



## RESEARCH ARTICLE

### STATISTICAL STUDY OF WAVE PARAMETERS: SEA STATES IN THE DEEP WATERS (OFFSHORE) OF THE GULF OF GUINEA IN BENIN

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#### ABSTRACT

Studying the evolution of swell characteristics as they propagate in coastal zone is essential to understanding the physics of the phenomenon. Control of wave parameters and knowledge of sea states are fundamental elements, both for the development of maritime activities and the prevention and fight against coastal disasters. Our study is based on measurements of data acquired from 2011 to 2014 as part of the extension of the Autonomous Port of Cotonou by the Millennium Challenge Account (MCA-Benin). This work, carried out on the coast of Benin, aims to fill the lack of information on the direction of the swells and to provide statistics representative of their heights in the deep waters of the zone close to the coast, where the swell is not yet submitted to the action of the winds. In these deep waters (offshore) of the coastal zone, based on measurements of wave data taken at five-minute intervals, on a regular basis over a period of four consecutive years (June 2011 to April 2014), the statistical distribution of the characteristic parameters of the swell (height, direction of propagation, wavelength and period) is worked out. Thus, the significant height  $H_s$ , the peak period or the stable mean period of the swell  $T_p \approx T_{m_s}$  are evaluated. The frequent incident direction  $D_f$  is evaluated to define the spectral directional distribution of the sea states in the deep waters of the Beninese coastal zone are known through the Beaufort scale used in maritime environments.

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## INTRODUCTION

Understanding the generation mechanism and the physics that accompanies wave propagation are of great interest to the scientific community (Nerzic, 2007). Knowledge of wave and/or swell parameters and mastery of sea states are fundamental elements, both for the design and construction of coastal structures (Noël, 1997). Waves are the most important phenomenon to consider among the environmental conditions affecting maritime structures, because they exert the greatest influence. Studies already carried out in the Gulf of Guinea have shown that three wave systems predominate in West Africa: the main swell, the secondary swell and the wind sea (Nerzic, 2007). The presence of waves makes the design process for maritime structures on earth (Peltier, 1994). Since waves are one of the most complex and variable phenomena in nature, it is not easy to achieve a full understanding of their fundamental character and behavior. Under the effect of an external disturbance (wind, seabed, etc.), the profile of a swell can be modified. The phenomenon of wave propagation is therefore dispersive (Houekpoheha, 2014). In addition, the swell do not all propagate in the same direction, resulting in a sometimes chaotic appearance of the surface state of the sea. The waves are therefore characterized by energy spectra, which reveal characteristic quantities, with for example a significant height  $H_s$ , a significant period  $T_s$ , etc (Oswald, 2022). These spectra are broad for wind waves, narrower for an already formed swell that continues to propagate away from its wind-generated zone. It can be subjected to movements, such as: shoaling, breaking, refraction, diffraction (Houekpoheha, 2015; Kounouhewa, 2014). This observation is all the more true for the Beninese coast, exposed to particularly energetic swells in the coastal zone. In terms of means of measurement, this portion of the Atlantic coast of the Gulf of Guinea has long been under-equipped. Indeed, non-directional measures are few in number and directional measures are rare and very recent. The incident direction of the swell is also a fundamental parameter in the calculation of the littoral drift, likely to move a significant quantity of sediment parallel to the coast (Abbasov, 2012; Tokpohozin, 2015). The Bight of Benin is an open, microtidal, wave-dominated coast forming a 500 km-long mild embayment in the Gulf of Guinea, in West Africa, between the Volta River delta in Ghana, to the west, and the western confines of the Niger River delta in Nigeria to the east.

The bight is exposed to energetic swells from the South Atlantic, and is characterized by Holocene sand barriers bounding lagoons (Laïbi, 2014). Due to the random nature of the swell, the state of the sea is described by statistical parameters, such as the average heights, periods or directions without forgetting the Beaufort scale which is an empirical measurement scale of the speed of the wind, comprising 13 degrees (from zero to 12), used in maritime environments. This work aims to fill the lack of information on the direction of the swells and to provide statistics representative of their parameters in deep waters, close to the coast, where the swell is not yet subjected to the action of the funds. It is thus hoped to cover a fairly large coastal fringe while avoiding the local distortions which may be caused by this action at the bottom.

## MATERIALS AND METHODS

**Presentation of the study site:** Benin is a Gulf of Guinea country situated between the parallels  $6^{\circ}15'$  and  $12^{\circ}30'$  of North latitude on the one hand and the meridians  $1^{\circ}$  and  $3^{\circ}40'$  East longitude on the other hand (Figure 1A). It benefits from access to the ocean over a distance of approximately 125 km and goes from Hilla-Condji in the West to Kraké in the East. The coastline of Benin is more or less linear and interrupted in two places, namely the Bouche du Roy and the mouth of the Cotonou Canal. In this coastal zone, it receives an abundance of regular sea swell of low amplitude compared to their wavelength (Houekpoheha, 2014). Their amplitude varies according to the period of the year on the one hand and on a daily basis on the other hand. As part of the extension of the Autonomous Port of Cotonou, the swell measurements carried out (Figure 1B) over four consecutive years (from June 2011 to April 2014) in five-minute steps by the Millennium Challenge Account (MCA-Benin) and the bathymetric map of Benin obtained (Figure 1C) near Institute of Fishery and Oceanological Research of Benin (IRHOB)/Benin Center for Scientific Research and Innovation (CBRSI), have made it possible to define the average stable daily, monthly and annual wave data for this site (Houekpoheha, 2015).

