RESEARCH ARTICLE

A REVIEW OF MATHEMATICAL MODELS OF ANAEROBIC DIGESTION AND THEIR CON-TRIBUTIONS TO ENERGY CONVERSION OF WASTES FROM THE PROCESSING OF AGRO-FOOD PRODUCTS

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ABSTRACT

The objective of this work is to take advantage of existing mathematical models of anaerobic digestion processes in order to devise a new model that will optimize the methanization of biomass waste from the processing of agro-food products in Benin. Our arguments are based, on the one hand, on principles of energy conversion processes, the application of physical-chemical laws and the law of conservation of matter, and on the other hand, on the features of existing models that yield optimal energy conversion of waste from the processing of agro-food products. Several mathematical models of anaerobic digestion have been identified, described and analyzed in the context of energy conversion of agro-food waste. These models differ in the number of digestion steps considered and/or in the digestion operating mode. The analysis shows that for some models, the biomasses used are agro-food products or waste products, while for other models no biomass is used. Moreover, when the growth rate of bacteria is an increasing, positive and bounded function, we observe that the energy conversion rate is optimal shortly before half of the conversion time. For other models, the dilution rate, the substrate concentration and the volatile fatty acid concentration are the optimization parameters for energy conversion.Finally,all the models identified, described and analysed in the context of energy conversion of agri-food waste, those of Arras and Salih are developed when the digester is in batch operation. These models differ in the digestion mode and the number of digestion steps considered.

INTRODUCTION

In Benin, the processing of agro-food products generates enormous waste in nature. This waste is stocked inappropriately and becomes harmful to health and the environment. At the same time, the population has difficulties in accessing this energy resources. In fact, this waste is a source of renewable energy and it is good to transform it, as much as possible, to energy. Considerable efforts have been made to value really this waste into biogas in order to solve the problem of waste management on the one hand, and that of lack to energy on the other hand. In this line, we have the pyrolysis of waste into gas, biochar and pyrolysis oil (Godjo, 2015); the gasification of waste into syngas and electricity (Godjo, 2017) and the bio digestion of waste into biogas (Arras, Bernard *et al*., 2001; Ouchtout). Despite of some results (a production of 27-93.7% by bio digester and 80% by gasifier (Godjo, 2017), it must be noted that, on the engineering level, there are serious problems of low energy production and low methanogenic power of biogas.

Therefore, engineers are striving to develop new technologies but their design and validation require enormous efforts in terms of economy and work time in absence of mathematical models that are able to predict the performance of these technologies before the implementation and experimentation phase. The purpose of this paper is to develop an analytical tool to help engineering optimize the energy conversion process of these wastes, by starting with the identification, description and analysis of existing tools. In particular, we shall focus on the mathematical models of anaerobic digestion, which is one of the processes of energy conversion of biomass. Also called methanization, anaerobic digestion is a natural process of transformation of organic matter by distinct micro-organisms in the absence of oxygen (Maystre, 1994). It consists generally of four phases: hydrolysis, acidogenesis, acetogenesis and methanogenesis and therefore allows the reduction of waste to produce energy. It is influenced by many factors such as the PH, the temperature, the dampness of wastes biomass and their physical and chemical characteristics (Rouez, 2008).

Different configurations of digesters can be adopted in anaerobic digestion. Depending on the origin of material and the desired result at the end of digestion, the biomass can be digested continuously or batchwise, in low or high temperature, in one, two, three or four stages (Arras, 2017). In continuous anaerobic digestion mode, the digester is usually fed with a constant flow rate and the digestate is discharged by mechanical action. This technology is ideal for large installations. The digesters used can be arranged both vertically and horizontally. In semi-continuous mode, the digesteris gradually filled according to the progress of the reactions to avoid overloading and promote growth (Hess). In batch operation of anaerobic digestion, the substrate is put into the digester at the beginning and the digester is hermetically sealed during the whole transformation period. When the amount of biogas drops or when there is no more biogas production, the digester is emptied and the process is started again (Donoso-Bravo, 2011). This is how anaerobic digestion is mostly carried out today in most developing countries and particularly in Benin. In controlling and optimising anaerobic digestion, mathematical models have been developed. Thus, some models consider one or two stages (Bernard, 2001; Hess; Rincón, 2014; Ouchtout), others three, four or five stages depending on the case (Fekih-Salem, 2011; Jérôme, 2014 ; Daoud, Harmand, 2014). The majority of these models are developed when the digester operates continuously. In the following, we will present, descriptions and contribution to support, these existing mathematical models of anaerobic digestion.

Model formulated by Bernard *et al*., (2001): The dynamics inside a digester were modeled by (Bernard, 2001). They assumed bacterial population is divided into two groups of bacteria with homogeneous characteristics. Anaerobic digestion is described by a two-step process (acidogenesis and methanogenesis). It's assumed biomass is uniformly distributed in the digester. Their model governing the interactions between populations in the digester is as follows:

$$
\begin{aligned}
\frac{dX_1}{dt} &= (\mu_1(S_1) - \alpha D)X_1 \\
\frac{dX_2}{dt} &= (\mu_2(S_2) - \alpha D)X_2 \\
\frac{dZ}{dt} &= D(Z_{in} - Z) \\
\frac{dS_1}{dt} &= D(S_{1in} - S_1) - k_1 u_1(S_1)X_1 \\
\frac{dS_2}{dt} &= D(S_{2in} - S_2) + k_2 u_1(S_1)X_1 - k_3 u_2(S_2)X_2 \\
\frac{dC}{dt} &= D(C_{in} - C) - q_C + k_4 u_1(S_1)X_1 + k_5 u_2(S_2)X_2 \\
q_M &= k_6 u_2(S_2)X_2 \\
pH &= -l_{10}\left(K_b \frac{C - Z + S_2}{Z - S_2}\right)
\end{aligned}
\tag{1}
$$

