i:m:n}} e^{-\alpha \sum_{i=1}^{m} (e^{\gamma x_{i:m:n}}-1)^{\theta}} \prod_{i=1}^{m} \left[(e^{\gamma x_{i:m:n}} - 1)^{\theta-1} \left(e^{-\alpha (e^{\gamma x_{i:m:n}}-1)^{\theta}} \right)^{R_i} \right], \tag{3.7}$$

Squared error (SE) loss function: The main symmetric loss function is the SE loss function that defined by $\ell(\widetilde{\Theta}, \Theta) = (\widetilde{\Theta} - \Theta)^2$. Bayes rule leads to the estimator Θ which is called Bayes estimator, if the SE loss function is applied. The usual estimator of the parameters under the SE loss function is the posterior mean. Therefore, the Bayesian estimators of the parameters Θ under SE, say $\widetilde{\Theta}$ are obtained, as, posterior mean as follows:

$$\tilde{\alpha} = \int_0^{\infty} \alpha \Pi(\Theta|x_{i:m:n}) d\alpha, \tilde{\gamma} = \int_0^{\infty} \gamma \Pi(\Theta|x_{i:m:n}) d\gamma, \tilde{\theta} = \int_0^{\infty} \theta \Pi(\Theta|x_{i:m:n}) d\theta$$

These integrals are very hard to be solved analytically, so the MCMC approach will be used. An important sub-class of the MCMC techniques is Gibbs sampling and more general Metropolis within Gibbs samplers. Metropolis et al. (1953) and Hastings (1970) were first introduced this algorithm. The Metropolis-Hastings (MH) algorithm together with the Gibbs sampling are the two most popular two examples of a MCMC method. It's similar to acceptance-rejection sampling, the MH algorithm consider that, to each iteration of the algorithm, a candidate value can be generated from a proposal distributions. The MH algorithm generates a sequence of draws from WGED under PTIICS as follows:

Start with any initial values $\left(\Theta_l^{\langle 0 \rangle}\right); \Theta = (\alpha, \gamma, \theta); l = 1,2,3$ satisfying $\pi\left(\Theta_l^{\langle 0 \rangle}\right) > 0$.

Using the initial value, sample a candidate point $\left(\Theta^*\right)$ from proposal $q\left(\Theta^*\right)$.

For $t = 0$ to N (a huge number 10,000, for example)

Given the candidate point $\left(\Theta^*\right)$, calculate the acceptance probability

$$A_l = \min\left(1, \frac{L_1(\Theta_l^*|x)\pi\left(\Theta_l^*\right)}{L_1(x|\Theta_l)\pi\left(\Theta_l\right)} \frac{q\left(\Theta_l\right)}{q\left(\Theta_l^*\right)}\right); l = 1,2,3$$

Draw a value of u from the uniform (0,1)distribution, $\Theta_l^{\langle t+1 \rangle} = \begin{cases} \Theta_l^* & if \quad u \le A_l \\ \Theta_l^{\langle t \rangle} & if \quad u \le A_l \end{cases}$.

Repeat steps 2-5 $t+1$ times until we get N draws.

The Bayes estimate of Θ_l, with respect to squared errorloss function is $\sum_{t=1}^{N} \frac{\left(\Theta_l^{\langle t-1 \rangle}\right)_t}{N}$.

Repeat this steps l to get Bayesian estimate of Θ_l.

4. SIMULATION STUDYN

In this section; Monte Carlo simulation is done for comparison between maximum likelihood and Bayesian estimation methods under censoring scheme, for estimating parameters of WGED in life time by R programme. Monte Carlo experiments were carried out based on the following data- generated from WGED, where xare distributed as WGED for different shape parameters:

Case-I: $(\alpha = 4;\ \gamma = 2;\ \theta = 3.5)$, case-II: $(\alpha = 0.5;\ \gamma = 1.2;\ \theta = 1.5)$.

For different sample size $n = 50, 100\ and\ 200$, different censored sample sizes m as 30, 40; 70, 80, 90; 150, 165, 175, 185 and set of different samples schemes, where

- Complete: $R_1 = R_2 = \cdots = R_{m-1} = 0, and\ R_m = 0$. It is complete scheme
- Scheme I: $R_1 = R_2 = \cdots = R_{m-1} = 0, and\ R_m = n - m$. It is type-II scheme
- Scheme II: $R_1 = n - m\ and\ R_2 = R_3 = \cdots = R_{m-1} = 0$.