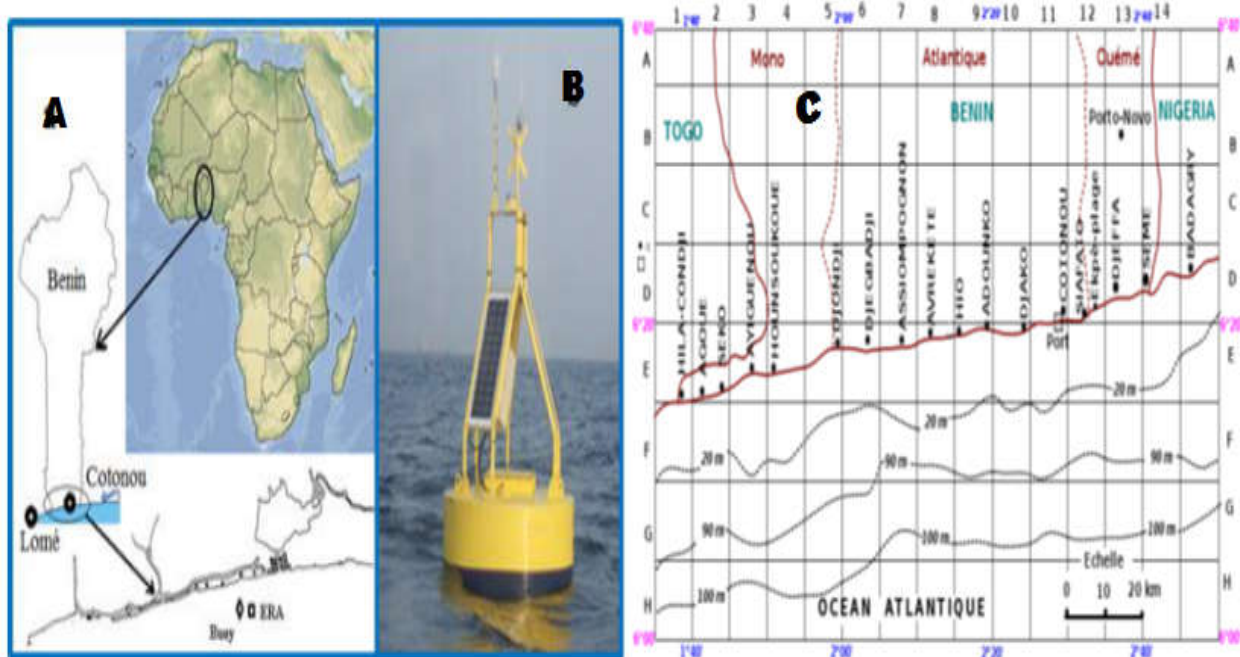


Figure 1. Geographical location, location of the oceanographic buoy/IRHOB and the bathymetric map of the Benin coastal zone

In this zone where gravity is  $g = 9,79 \text{ N/kg}$  and the density of ocean water is  $\rho = 1025 \text{ kg/m}^3$ , we observe:

- The dominant swells are long swells which have a period  $T$  ( $8 \text{ s} \leq T \leq 16 \text{ s}$ ) of which the stable average value  $T_m \approx 12 \text{ s}$  and a wavelength  $L_o$  of approximately 200 m in the deep waters of the coastal zone of the Gulf of Guinea (Kounouhewa, 2014a; Kounouhewa, 2014b).
- The bathymetric map of Benin (Figure 1C) shows the evolution of the seabed in the coastal zone and allows to predict the average slope and the macroscopic variability of the seabed. This map shows that the seabed in the coastal zone of Benin is almost flat with the inclination. It is a low slope seabed  $p = \tan \beta$ , such as  $0.001 < \tan \beta < 0.1$ . The average of this slope in the study area is  $\beta_m \approx 90 / 2000 = 0.045 \Rightarrow \tan \beta \approx \beta$ .
- In Benin, the swells are regular. They have a wavelength of approximately  $L = 200 \text{ m}$  in deep waters and their period varies between 10 s and 18 s with a stable mean value (Oswald, 2022; Guy Hervé Houngouè, 2018).
- Short swells (wind seas) of wavelength  $L_c \approx 50 \text{ m}$  and period  $T_c \approx 6 \text{ s}$  which are generated by local winds. These swells are rare (Guy Hervé Houngouè, 2018; Guy Hervé Houngouè, 2018).
- The average stable wind speed  $\vartheta$  in the Gulf of Guinea at Cotonou, during a day, varies between 4.25 m/s and 6.75 m/s ( $4,25 \text{ m} \cdot \text{s}^{-1} \leq \vartheta \leq 6,755 \text{ m} \cdot \text{s}^{-1}$ ) (Houekpoheha, 2014).
- Based on wave data measurements carried out, at five-minute intervals over four consecutive years (June 2011 to April 2014) by the Millennium Challenge Account (MCA-Benin) as part of the extension of the Autonomous Port of Cotonou, we have:
- Realized the statistics of the values of the crest-to-trough heights of the swells according to the Beaufort scales near the coasts in order to identify the dominant sea states prevailing on the study site.
- Evaluated the annual distributions of these heights during each year to determine the significant heights  $H_s$  of these waves.
- Defined a typical day for each month by averaging the measurements that are taken at the same time every day of the month. This typical day makes it possible to know the variations of the almost stable values of these heights during a day.
- Represented the distribution diagrams of the swell propagation directions and established the frequency and direction spectra of these swells as a function of the stable average wind speed on the site.

**Beaufort scales in the coastal zone:**

In order to characterize the different sea states in the coastal zone of Benin in the Gulf of Guinea, it is used in this work, the Beaufort scales which characterize the swells in the deep waters near the coasts. These scales are summarized in the table below.