The results of 70 days of experiments are used to estimate the parameters of the model in order to validate it. First, the acidogenesis bacteria (X_1) consume the organic substrate (S_1) and produce CO_2 and volatile fatty acids (VFAs) (S_2). The biological reaction of acidogenesis is as follows:

$k_1 S_1 \xrightarrow{\mu_1 X_1} X_1 + k_2 S_2 + k_4 CO_2$ where $\mu_1 X_1$ is the rate of the reaction and μ_1 the growth rate of (X_1). Methanogenic bacteria (X_2) use the volatile fatty acids (VFAs) (S_2) for growth, and produce (CO_2) and methane (CH_4). The biological reaction of methanogenesis as:

$k_3 S_2 \xrightarrow{\mu_2 X_2} X_2 + k_5 CO_2 + k_6 CH_4$
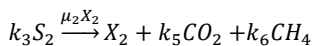
$\mu_i X_i, i = 1; 2$ refers to reaction rates and $\mu_i; i = 1; 2$ the specific growth rates of the X_i. $K_i; i = 1; 2; 3; 4; 5; 6$ are the yield coefficients associated with the reactions. q_C, q_M and pH are the flow rate of carbon dioxide, methane and pH of the medium respectively. $X_1(t), X_2(t), S_1(t), S_2(t), C(t)$ et $Z(t)$ are function of time the concentration of acidogenic bacteria (g/l), concentration of methanogenic bacteria (g/l), concentration of organic substrate(g/l), concentration of volatile fatty acids (VFAs) ($mmol/l$), the total inorganic carbon concentration($mmol/l$) and total alkalinity (this is the total concentration of dissociated acids in the medium). It is expressed as ($mmol/l$). D is the dilution ratio (day^{-1}). S_{1in}, S_{2in}, C_{in} et Z_{in} are respectively, at the inlet, concentration of organic substrate (g/l), the concentration of volatile

fatty acids (VFAs) ($mmol/l$)the concentration of inorganic carbon ($mmol/l$) and the total alkalinity ($mmol/l$). α is the fraction of bacteria in the liquid phase that leaves the reactor and K_b the acid-base equilibrium constant. For the growth of acidogenic bacteria, Monod's kinetics is considered. Thus, $\mu_1(S_1) = \mu_{1max} \frac{S_1}{S_1 + K_{S_1}}$ where μ_{1max} is the maximum growth rate of acid forming bacteria (day^{-1}), K_{S_1} is the concentration in(g/l) of S_1 for which the growth rate of acid forming bacteria is the half of the maximum (Monod). Since there are accumulation of (VFAs) that can inhibit the growth of methanogenic bacteria, Haldane kinetics is considered and thus $\mu_2(S_2) = \mu_{2max} \frac{S_2}{S_1 + K_{S_2} + \frac{S_2^2}{K_{I_2}}}$ where μ_{2max} is maximum the growth rate of the methanogenic bacteria without inhibition (day^{-1}), K_{S_2} is the concentration in($mmol/l$) of S_2 for which the growth rate of methanogenic bacteria is the half of the maximum and K_{I_2} the inhibition constant associated with S_2 (Haldane, 1964). It is expressed in($mmol/l$). Analysis of the Bernard's model in the context of energy conversion of agri-food waste. Bernard's modelis a two-step model (acidogenesis and methanogenesis). Therefore, the other stages of anaerobic digestion are neglected. The model is developed and validated under mesophilic conditions when the digester is in continuous operation using as biomass industrial wine distillery residues from local wineries in the Narbone region of France. The model is therefore developed and validated in the context of energy conversion of agri-food waste including industrial wine distillery residues. It can thus govern the anaerobic digestion of other biomasses, under the same or different conditions, from other localities or other countries.

Contribution of Bernard's model toenergy conversion: The energy conversion of these wine distillery residues to biogas and the solution dynamics of the model predicts well the results of this production for this type of biomass under the conditions of the experiment. This is of great benefit and can contribute to the optimization of this conversion. Indeed, between the 22^e and 27^e days of experiments, a maximum production of biogas is observed:100 litres per hour of CO_2and150 litres per hour ofCH_4. The conditions and reasons for this maximum production are not determined in the work of Bernard *et al* (2001). However, a mathematical analysis of their model would determine these conditions for optimizing this energy conversion.

Models formulated by Lemesle: According to Lemesle (Lemesle), the modelling of anaerobic digestion in continuous operation takes into account two parts: the physical part which describes the material flows due to the circulation of the liquid; and the biological part which describes the biological processes taking place inside the digester. Lemesle's first model is the following:

$$
\begin{cases}
\frac{dx(t)}{dt} = \mu(s)x - dx \\
\frac{ds(t)}{dt} = -\sigma(s)x + ds_{in} - ds \\
x(0) > 0 \ ets(0) \geq 0
\end{cases}
\tag{2}
$$

A second model was developed by Lemesle taking into account the mortality of cells (bacteria). By cell death, we mean degeneration of vital functions such as division. The model presented as:

$$
\begin{aligned}
\frac{dx(t)}{dt} &= \mu(s)x - mx - dx \\
\frac{ds(t)}{dt} &= -\alpha\mu(s)x + ds_{in} - d \\
x(0) &> 0 \ ets(0) \geq 0
\end{aligned}
\tag{3}
$$