We could define the best scheme as the scheme, which, it has minimizes the mean squared error ($MSE\ (\Theta)$) and bias of estimation. The CI of MLE and Bayesian estimation.

The simulation outcomes are recorded in Tables 4 and 5, Appendix I. The following concluding remakes are noticed based on these tables as follows

- As m increases and for fixed values of n, the Bias and MSE associated with the parameter estimates decrease for both methods of estimation.
- For fixed values of m the Bias and MSE associated with the parameter estimates decrease for both methods of estimation as n increases.
- In approximately most of situations, we notice that the measures of Bayesian estimates are preferable than the measures of MLE estimates.

5. APPLYING WITH REAL INSURANCE DATA:

We will use retention limits data according to the insurance activity of the Saudi insurance market (2018) to present the numerical results of the parameter estimation of WGED under ATIIP. These data are as follow:

Table 1: Retention limits percentages by insurance activity for Saudi insurance market (2018)

Insurance branch	Retention Limit (%)
Accidentand liabilities insurance and others	47
Vehicle insurance	94
Property and fire insurance	18.5
Marine insurance	28.5
Aviation insurance	3.6
Energy insurance	0.7
Engineering insurance	17.1
Health insurance	97.2
Protection and savings	72.1

We computed the KS distance between the fitted and the empirical distribution functions for the data is 0.1351 and the corresponding p-value is 0.9886. In the following Figure (1) are discussed plot of max distance between two ecdf curves, histogram, PP-plot and QQ-plot for WGED. Therefore, it indicates that WGED can be fitted to the insurance data set.

Table 2-a: Estimation of coefficient and stander error for Complete for insurance data

	MLE		Bayesian	
	Estimate	St.E	Estimate	St.E
α	0.4040	0.3922	0.5831	0.3205
γ	0.0357	0.03349	0.03315	0.0183
θ	0.5416	0.29707	0.54197	0.1990

Using the estimated model parameters under the complete sample, some probabilities can be calculated. For example, a researcher wants to know what the probability of the retention rate for accidents is? The probabilities related to these different cases are calculated for the retention rate by activity type in insurance data of the Saudi insurance market. The prediction of this probabilities is reported in Table 2-b.

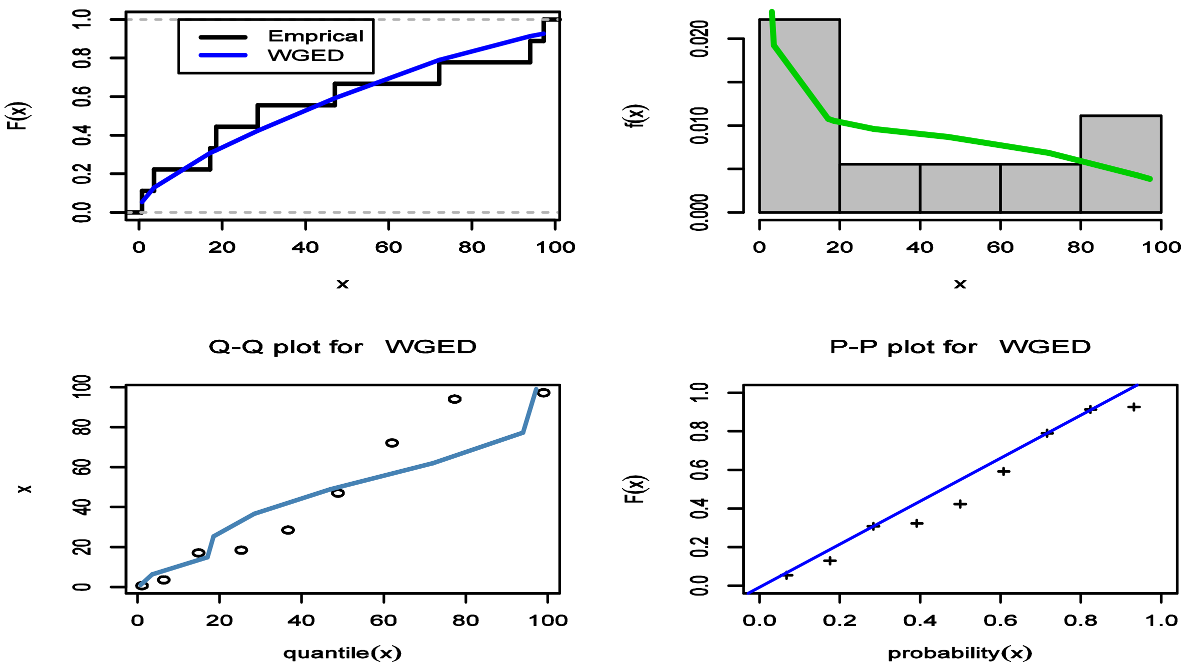

Figure 1. Plot of max distance between two ecdf curves, histogram, PP-plot and QQ-plot.