**Table 1. Beaufort scales characterizing swells in deep waters**

Degree or force	Appellation Terms (Description)	Wave heights near the coast (m)	Sea states
B <sub>0</sub>	Calm	0	The sea is like a mirror, smooth and without waves.
B <sub>1</sub>	Very light breeze (rippled)	(0 – 0,1)	A few wrinkles resembling fish scales but without any foam (Ridled Sea).
B <sub>2</sub>	light breeze (beautiful)	(0,1 – 0,15)	Ripples not breaking.
B <sub>3</sub>	Small breeze (little agitated)	(0,15 – 0,3)	Very small waves. The ridges begin to break. Glassy-looking scum.
B <sub>4</sub>	Pretty broken (agitated)	(0,3 – 0,46)	Small waves of many sheep.
B <sub>5</sub>	Good breeze (Strong)	(0,46 – 0,76)	Moderate waves, sheep, spray.
B <sub>6</sub>	Cool wind (Very strong)	(0,76 – 1,22)	Crests of white foam, spray waves.
B <sub>7</sub>	Big Fresh (Big)	(1,22 – 1,68)	Breaking waves, streaks of foam.
B <sub>8</sub>	Gust of wind (Very big)	(1,68 – 2,29)	Whirlwinds of foam at the crest of the waves, streaks of foam.
B <sub>9</sub>	Strong gust of wind (Huge)	(2,29 – 3,05)	Spray obscuring the view, you can't see farther.
B <sub>10</sub>	Storm	3,05 and more	- Very large blades with a long crest in a plume, the surface of the waters seems white. Reduced visibility. - The sea is completely covered with foam banks. Reduced visibility. - The air is full of foam and spray. Visibility greatly reduced.

**Swell heights**

Sea states are generally characterized by the different values of wave heights such as: Minimum heights  $H_{min}$ , Maximum heights  $H_{max}$ , Average heights  $H_m$  and Significant heights  $H_s$  obtained after processing data from the buoy (Figure 1B). From the analysis of the data, it appears that the extreme values of these heights, their average and their significant value, during each of the months of effective measurements, are presented in the table below.

**Table 2. Swell height values according to the different months of the years 2011 to 2014**

Years	Month	Swell height values (m)			
		Minimum heights $H_{min}$	Maximum heights $H_{max}$	Average heights $H_m$	Significant heights (frequent values) $H_s$
2011	June	0.15	1.65	1.02	$0.6 \leq H_s \leq 1.50$
	July	0.20	1.70	0.95	$0.5 \leq H_s \leq 1.50$
	August	0.10	2.05	1.02	$0.5 \leq H_s \leq 1.55$
	September	0.10	2.10	1.01	$0.5 \leq H_s \leq 1.40$
	October	0.10	2.05	1.05	$0.5 \leq H_s \leq 1.35$
	November	0.40	1.60	0.95	$0.6 \leq H_s \leq 1.30$
	December	0.05	1.60	0.83	$0.4 \leq H_s \leq 1.20$
2012	January	0.05	1.65	0.82	$0.4 \leq H_s \leq 1.20$
	February	0.10	1.80	0.88	$0.4 \leq H_s \leq 1.30$
	March	0.05	1.85	0.86	$0.4 \leq H_s \leq 1.25$
	April	0.10	1.85	0.88	$0.4 \leq H_s \leq 1.35$
	May	0.05	1.60	0.83	$0.35 \leq H_s \leq 1.2$
	June	---	---	---	---
	July	---	---	---	---
	August	---	---	---	---
	September	---	---	---	---
	October	---	---	---	---
	November	0.10	2.10	1.05	$0.6 \leq H_s \leq 1.60$
	December	0.10	1.80	0.90	$0.4 \leq H_s \leq 1.40$
2013	January	0,15	1.62	0.85	$0.4 \leq H_s \leq 1.27$
	February	0.06	1.70	0.80	$0.40 \leq H_s \leq 1.32$
	March	---	---	---	---
	April	---	---	---	---
	May	---	---	---	---
	June	---	---	---	---
	July	---	---	---	---
	August	0.15	1.85	0.89	$0.53 \leq H_s \leq 1.4$
	September	0.20	2.20	1.06	$0.5 \leq H_s \leq 1.45$
	October	0.11	1.95	1.01	$0.45 \leq H_s \leq 1.5$
	November	0.10	2.15	1.02	$0.55 \leq H_s \leq 1.6$
	December	0.10	1.83	0.95	$0.4 \leq H_s \leq 1.45$
2014	January	0.10	1.76	0.93	$0.4 \leq H_s \leq 1.25$
	February	0.15	1.82	0.85	$0.35 \leq H_s \leq 1.3$
	March	0.12	1.75	0.89	$0.4 \leq H_s \leq 1.25$
	April	0.13	1.81	0.85	$0.4 \leq H_s \leq 1.35$
Balance sheet		0.05	2.20	1	$0.45 \leq H_s \leq 1.40$

**Frequency and directional swell spectra:** The frequency spectrum of Pierson-Moskowitz (1964) is deduced from a series of measurements made in the Atlantic for established seas. It corresponds to the area of gravity waves. It is also a function of the wind speed  $v$ . This spectrum is currently represented by one of the best empirical formulas available (2):

$$S_f(\omega) = \frac{bg^2}{\omega^5} e^{-\left(\frac{a\omega}{\omega_0}\right)^4} \quad (1) \quad \text{with} \quad \begin{cases} a = 0,93 \\ b = 0,0081 \\ \omega = \frac{2\pi}{T} \end{cases} \quad (1)$$

Thus, in the coastal zone of the Gulf of Guinea, we have:

$$S_f(\omega) = S_f(T) = \frac{0,0243T^5}{\pi^5} e^{-\left(\frac{4,55T}{\pi\vartheta}\right)^4} \quad (2)$$

$$\text{with} \quad \begin{cases} 4,25 \text{ m} \cdot \text{s}^{-1} \leq \vartheta \leq 6,755 \text{ m} \cdot \text{s}^{-1} \\ 6\text{s} \leq T \leq 18\text{s} \end{cases}$$

This frequency spectrum makes it possible to define the directional spectrum  $S_D(\omega, \theta)$  by the relationship:

$$S_D(\omega, \theta) = S_f(\omega) \cdot G(\theta) \quad \text{with} \quad G(\theta) = \begin{cases} \frac{2}{\pi} \cos^2(\theta) & \text{if } \theta_i \leq \theta \leq \theta_f \\ 0 & \text{some where else} \end{cases} \quad (3)$$