Lemesle's third model takes into account maintenance, and by maintenance we mean taking into account biological mechanisms other than growth, such as survival. This phenomenon is defined as the energy required by an organism to live, excluding the processes of growth, reproduction and development. The maintenance factor is not necessarily constant and can also depend on the substrate. We thus have the third Lemesle model as:

$$\frac{dx(t)}{dt} = \mu(s)x - dx$$

$$\frac{ds(t)}{dt} = -(\alpha + M_s)\mu(s)x + ds_{in} - ds \qquad (4)$$

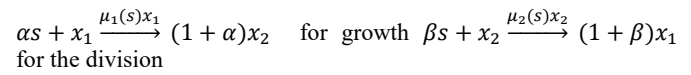$$x(0) > 0 \; et \; s(0) \geq 0$$

Finally, in Lemesle's thesis, the size-structured model of (Arino) is reported. A structured model is a dynamic system in which we choose to describe the state variables according to one or more structuring variables (size, age, etc.). This structuring can be discrete or continuous, and is due to the growth and cell division in bacteria. It should be noted that this notion of structuring is theoretical and has a meaning. It had been addressed by (Pascual, 1997; Droop, 1968; Droop, Cazzador, 1991) before its development into a size structured model by Arino (Arino). The model of (Arino) reported by Lemesle (Lemesle) is presented as follows:

$$\frac{dx_1(t)}{dt} = -\mu_1(s)x_1 + (1+\beta)\mu_2(s)x_2 - dx_1$$

$$\frac{dx_2(t)}{dt} = (1+\alpha)\mu_1(s)x_1 - \mu_2(s)x_2 - dx_2 \qquad (5)$$

$$\frac{ds(t)}{dt} = -\alpha\mu_1(s)x_1 - \beta\mu_2(s)x_2 - ds + ds_{in}$$

$$x_1(0) > 0, x_2(0) > 0 \; et \; s(0) \geq 0$$

Description of the model (2): The model takes into account two parts: The physical part and the biological part. During the physical part, the variation per unit time of a component P present in the digester, at concentration p is the difference between the quantity entering the chamber at the rate d at concentration p_{in} and the quantity leaving at the same rate, at concentration p. So $\frac{dp(t)}{dt} = dp_{in} - dp$. For bacteria X at concentration x, since there is no entry of bacteria through the flow but rather an exit, we have: $\frac{dx(t)}{dt} = -dx$

For the substrate S at the concentration s in the chamber and s_{in} at the inlet, we have: $\frac{ds(t)}{dt} = ds_{in} - ds$. For the biological part, $\mu(s)$ means the specific growth rate of the bacteria, corresponding to the absorption of a quantity s of substrate. Thus, we have: $\frac{dx(t)}{dt} = \mu(s)x$ The consumption of substrate by organisms results in a decrease in the amount of this nutrient at a rate $\sigma(s)$ called the rate of consumption. Thus $\frac{ds(t)}{dt} = -\sigma(s)x$. The grouping of these two parts gives the first model noted (1)of Lemesle. He used only Monod type functions in these models, since there is no inhibition. Description of the model (3): Theprocess of degeneration of vital functions results in a premature death of the cells. That is to say, before their "disappearance" from the digester carried away by the digestion process. It is then assumed that the residence time of the cells in the digester is longer than their life span. To define this residence time, the following concepts are used. The dilution rate d given by the ratio $\frac{Q}{V}$ where Q is the flow velocity and V the volume of the digester. The amount $T_{res} = \frac{V}{Q} = \frac{1}{d}$ is then called the residence time of the cells in the digester. Thus, if T_{res} is no longer negligible compared to the average cell lifetime, then cell mortality must be taken into account. By denoting by m the mortality rate with $0 < m < 1$ and adding the term $-mx$ to the first equation of the model (2) Lemesle obtained the model (3). The growth and consumption functions being the same, α is a positive constant that corresponds to a conversion term between the unit of substrate and the unit of bacterial concentration.

Description of the model (4): The maintenance phenomenon is modelled and taken into account by considering that more substrate is consumed. The term $-M_s\mu(s)x$ is added to the second equation of the model (2) to obtain the model (4). M_s is called the maintenance factor and $\mu(s)$ the consumption function. Description of the model (5): Here again, two parts are taken into account in the modelling (the physical part and the biological part) and two size classes of bacteria x_1 and x_2 have been chosen. The following reaction schemes represent the growth and division phenomena:

$$\alpha s + x_1 \xrightarrow{\mu_1(s)x_1} (1+\alpha)x_2 \quad \text{for growth} \quad \beta s + x_2 \xrightarrow{\mu_2(s)x_2} (1+\beta)x_1$$
for the division

For the biological part, during the growth phenomenon, α units of substrate and one unit of bacteria x_1 are consumed by the reaction when $(1+\alpha)$ units of bacteria x_2 are produced at the rate $\mu_1(s)x_1$. Therefore: $\frac{1}{-\alpha}\dot{s} = \frac{1}{-1}\dot{x}_1 = \frac{1}{1+\alpha}\dot{x}_2 = \mu_1(s)x_1$.

During the division phenomenon, β units of substrate and one unit of bacteria x_2 are consumed by the reaction when $(1+\beta)$ units of $x_1$are produced at the rate $\mu_2(s)x_2$. Therefore: $\frac{1}{-\beta}\dot{s} = \frac{1}{-1}\dot{x}_2 = \frac{1}{1+\beta}\dot{x}_1 = \mu_2(s)x_2$.

The physical part depends on material flows. Thus, $\dot{x}_1 = -dx_1$, $\dot{x}_2 = -dx_2$ et $\dot{s} = -ds + ds_{in}$

The size-structured model reported by Lemesle and noted (5) in this work is the combination of these two parts.