Table 2-b: Possibilities for change and risk factors

	MLE			Bayesian		
X	$P(x_{i-1} < X < x_i)$	$P(X > x_i)$	Risk	$P(x_{i-1} < X < x_i)$	$P(X > x_i)$	Risk
0.7	0.0537	0.9463	0.0432	0.0735	0.9265	0.0598
3.6	0.0751	0.8713	0.0221	0.0999	0.8267	0.0304
17.1	0.1792	0.6921	0.0156	0.2222	0.6044	0.0209
18.5	0.0149	0.6771	0.0156	0.0174	0.5870	0.0209
28.5	0.1002	0.5769	0.0167	0.1123	0.4747	0.0219
47	0.1690	0.4079	0.0213	0.1715	0.3032	0.0272
72.1	0.1980	0.2100	0.0327	0.1707	0.1324	0.0400
94	0.1229	0.0871	0.0489	0.0865	0.0460	0.0579
97.2	0.0130	0.0741	0.0519	0.0080	0.0380	0.0612

The data under progressive censoring when m=7 and R is 1, 0, 1, 0*4. The $x_{i:m:n}$ is 0.7, 3.6, 17.1, 28.5, 47.0, 72.1, 94.0. The time of adaptive model is 50. By using KS test and MLE estimate the distance of KS test is 0.1612 and the P-value is 0.9453.

Table 3-a: Estimation and stander error for WGED based on PTIICS for insurance data

	MLE		Bayesian	
	Estimate	St.E	Estimate	St.E
α	0.3693	0.3852	0.52416	0.37966
γ	0.0407	0.0407	0.05310	0.03861
θ	0.5313	0.3130	0.4985	0.2515

Table 3-b: Possibilities for change and risk factors

	MLE			Bayesian		
y	$P(x_{i-1} < X < x_i)$	$P(X > x_i)$	Risk	$P(x_{i-1} < X < x_i)$	$P(X > x_i)$	Risk
0.7	0.0546	0.9454	0.0433	0.0974	0.9026	0.0744
3.6	0.0747	0.8707	0.0220	0.1168	0.7857	0.0367
17.1	0.1802	0.6905	0.0160	0.2569	0.5288	0.0283
18.5	0.0153	0.6751	0.0161	0.0206	0.5081	0.0286
28.5	0.1040	0.5712	0.0176	0.1346	0.3736	0.0334
47	0.1795	0.3917	0.0238	0.1985	0.1751	0.0503
72.1	0.2100	0.1817	0.0390	0.1448	0.0303	0.0946
94	0.1200	0.0616	0.0616	0.0285	0.0019	0.1676
97.2	0.0114	0.0503	0.0659	0.0008	0.0011	0.1823

4. CONCLUSION AND RECOMMENDATION

In this paper, to evaluate how data of retention limits according to the insurance activity of the Saudi insurance market can be modelled, we discussed MLE and Bayesian estimation to estimate parameters problem of the WGED based on PT IICS. We used Bayesian estimation under square error loss function to estimate the unknown parameters for WGED under the assumption of independent gamma priors. The performance of the different estimator's optimal censoring schemes is compared based on simulation study to determine the optimal censoring schemes by using MSE and the bias. It is noticeable that the Bayesian estimation is better and more efficient than the MLE estimation. The application study shows that data of retention limits by activity for Saudi insurance market are fitting to the model and how the schemes work in practice. According to both MLE and Bayesian, the results reveal an increase in the risk factors for instability in the retention rates with the most extreme values, whether the rate is more extreme, increased or decreased. Represented in branches; Health insurance, Energy insurance, Vehicle insurance, Protection and savings. The results also reflect the expectation of an increase in the rate of retention for branches; Energy insurance and Aviation insurance. Also, the results indicate that most of the insurance branches whose retention rate is expected to decrease are Health insurance followed by Vehicle insurance. Effective reinsurance implementation ensures that the Direct Insurance Institute has a balanced financial performance and ensures a stable profitability level. This stems from the premise of the role of the reinsurance program in reducing underwriting risks, as well as in the face of fluctuations in loss rates, therefore, the most important recommendation that is the focus of this study is building an ideal reinsurance program means a successful "financial performance" of the insurance branch.