On the site, taking the origin of the angles of incidence of the swells in the SSW direction, we have

$$\theta_i \approx -\frac{\pi}{4} \text{ rad} \quad \text{and} \quad \theta_f \approx \frac{\pi}{4} \text{ rad}, \quad \text{then we get:}$$

$$S_D(\omega, \theta) = S_D(T, \theta) = \frac{0,0486T^5}{\pi^6} e^{-\left(\frac{4,55T}{\pi\vartheta}\right)^4} \cos^2(\theta) \quad (4)$$

$$\text{with} \quad \begin{cases} 4,25 \text{ m} \cdot \text{s}^{-1} \leq \vartheta \leq 6,755 \text{ m} \cdot \text{s}^{-1} \\ -\frac{\pi}{4} \text{ rad} \leq \theta \leq \frac{\pi}{4} \text{ rad} \end{cases}$$

Comparison of crest height statistics from the sensors for individual sea states is difficult due to the different locations of the sensors and to the great variability in time and space of the swell field in wind sea conditions. By combining several hours of data with similar wave heights we can eliminate some of the sampling variability in single data records while still being able to see variations due to different ranges of swell heights (15;16).

$$R_{ay}(H_c > H) = e^{-\left(\frac{8H^2}{H_s^2}\right)} \quad (5)$$

$$\text{with} \quad \begin{cases} 0,2\text{m} \leq H \leq 2,1\text{m} \\ 0,8\text{m} \leq H_s \leq 1,5\text{m} \end{cases}$$

### Variation of Swell Parameters

The Airy or Stokes swell dispersion relationship is (4):

$$\omega^2 = \frac{4\pi^2}{T^2} = gk \tanh(kd) \quad (6)$$

$$\begin{cases} C_\varphi = \frac{\omega}{k} = \sqrt{\frac{g}{k} \tanh(kd)} \\ C_g = \frac{\partial \omega}{\partial k} = \frac{1}{2} \frac{\omega}{k} \left(1 + \frac{2kd}{\sinh(2kd)}\right) \end{cases} \quad (7)$$

- In the deep waters (offshore),  $C_g = \frac{1}{2} C_\varphi = \frac{gT}{4\pi}$  or  $L = \frac{gT^2}{2\pi}$
- In the Shoaling zone,  $C_g = \frac{1}{2} \left(1 + \frac{2kd}{\sinh(2kd)}\right) \sqrt{\frac{g}{k} \tanh(kd)}$
- In the shallow waters,  $C_g = C_\varphi = \sqrt{gd}$
- In the end, we have:

$$C_g(d) = \begin{cases} \frac{gT}{4\pi} & \text{if } d > \frac{L}{2} \\ \frac{1}{2} \left(1 + \frac{2kd}{\sinh(2kd)}\right) \sqrt{\frac{g}{k} \tanh(kd)} & \text{if } \frac{L}{25} \leq d \leq \frac{L}{2} \\ \sqrt{gd} & \text{if } 0 \leq d \leq \frac{L}{25} \end{cases} \quad (8)$$

Or

$$C_g(d) = \begin{cases} \frac{gT}{4\pi} & \text{if } d > \frac{L}{2} \\ \frac{1}{2} \left(1 + \frac{gdT^2/2\pi^2}{\sinh(gdT^2/2\pi^2)}\right) \sqrt{\frac{4\pi^2}{T^2} \tanh(gdT^2/4\pi^2)} & \text{if } \frac{L}{25} \leq d \leq \frac{L}{2} \\ \sqrt{gd} & \text{if } 0 \leq d \leq \frac{L}{25} \end{cases}$$

The expressions which translate the variations of the period  $T$ , of the phase velocity  $C_\phi$  and of the group velocity  $C_g$  are:

$$\left\{ \begin{array}{l} T = \frac{2\pi}{\sqrt{gk \tanh(kd)}} \\ C_\phi = \sqrt{\frac{g}{k} \tanh(kd)} \\ C_g = \frac{1}{2} \left( 1 + \frac{2kd}{\sinh(2kd)} \right) \sqrt{\frac{g}{k} \tanh(kd)} \end{array} \right. \text{ with } \left\{ \begin{array}{l} L_o = \frac{gT_o^2}{2\pi} \\ C_{\phi_o} = \frac{gT}{2\pi} \\ C_{g_o} = \frac{gT}{4\pi} \end{array} \right. \quad (9)$$

In deep waters (offshore) ( $d \geq \frac{L_o}{2} \Rightarrow \mu \geq \frac{1}{2}$ ),  $\tanh(kd) \approx 1$ , then all the parameters are constant in this propagation zone. Taking into account the previous conditions ( $\eta \ll d$ ,  $\varepsilon \ll 1$  and  $\alpha < 1$ ), the Peregrine equations (Boussinesq) which correspond to the study sites are (6):

$$\left\{ \begin{array}{l} \frac{\partial \eta}{\partial t} + \frac{\partial(du)}{\partial x} = 0 \\ \frac{\partial u}{\partial t} + g \frac{\partial \eta}{\partial x} = \alpha \left[ \frac{d}{2} \frac{\partial^2}{\partial x^2} \left( d \frac{\partial u}{\partial t} \right) - \frac{d^2}{6} \frac{\partial^3 u}{\partial x^2 \partial t} \right] \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \frac{\partial \eta}{\partial t} + \frac{\partial(du)}{\partial x} = 0(i) \\ \frac{\partial u}{\partial t} + g \frac{\partial \eta}{\partial x} = \alpha \frac{d^2}{3} \frac{\partial^3 u}{\partial x^2 \partial t} (ii) \end{array} \right. \quad (10)$$

Substituting(ii)in the derivative of(i)with respect to time t, we get:

$$\frac{\partial^2 \eta}{\partial t^2} + \frac{\partial}{\partial x} \left[ -g\mu L_o \frac{\partial \eta}{\partial x} + \mu^5 \frac{L_o^3}{3} \frac{\partial^4 u}{\partial x^2 \partial t^2} \right] = 0 \text{ with } \left\{ \begin{array}{l} \eta = \eta(x, t) = \eta_a e^{-i\omega t} \\ \eta_a = \frac{H(x)}{2} e^{ikx} \\ d = -\beta x = \mu L_o \\ \alpha = \mu^2 \end{array} \right. \quad (11)$$