Analysis of Lemesle models in the context of energy conversion of agri-food waste

The Lemesle models are one-step and are developed when the digester is in continuous operation. They are based on purely theoretical considerations, including maintenance and structuring, and can contribute to the mathematical modelling of the energy conversion process of agri-food waste. Only experiments can confirm or deny these theoretical considerations. It should be noted that the parts of anaerobic digestion considered by Lemesle for the development of these models allow a better understanding of the construction process of anaerobic digestion models in general. The simulation resultsof the model (5) show the dynamics of its solutionsfor three different initial conditions.They also show that the non-trivial equilibrium point is globally convergent.

Contribution of Lemesle's models to energy conversion: The mathematical analysis of these models has identified the equilibrium points of the models and the conditions for optimizing the conversion. Thus, for model (2), the leaching equilibrium point is stable when $(s_{in}) \leq d$. Similarly, for model (5), the leaching equilibrium point is stable when $\mu_i(s)$ is an increasing, positive and bounded function with $g(s_{in}) \leq d^2$; $\mu_1 \neq \mu_2$ and $g(s) = \mu_1(s)\mu_2(s)(\alpha + \beta + \alpha\beta) - (\mu_1(s) + \mu_2(s))d$. The leaching equilibrium point is the state of the system where the population of bacteria present in the chamber disappears. That is, all the bacteria present are transformed into the final product. The simulation resultsof the model (5) show the dynamics of its solutions for three different initial conditions. They also show that the non-trivial equilibrium point is globally convergent.Given these results, it goes without saying that Lemesle models contribute well to the optimization of the conversion.

Model formulated by Radhouane: In order to find a suitable model and intended to be used in control problems to optimize anaerobic digestion processes, a three-stage model (hydrolysis, acidogenesis and methanogenesis) has been developed and analyzed by **(12).** The three-stage model is written:

$$\dot{X}_0 = DX_{0in} - \alpha DX_0 - r_0$$
$$\dot{S}_1 = D(S_{1in} - S_1) + k_0 r_0 - k_1 u_1(S_1)X_1$$
$$\dot{X}_1 = (\mu_1(S_1) - \alpha D)X_1$$
$$\dot{S}_2 = D(S_{2in} - S_2) + k_2\mu_1(S_1)X_1 - k_3 u_2(S_2)X_2 \qquad (6)$$
$$\dot{X}_2 = (\mu_2(S_2) - \alpha D)X_2$$