ACKNOWLEDGEMENTS: We Also Thank Anonymous For Their Encouragement And Support.

COMPETING INTERESTS: The Authors Declare That They Have No Known Competing Financial Interests Or Personal Relationships That Could Have Appeared To Influence The Work Reported In This Paper.

AUTHORS' CONTRIBUTIONS

'Author A' designed the study, wrote the protocol, managed the literature searches, and wrote the first draft of the manuscript. 'Author B' and 'Author C' performed the statistical analysis. All authors managed the analyses of the study & read and approved the final manuscript.

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APPENDIX

Table 4. Parameter estimation by using MLE and Bayesian method for WGED under PTIICS in case-I and II

				$\alpha = 4; \gamma = 2; \theta = 3.5$				$\alpha = 0.5; \gamma = 1.2; \theta = 1.5$			
				MLE		Bayesian		MLE		Bayesian	
n	scheme	m		Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
			α	0.081	0.189	0.0719	0.1466	0.4942	0.5943	0.4568	0.4257
		30	γ	-0.6865	0.4983	-0.679	0.4849	-0.6389	0.4828	-0.6296	0.4587
	I		θ	-0.898	0.985	-0.8917	0.9551	-0.2245	0.0948	-0.2345	0.081
			α	0.0719	0.1329	0.0619	0.1092	0.4589	0.4612	0.3957	0.3524
		40	γ	-0.422	0.197	-0.4144	0.1877	-0.5062	0.2807	-0.498	0.2684
			θ	-0.6731	0.6307	-0.6717	0.596	-0.1481	0.0569	-0.1364	0.0453
			α	-0.0296	0.4369	-0.0046	0.351	0.0578	0.0586	0.0798	0.0243
50		30	γ	0.0451	0.0604	0.0335	0.0586	-0.0107	0.0698	-0.0844	0.018
	II		θ	0.2741	0.4071	0.2571	0.3439	0.1462	0.1563	0.1238	0.1218
			α	0.0259	0.1569	0.0013	0.264	0.0187	0.0839	0.0176	0.0169
		40	γ	0.0187	0.0589	0.0119	0.0111	-0.0122	0.0241	-0.0321	0.0152
			θ	0.165	0.2136	0.1466	0.2458	0.1295	0.0981	0.1022	0.0945
			α	-0.0132	0.1162	-0.0132	0.1052	0.0749	0.1083	-0.047	0.0283
	complete	50	γ	0.0426	0.0284	0.0306	0.0103	0.0875	0.1867	0.0753	0.095
			θ	0.0862	0.2031	0.0775	0.1905	0.1046	0.041	0.0665	0.0355
			α	0.0439	0.1584	0.0251	0.0109	1.0879	1.4719	1.0416	1.2968
		70	γ	-0.6327	0.4148	-0.6292	0.4081	-0.7374	0.5554	-0.7313	0.5509
			θ	-1.0249	1.128	-1.0162	1.1014	-0.2117	0.0631	-0.2011	0.0562
			α	0.0598	0.1429	0.052	0.0079	0.9185	1.3903	0.9146	1.1724
	I	80	γ	-0.4759	0.2385	-0.4692	0.2299	-0.6714	0.465	-0.6614	0.4583
			θ	-0.8765	0.8514	-0.8677	0.8182	-0.1527	0.0389	-0.1428	0.0332
			α	0.032	0.0851	0.0305	0.0067	0.928	1.1976	0.9128	1.102
		90	γ	-0.2733	0.0829	-0.2704	0.0795	-0.5251	0.2982	-0.5117	0.2909
			θ	-0.5932	0.4407	-0.5885	0.4127	-0.0608	0.0204	-0.0464	0.015
			α	0.0624	0.1462	0.0614	0.0566	0.1312	0.1108	0.1342	0.0992
100		70	γ	0.0163	0.0061	0.0107	0.0046	-0.0311	0.0716	0.0042	0.0699
			θ	0.1226	0.1387	0.1054	0.1277	0.0999	0.0502	0.0863	0.0492
			α	-0.0035	0.1253	-0.0251	0.0316	0.306	0.0918	0.2571	0.0814
	II	80	γ	0.0118	0.0057	0.011	0.0037	-0.1375	0.0639	-0.0922	0.061
			θ	0.1007	0.1222	0.0924	0.0952	0.1769	0.0467	0.155	0.0386
			α	-0.0023	0.0843	0.0007	0.03	0.2108	0.0823	0.0675	0.071
		90	γ	0.009	0.0046	0.0029	0.0032	0.0538	0.0611	0.0303	0.0578
			θ	0.0739	0.118	0.0538	0.0876	0.1005	0.0409	0.0602	0.0274
			α	-0.0081	0.0791	-0.008	0.0155	-0.1418	0.0325	-0.0869	0.027
	complete	100	γ	0.0089	0.0042	0.0063	0.0028	0.0288	0.0165	0.0216	0.0124
			θ	0.0509	0.0936	0.0395	0.0775	-0.0512	0.0213	-0.0454	0.0167