At the limit of deep waters (offshore) and the shoaling zone where  $\mu = \frac{1}{2}$ , we have :

$$H\left(\mu = \frac{1}{2}\right) \approx H_o \text{ and } L\left(\mu = \frac{1}{2}\right) \approx L_o \quad (12)$$

The equation for the tangent to the seabed is  $d = -x \tan \beta = \mu L_o$ (16; 18 ;19), or at the study site,

$$\tan \beta \approx \beta \Rightarrow d = -\beta x = \mu L_o \quad (13)$$

By making a change of variable  $\mu = -\frac{\beta}{L_o} x$  and using the approximation  $\mu^5 = \sqrt{\alpha^5} = \mathcal{O}(1)$ (6), the previous equation becomes:

$$\frac{\partial^2 \eta_a}{\partial \mu^2} + \frac{1}{\mu} \frac{\partial \eta_a}{\partial \mu} + \frac{2\pi}{\beta^2 \mu} \eta_a = 0 \quad (14)$$

The general solution of this equation is  $\eta_a = \gamma Y_0(z)$ , where  $z = \frac{2}{\beta} \sqrt{2\pi\mu}$  and  $Y_0(z)$  is the modified Bessel function of the second kind and whose complex asymptotic formula is:  $Y_0(z) = \sqrt{\frac{2}{\pi z}} e^{-i(z - \frac{\pi}{4})}$

Either  $\eta_a = \gamma \sqrt{\frac{2}{\pi z}} e^{-i(z - \frac{\pi}{4})}$  with  $z = \frac{2}{\beta} \sqrt{2\pi\mu}$ ; the physically acceptable solution is:

$$\eta_a = \gamma \sqrt{\frac{\beta}{\pi \sqrt{2\pi\mu}}} \cos \left[ \frac{2}{\beta} \sqrt{2\pi\mu} - \frac{\pi}{4} \right] \quad (15)$$

From the previous boundary conditions, we get:

$$\left\{ \begin{array}{l} H = H_o \left( \frac{1}{2\mu} \right)^{1/4} = H_o \left( \frac{2d}{L_o} \right)^{-1/4} \\ L = L_o (2\mu)^{1/2} = L_o \left( \frac{2d}{L_o} \right)^{1/2} \Rightarrow kd = \pi \sqrt{2\mu} \end{array} \right. \quad (16)$$

According to this theory, which takes into account the variability of the bathymetry, the different coefficients of variation of the parameters previously studied in the shoaling zone ( $\mu_b \leq \mu \leq \frac{1}{2}$ ), are:



$$\left\{ \begin{array}{l} A = \frac{L}{L_0} = \sqrt{2\mu} \\ B = \frac{H}{H_0} = \left(\frac{1}{2\mu}\right)^{\frac{1}{4}} \\ E = \frac{T}{T_0} = \sqrt{\frac{\sqrt{2\mu}}{\tanh(\pi\sqrt{2\mu})}} \\ C = \frac{C_{\varphi}}{C_{\varphi_0}} = \sqrt{\sqrt{2\mu}\tanh(\pi\sqrt{2\mu})} \\ D = \frac{C_g}{C_{g_0}} = \left(1 + \frac{2\pi\sqrt{2\mu}}{\sinh(2\pi\sqrt{2\mu})}\right) \sqrt{\sqrt{2\mu}\tanh(\pi\sqrt{2\mu})} \end{array} \right. \text{with } \left\{ \begin{array}{l} L_0 = \frac{gT_0^2}{2\pi} \\ C_{\varphi_0} = \frac{gT_0}{2\pi} \\ C_{g_0} = \frac{gT_0}{4\pi} \end{array} \right. \quad (17)$$

**Presentation of the results:** The statistical representations in Figures 2 below show the distribution of height values crest to trough swells in the Gulf of Guinea in Benin according to the Beaufort scales, near the coasts, during each month. The curves in Figure 3 reveal the annual statistical distribution of swell heights in the deep waters of the coastal zone from the Gulf of Guinea to Cotonou from 2011 to 2014.

The curves in Figures 4 below show the variations in swell heights, in the deep waters of the coastal zone of the Gulf of Guinea in Benin, as a function of time (GMT hours) during each typical day. The diagrams in Figures 5 below show the annual variations in maximum and minimum swell heights on the Benin coast. The curve in Figure 6 indicates the directional frequency spectrum of the swells in the study zone. It makes it possible to describe the state of the sea beyond the significant height and the average direction observed. A strong frequency spread corresponds to short groups of waves and very irregular series.

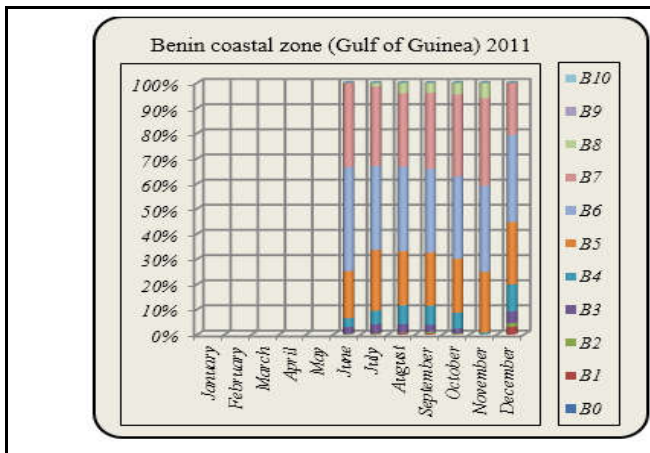


Figure 2a. Monthly Ocean states in Coastal zone of Gulf of Guinea at Cotonou in 2011