where $r_0 = k_{hyd}X_0$ or $r_0 = \mu_0(X_0)X_1$. As the modelling of hydrolysis is still debated, both visions, in practice, were considered: Thus, they assumed, initially, that hydrolysis is a purely enzymatic phenomenon without a hydrolytic microbial compartment. The speed of the reaction is then $r_0 = k_{hyd}X_0$ where k_{hyd} is a constant and X_0 the concentration of the slowly biodegradable substrate. Indeed, the substrate compartment is divided into two parts.
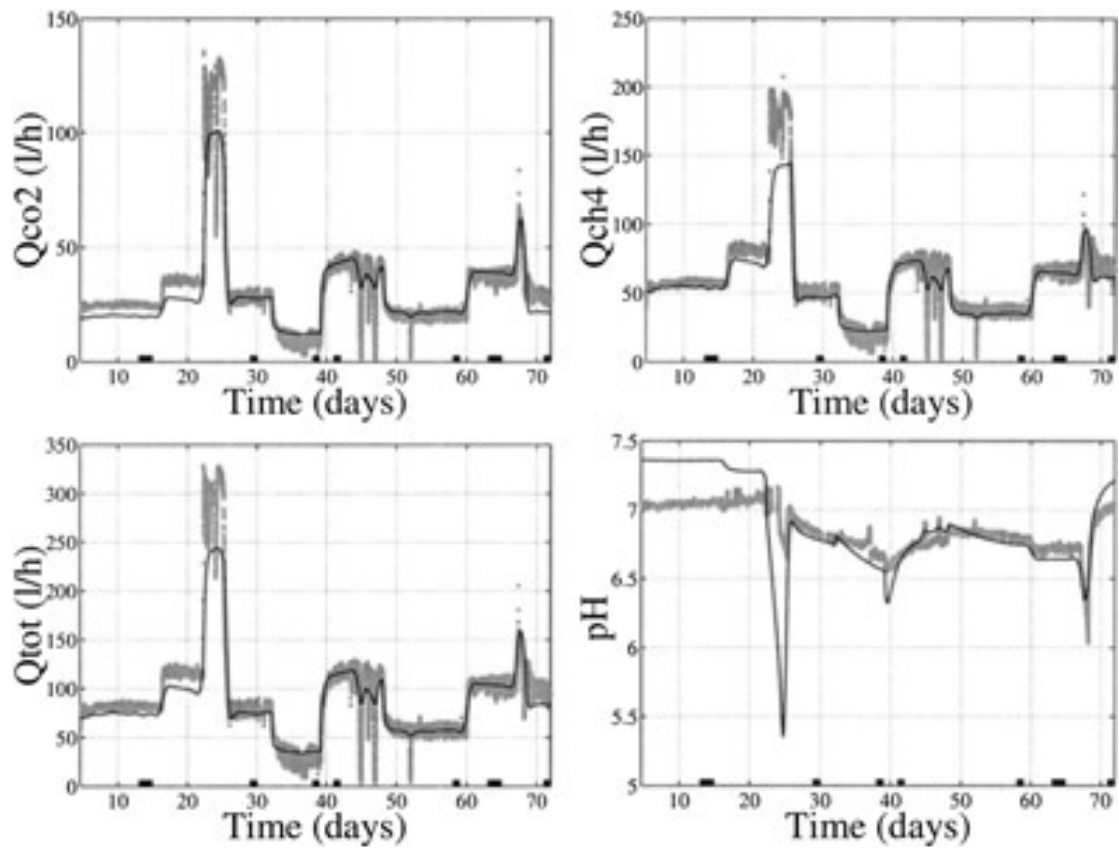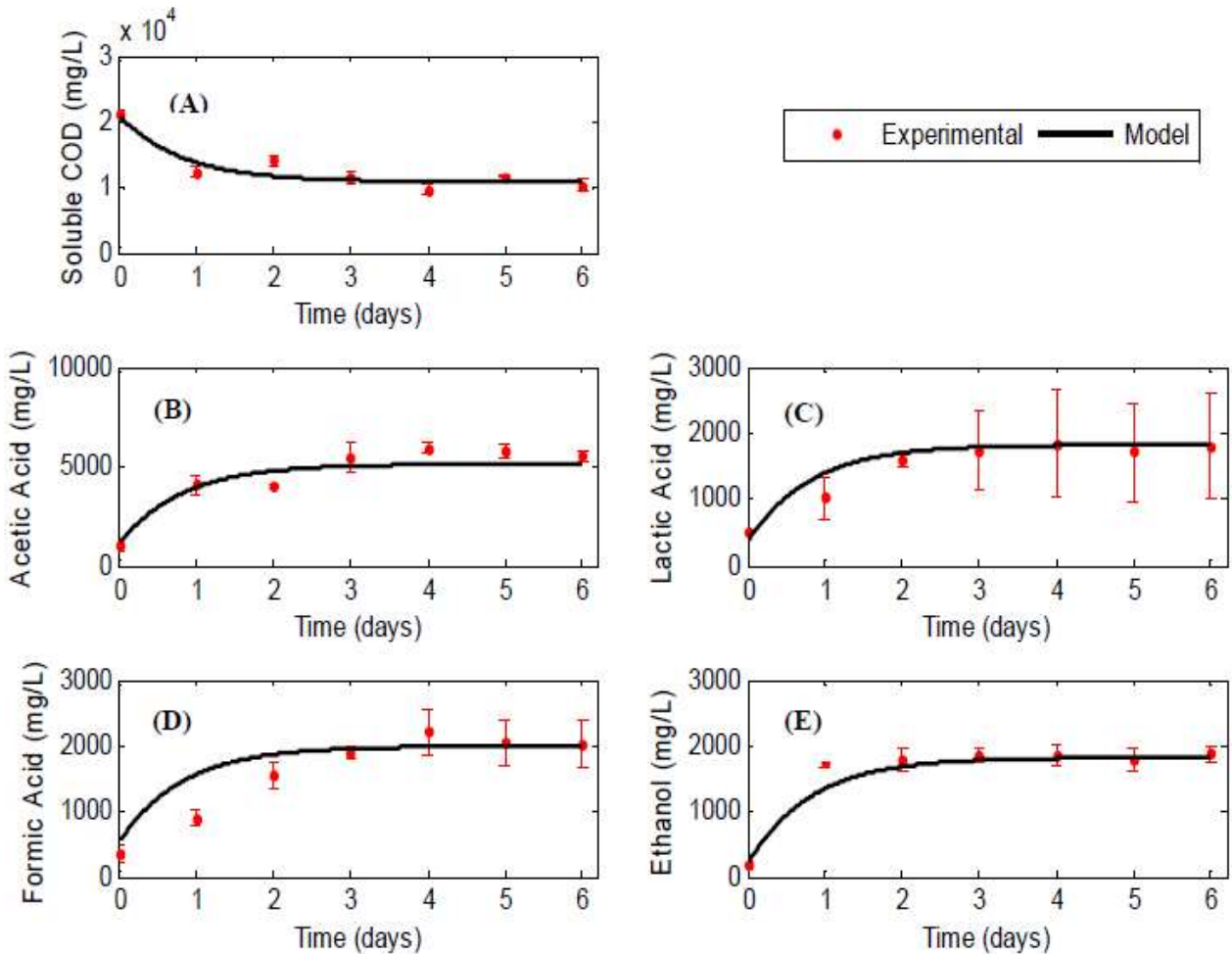
Figure 1. The gaseous flow rates and the PH (Bernard *et al*., 2001)

Figure 2. The simulation and the experimental results of the system (7) (Arras, 2017)
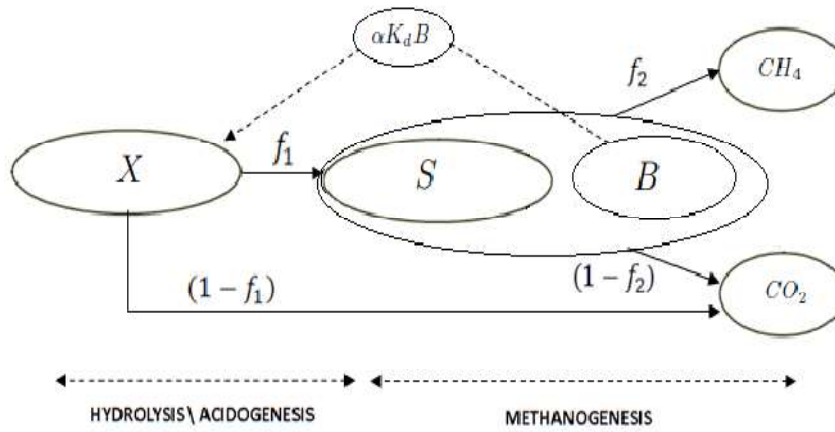
Figure 3. Schematic view of the Salih's model (Salih, 2021)