Table 5. Parameter estimation by using MLE and Bayesian method for WGED under PTIICS in case-I and II when n=200

			$\alpha=4;\ \gamma=2;\ \theta=3.5$				$\alpha=0.5;\ \gamma=1.2;\ \theta=1.5$			
			MLE		Bayesian		MLE		Bayesian	
scheme	m		Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
I	150	α	0.9338	2.3077	0.8932	2.0675	1.0798	1.3039	0.9867	1.0348
		γ	-0.6798	0.4786	-0.674	0.4693	-0.7258	0.5337	-0.7241	0.5277
		θ	-1.0438	1.128	-1.0411	1.1177	-0.2348	0.0647	-0.231	0.061
	165	α	0.6676	2.0651	0.6175	1.9661	1.0697	1.1309	0.9502	1.0119
		γ	-0.523	0.29	-0.5206	0.2861	-0.6604	0.439	-0.6567	0.44
		θ	-0.9075	0.8636	-0.9	0.8444	-0.1794	0.0412	-0.1742	0.0372
	175	α	0.4713	0.9315	0.4582	0.8606	0.9572	0.9085	0.9383	0.8762
		γ	-0.4082	0.1803	-0.4063	0.1769	-0.5932	0.3503	-0.5831	0.3564
		θ	-0.7822	0.6581	-0.7775	0.6377	-0.1258	0.0232	-0.1147	0.0186
	185	α	0.2035	0.3332	0.2013	0.3058	0.8145	0.7928	0.7987	0.7379
		γ	-0.2651	0.0777	-0.2628	0.0753	-0.4982	0.2597	-0.4969	0.2572
		θ	-0.5962	0.4033	-0.5888	0.3813	-0.0562	0.0122	-0.0518	0.0097
II	150	α	-0.0133	0.2117	-0.0058	0.0757	1.0191	1.217	0.9922	1.1601
		γ	0.0137	0.0053	0.0132	0.0034	0.01	0.3528	-0.0195	0.341
		θ	0.0643	0.0716	0.0614	0.0604	0.0599	0.0863	0.0605	0.0833
	165	α	-0.0059	0.1356	0.0013	0.0212	0.9407	0.9771	0.9144	0.9376
		γ	0.003	0.0034	0.0029	0.0017	-0.0079	0.3104	-0.0085	0.324
		θ	0.0298	0.0616	0.0276	0.0463	0.0574	0.0786	0.0546	0.0802
	175	α	-0.0099	0.0972	-0.0091	0.0138	0.6108	0.514	0.5913	0.4822
		γ	0.0053	0.0029	0.005	0.0017	0.026	0.291	0.0312	0.3084
		θ	0.0144	0.0594	0.0143	0.0405	0.0381	0.0725	0.0317	0.0755
	185	α	-0.0012	0.0346	0.0013	0.0064	0.4212	0.4054	0.4122	0.3911
		γ	0.0049	0.0023	0.0016	0.0016	0.0405	0.2338	0.0376	0.2134
		θ	0.026	0.056	0.0144	0.0402	0.0366	0.0671	0.0329	0.0658
complete	200	α	0.0077	0.0092	-0.0009	0.0043	0.0414	0.0755	0.4011	0.0745
		γ	0.0024	0.0021	0.0023	0.0012	0.2246	0.0356	0.2152	0.0329
		θ	0.0085	0.0422	0.0081	0.0366	-0.0052	0.0102	-0.0049	0.0072

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