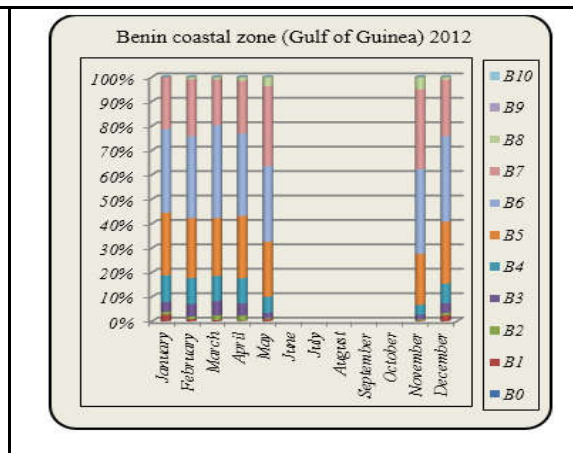


Figure 2b. Monthly Ocean states in Coastal zone of Gulf of Guinea at Cotonou in 2012

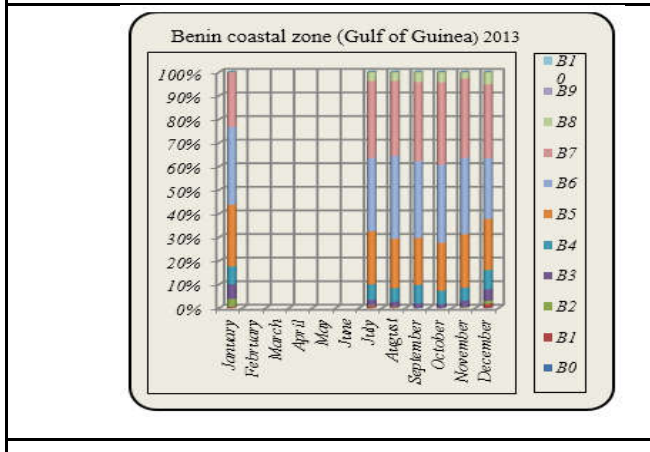


Figure 2c. Monthly Ocean states in Coastal zone of Gulf of Guinea at Cotonou in 2013

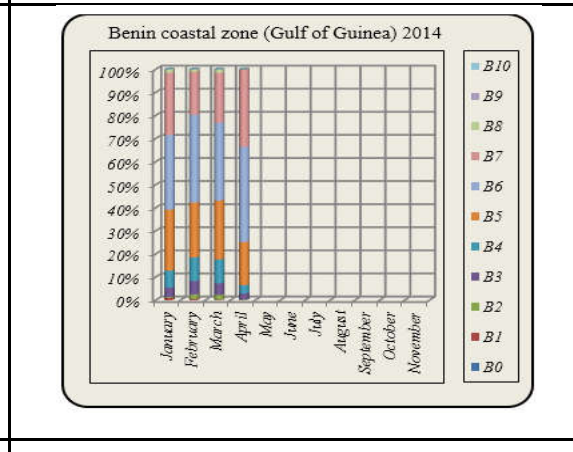


Figure 2d. Monthly Ocean states in Coastal zone of Gulf of Guinea at Cotonou in 2014