The slowly biodegradable substrate X_0 and the readily biodegradable substrate S_1. Thus, while X_1 (acid forming bacteria) degrade S_1, X_0 slowly transforms into S_1 with the speed r_0. In a second step, they explicitly involved a hydrolytic microbial compartment. That is to say that the presence of acidogenic bacteria X_1 catalyzes the hydrolysis reaction. Thus, the speed of the reaction is then $r_0 = \mu_0(X_0)X_1$ where μ_0 is the specific growth rate of X_1 on X_0. The different reaction steps of anaerobic digestion that they considered are summarized as follows:
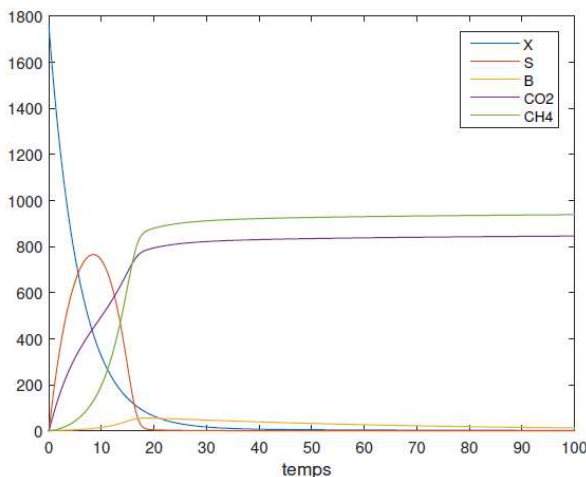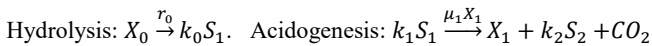

Figure 4. Evolution of the variable of the model (9) (Salih, 2021)

Hydrolysis: $X_0 \xrightarrow{r_0} k_0 S_1$.   Acidogenesis: $k_1 S_1 \xrightarrow{\mu_1 X_1} X_1 + k_2 S_2 + CO_2$

Methanogenesis $k_3 S_2 \xrightarrow{\mu_2 X_2} X_2 + CO_2 + CH_4$ where $\mu_i X_i; i = 1; 2$ are the reaction rates and μ_i the specific growth rates of X_i on S_i with S_2 the concentration of the metabolic intermediates volatile fatty acids and $X_2(t)$ the concentration of methanogenic bacteria. Finally, $k_i; i = 0; 1; 2; 3$ are the efficiency coefficients associated with the biological reactions. The digester being in continuous operation, $D = \frac{Q_{in}}{V} = \frac{Q_{out}}{V}$ (where Q_{in} is the incoming flow, Q_{out} the outflow and V the volume of the digester) is the dilution rate. Proposed by (8), $\alpha \in [0; 1]$ represents the fraction of the (biomass/bacteria) that leaves the reactor.

Analysis of the Radhouane's model in the context of energy conversion of agri-food waste: Only the acetogenesis step is not considered by the model of (Jérôme, 2014). It is a continuous operation anaerobic digestion model. It takes into an account the hydrolysis phase which other models have neglected. It can contribute to mathematical modelling of the energy conversion process of agro-food waste. Experiments can confirm or invalidate the modelling of this stage of the conversion process.

Contribution of the Radhouane's model toenergy conversion: The mathematical analysis of their model has shown the existence of at most nine equilibrium points, four of which are strictly positive and one of which is locally asymptotically stable (LAS). The numerical simulations are performed to illustrate the behavior of the solutions of the model (6) for different initial conditions. Indeed, according to the initial condition, there is either stability of the coexistence (of the two types of bacteria) state, stability of the lessivage (of the acidogenesis bacteria only) state, stability of the lessivage (of the methanogenesis bacteria only) state or the stability of the lessivage (of the two types of bacteria) state.

Model formulated by ARRAS: The effect of temperature on acid formation during the acidogenesis stage of municipal solid waste is studied by (Arras). The results showed that a one-day lag phase is observed at 35°C and 55°C, in contrast to 70°C, where solubilization starts from the first day. To predict the types of acids in this step of batch food waste acidogenesis, Arras developed a one-step hyper-thermophilic acidogenesis (HAM) model. The model parameters were estimated using experimental data obtained from the anaerobic batch reactor. The model was validated using data from a batchreactor where the organic loading rate is increased. It was then tested using data from a batch reactor at 35°C. The experimental results show good agreement with the model prediction, indicating that the model can accurately simulate the generation of organic acids at 70°C and 35°C. The model is based on the assumption that the products of fermentation at 70°C are composed of acetic acid, lactic acid, formic acid, ethanol and bacteria. It takes the following form:

$$\frac{dS}{dt} = -\frac{\mu}{Y_X} X$$
$$\frac{dX}{dt} = \mu X - k_d X$$
$$\frac{dAA}{dt} = \frac{Y_{AA}}{Y_X} \mu X$$
$$\frac{dLA}{dt} = \frac{Y_{LA}}{Y_X} \mu X \qquad (7)$$
$$\frac{dFA}{dt} = \frac{Y_{FA}}{Y_X} \mu X$$

$$\frac{dEOH}{dt} = \frac{Y_{EOH}}{Y_X} \mu X$$
$$\mu = \mu_{ma} \frac{s}{K_s + S}$$

S is the concentration of the substrate (mg/L), μ_{max} is the maximum growth rate $(1/Jour)$, K_S is the half-saturation constant (mg/l), X is the concentration of bacteria (mg/L), k_d is the bacterial mortality rate $(1/Jour)$ and Y_X the fraction of the substrate converted to bacteria $(mg - X/mg - S)$. Y_{AA}, Y_{LA}, Y_{FA}, Y_{EOH}, $(mg/mg - S)$ represents the fraction of the substrate

converted (yield coefficient) to acetic acid, lactic acid, formic acid and ethanol respectively.

Analysis of the Arras's model in the context of energy conversion of agri-food waste: The Arras model is a one-step model (acidogenesis) that does not take into account the hydrolysis, acetogenesis, methanogenesis steps. Developed and validated under mesophilic and thermophilic conditions when the digester is in batch operation, this model governs the acidogenesis stage of the energy conversion of food residues from the cafeteria of the École detechnologie supérieure (ÉTS) in Montreal. A complete model of batch operation of the digester would allow a complete understanding of the dynamics of the conversion system variables.

Contribution of the Arras's model to energy conversion: The study conducted by Arras **(5)** showed that under hyper-thermophilic conditions, the products of acidogenesis are obtained from the first day of the experiment. This would be of great importance in minimizing the digestion time. Furthermore, the mathematical study and large-scale numerical simulation of this model would determine the conditions for optimizing the final products of the acidogenesis stage of energy conversion. The followings are the simulation and the experimental results of the system (7).

Model formulated by Daoud

The effect of hydrolysis on the behavior of continuous anaerobic digestion process and biogas (methane and hydrogen) production was analyzed by (14). For this purpose, they developed five-step models. The model is as follows:

$$\frac{dX_0}{dt} = D(X_{0in} - X_0) - r_0$$

$$\frac{dS}{dt} = D(S_{in} - S) - \frac{1}{C_s}g_s(S)X_S + k_0r_0$$

$$\frac{dX_s}{dt} = (g_s(S) - D)X_S$$

$$\frac{dV}{dt} = -DV + \gamma_{sv}g_s(S)X_S - \frac{1}{C_v}g_V(V,H)X_V$$

$$\frac{dX_V}{dt} = (g_V(V,H) - D)X_V$$

$$\frac{dA}{dt} = -DA + \gamma_{sa}g_s(S)X_S + \gamma_{va}g_V(V,H)X_V - \frac{1}{C_a}g_A(A)X_A$$

$$\frac{dX_A}{dt} = (g_A(A) - D)X_A$$

$$\frac{dH}{dt} = -DH + \gamma_{sh}g_s(S)X_S + \gamma_{vh}g_V(V,H)X_V - \frac{1}{C_h}g_H(H,A)X_H$$

$$\frac{dX_H}{dt} = (g_H(H,A) - D)X_H$$

Anaerobic digestion is described as a process consisting of five stages: hydrolysis, acidogenesis, acetogenesis, hydrogenated methanogenesis and acetoclastic methanogenesis. In the first stage, complex organic molecules (X_0) are broken down into simple substrates (S). The two views in practice of hydrolysis, considered by (12) were considered here as well. During acidogenesis, the acidogenic bacteria (X_s) convert the substrate (S) into acetic acid (A)volatile fatty acid (V)alcohol, hydrogen (H) and carbon dioxide. Then, the volatile fatty acid and alcohols are consumed by the acetogenic bacteria (X_v) and they are converted into acetic acid (A) as well as to carbon dioxide and hydrogen (H). In the final phase, the methanogenic acetoclast bacteria (X_A) convert acetic acid (A) into methane and carbon dioxide, while the hydrogenating methanogenic bacteria (X_H) convert hydrogen (H) and carbon dioxide into methane. X_{0in} is the concentration of the solid (slowly biodegradable) substrate at the inlet of the chemostat. r_0 is the rate of the hydrolysis reaction which depends on the state variables according to the model considered. S_{in} is the concentration of the soluble (rapidly biodegradable) substrate at the inlet to the chemostat, and D is the dilution rate. c_S, c_v, c_a, c_h are the yield coefficients of the bacteria and $\gamma_{sv}, \gamma_{sa}, \gamma_{sh}, \gamma_{va}, \gamma_{vh}$ are ratios of product yield to biomass yield. $g_S(.), g_A(.), g_V(.;.)etg_H(.;.)$ are the specific growth rates.

Analysis of the Daoud's model in the context of energy conversion of agri-food waste: Daoud's models take into account all stages of anaerobic digestion in continuous operation. They allow a better understanding of all the steps of the energy conversion and the products resulting from these steps. They can contribute to the mathematical modelling of the energy conversion process of agri-food waste. Experiments to validate these models would confirm the assumptions on which the development of these models is based.

Contribution of Daoud's models to energy conversion: The models considered involve the inhibition of the growth of acetogenic (respectively methanogenic hydrogen) bacteria by hydrogen (respectively, by acetate). The study of the model without inhibition was done to analyze the effect of these inhibitions, in the presence of the hydrolysis step. The necessary and sufficient conditions for the stability of the equilibrium points are given. Then, they showed that modeling the hydrolysis phase by constant enzymatic activity affects the production of methane and hydrogen. In addition, the introduction of the hydrolytic microbial compartment reveals new equilibrium points and affects the stability regions. Finally, it was shown that the maximum methane and hydrogen production is reached at a stable equilibrium point.