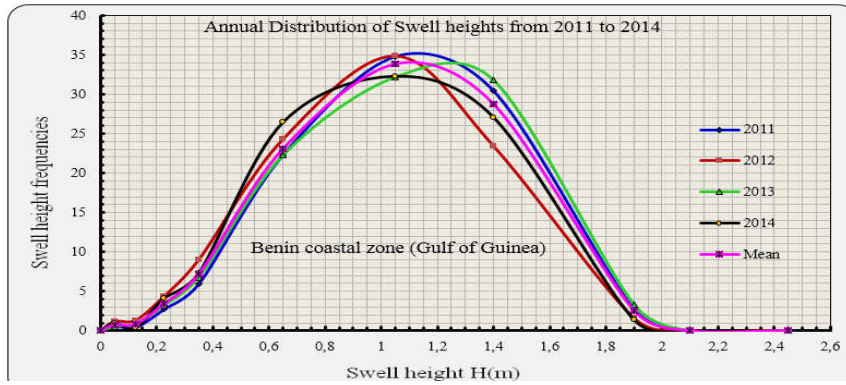


Figure 3. Annual distribution of wave heights from 2011 to 2014

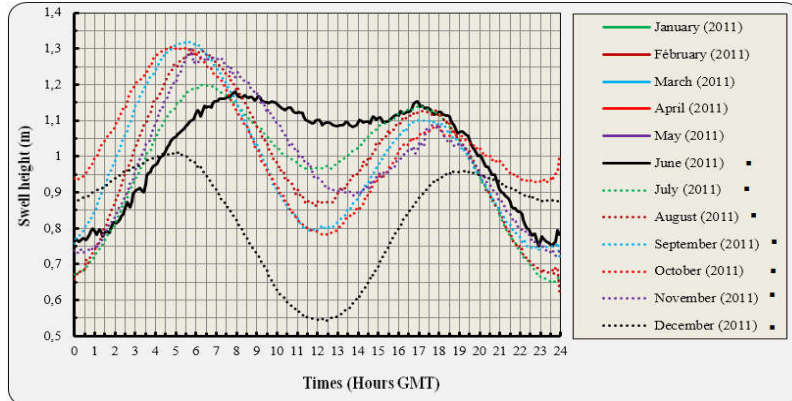


Figure 4a. Swell heights during each typical day in the months of 2011

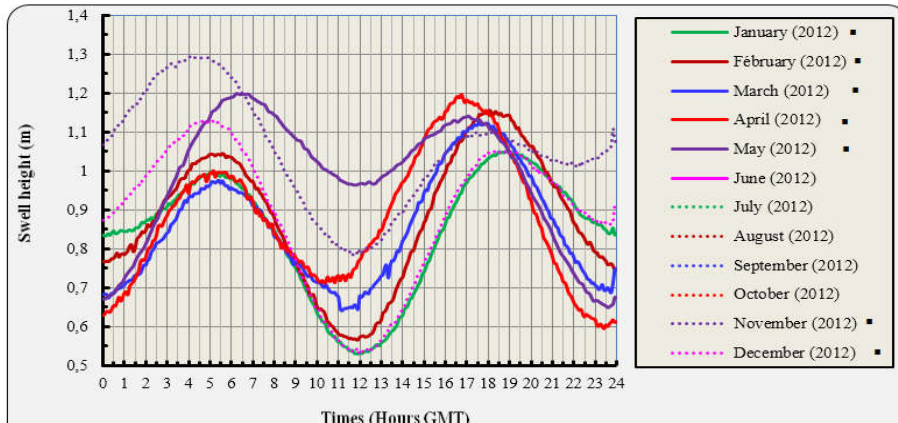


Figure 4b. Swell heights during each typical day in the months of 2012

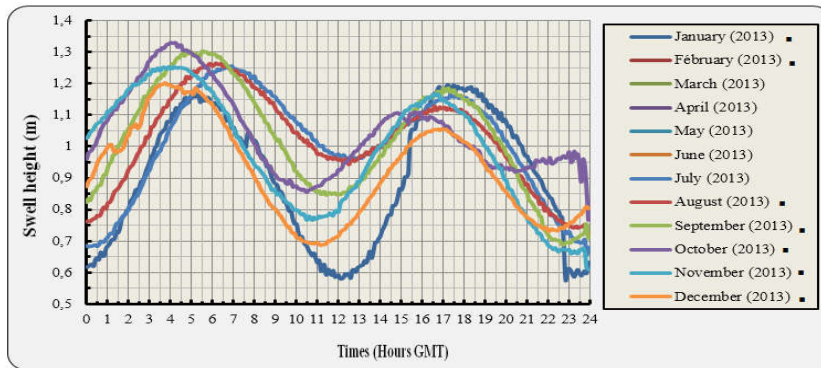


Figure 4c. Swell heights during each typical day in the months of 2013

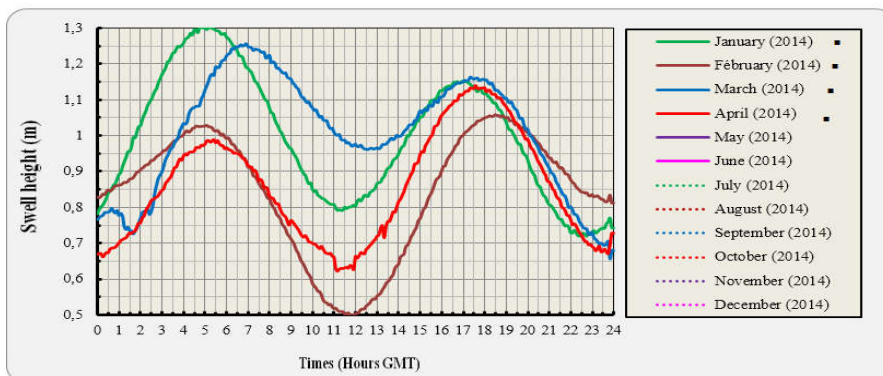


Figure 4d. Swell heights during each typical day in the months of 2014

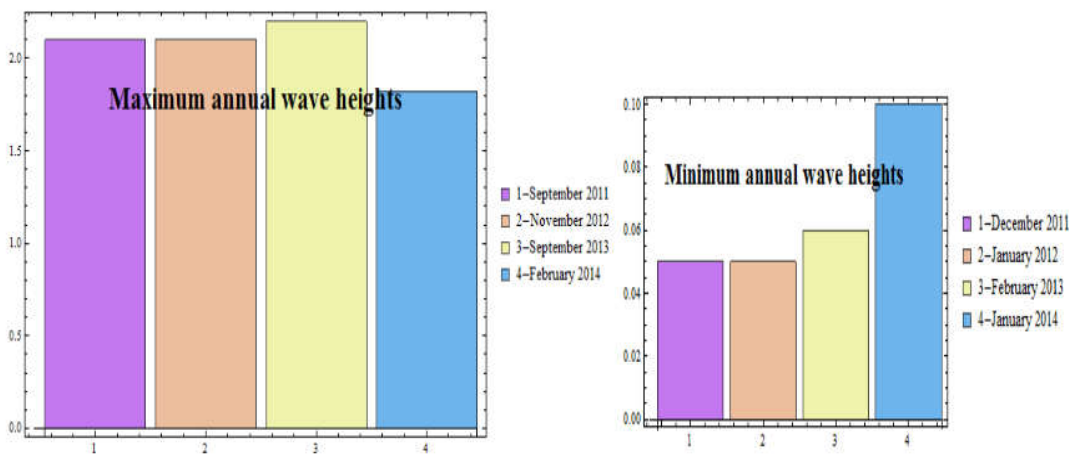


Figure 5. Maximum and minimum annual swell heights

High frequencies correspond to short-period waves. The diagrams in Figures 7 below represent the annual statistical distribution of swell propagation directions in the coastal zone of the Gulf of Guinea at Cotonou from 2011 to 2014.

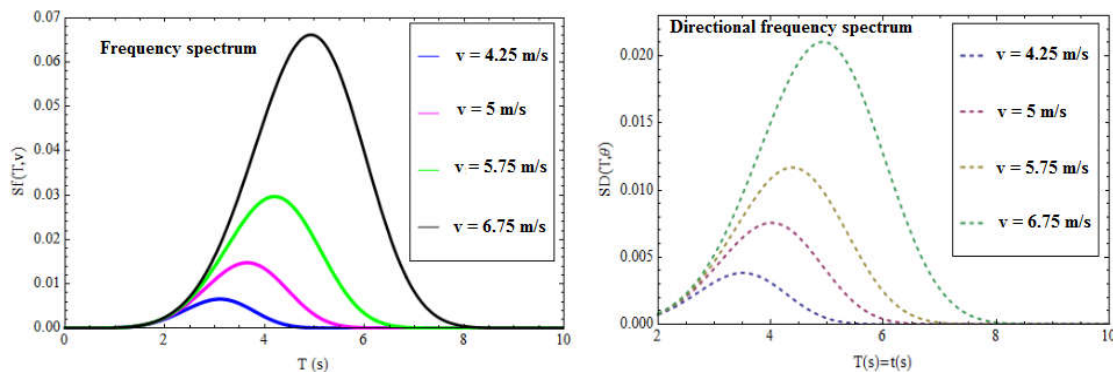


Figure 6. Representation of the directional frequency spectrum of swells

The direction from which the waves come is given by the angular position of the peak, while that the frequency is indicated by the distance to the center of the diagram.