Model formulated by Salih: To describe the operation of a landfill in which biodegradable waste is used to generate methane, Salih (10) in his thesis presented and studied, theoretically and numerically, a mathematical model of anaerobic digestion in two stages (hydrolysis and methanogenesis) taking into account the inhibition phenomena. The schematic view of the model and the model are as follows:

$$\frac{dX}{dt} = -K_hX + \alpha K_dB$$

$$\frac{dB}{dt} = (\mu(S) - K_d)B$$

$$\frac{dS}{dt} = f_1K_hX - \frac{1}{Y}\mu(S)B$$

$$\frac{d[CO_2]}{dt} = (1 - f_1)K_hX + (1 - f_2)\frac{1-Y}{Y}\mu(S)B$$

$$\frac{d[CH_4]}{dt} = f_2\frac{1-Y}{Y}\mu(S)B$$

X is the concentration of complex degradable organic matter. $B$is the concentration of methanogenic bacteria. S is the concentration of soluble organic matter. (CO_2) and (CH_4) are respectively the concentrations of carbon dioxide and methane. $K_h$and K_d are the hydrolysis constant and the mortality rate of the methanogenic bacteria, respectively. The dead bacteria can constitute a new substrate and are then recycled as material to be hydrolysed. This explains the term αK_dB in the first equation of the system with α the conversion constant between 0 and 1. $\mu(s)$ is the specific growth rate of the methanogenic bacteria, Y is the substrate utilization rate. The parameter f_1 is the stoichiometric coefficient which represents the part of the organic matter transformed into simple organic matter during the hydrolysis phase and $(1 - f_1)$ represents the part of the material transformed into CO_2. $f_2$is the stoichiometric coefficient that represents the part of the organic matter transformed into CH_4 during the methanogenesis phase and $(1 - f_2)$ is the part transformed into CO_2. Based on the principle of conservation of mass, the amount of substrate S degraded corresponds to the number of bacteria$B$ created plus the number of products $P(CO_2$ and $CH_4)$ formed. Thus. $\frac{-d}{dt} =$

$\frac{dB}{dt} + \frac{dP}{dt}$. Thus $\frac{d[CH_4]}{dt} = \frac{-dB}{dt} - \frac{dS}{dt}$ and $\frac{d[CO_2]}{dt} = \frac{-dB}{dt} - \frac{dS}{dt}$. Hence $\frac{d[CH_4]}{dt} = f_2 \frac{1-Y}{Y} \mu(S)B$ and $\frac{d[CO_2]}{dt} = (1 - f_2) \frac{1-Y}{Y} \mu(S)B$.

Analysis of the Salih's model in the context of energy conversion of agri-food waste: Since the acidogenesis and acetogenesis stages are not limiting stages, Salih's model (10) takes into account only the input stage, hydrolysis, and the output stage, methanogenesis. Developed in a batch operation of the anaerobic process, the model describes the operation of a landfill in which biodegradable waste (agro-food or not) is used to generate methane. This model is therefore an important tool for understanding and modelling the energy conversion process of agrifood waste. The other intermediate steps can be considered for the development of a complete batch model.

Contribution of the Salih's model toenergy conversion: The mathematical and numerical study of Salih's model (10) showed that for non-monotonic growth functions, such as the Haldane function the performance in terms of biogas production is discontinuous with respect to the initial condition. That is, the dynamics of the model solutions vary with the initial conditions set. Also, a too high initial loading of organic matter could be penalizing because there could be a significant amount of residual soluble matter, especially when the mortality rate is low. A control problem arises to find a better strategy for the introduction of organic matter in order to optimize biogas production. Furthermore, it has been shown that under the conditions

$(0 < \alpha \le 1 ; 0 < Y < 1 ; 0 < f_1 < 1; 0 < f_2 < 1; et \ 0 < K_d < \max_s \mu(s)$ where

μ is a bounded function of classe C^1 with $\mu(0) = 0$), all the organic matter is transformed into biogas. The dynamics of the solution of the model (9) is presented as follows.

We observed a stability in the production of CO2 and CH4 from 20 th day.

Recapitulation

Table 1. Recapitulation of existing models with steps and operation modes considered

Models formulated by	Step considered	Operation mode	Biomass used
Bernard *et al.*, (2001)	Two step: Acidogenesis and methanogenesis	Continuous under mesophilic conditions	Industrial wine distillery residues
Lemesle (2004)	One step including mortality, maintenance and structuration	Continuous under purely theoretical considerations	-
Radhouane (2014)	Three step: Hydrolysis, Acidogenesis and methanogenesis	Continuous under theoretical considerations	-
Arras (2017)	One step: Acidogenesis	Discontinuous (batch) under mesophilic, thermophilic and hyper-thermophilic conditions	Food residues from the cafeteria of (ETS-Montreal)
Daoud	Five steps: Hydrolysis Acidogenesis, Acetogenesis, hydrogenated methanogenesis and acetolacticmethano genesis	Continuous under theoretical considerations	-
Salih	Two steps: Hydrolysis and methanogenesis	Discontinuous (batch) under mesophilic, and thermophilic conditions	Household and similar waste.

CONCLUSION

The survey of existing mathematical models of anaerobic digestion processes was conducted. This is to optimise mechanisationof biomass waste from processing of agro-food products in Benin. The methodology used was based on review of research results conducted in Benin and elsewhere in the world. Models of energy conversion processes, application of physical-chemical laws and the law of conservation of matter and on the other hand. The analysis of models listed as well as their contribution to optimization of energy conversion of waste from the processing of agro-food products. In conclusion, all the models identified, described and analysed in the context of energy conversion of agri-food waste, those of Arras and Salih are developed when the digester is in batch operation. These models differ in the digestion mode and the number of digestion steps considered. Some of them are validated under mesophilic and hyper-thermophilic conditions with appropriate biomasses while others are developed with purely theoretical but meaningful considerations.

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