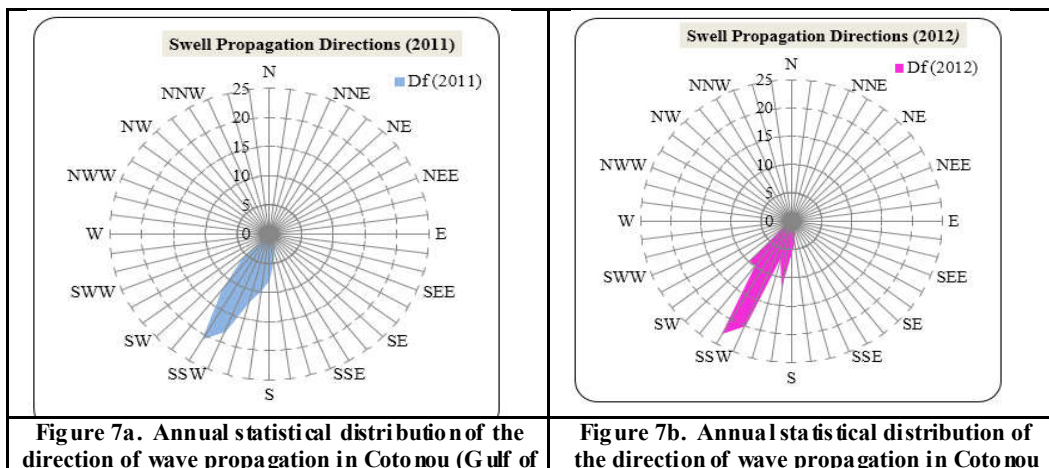
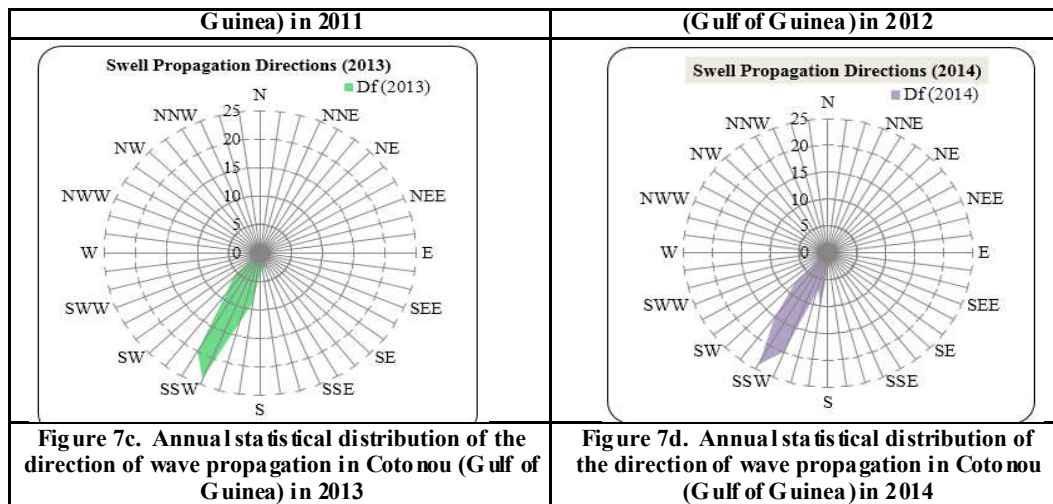


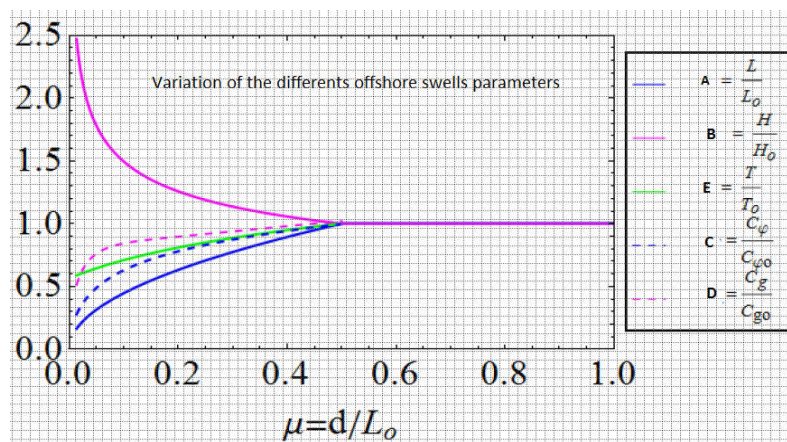
Figure 7a. Annual statistical distribution of the direction of wave propagation in Cotonou (Gulf of

Figure 7b. Annual statistical distribution of the direction of wave propagation in Cotonou





The curves in figure 8 below show the variations of the offshore swells parameters (deep waters) up to the breaking point.



**Figure 8. Variations in characteristic swells parameters at the study site**

#### Analysis and discussion of the results

The curves in Figures 2 and 3, which reflect the annual statistical distribution and distribution of wave heights according to the Beaufort scales, reveal that the annual proportions of:

- Wavelets and sheep ( $0 < H < 0,5m$ ) in Benin are less than 15%,
- Moderate waves with foam and spray with swirls at the crest of the blades of streaks of white foam whose heights vary between  $0,5m$  and  $1,8m$  ( $0,5m \leq H \leq 1,8m$ ) make approximately 80%.
- Extreme swells with very large waves with long plume crests, the water surface appears white with reduced visibility ( $H > 1,8 m$ ), represent less than 5%.

This distribution of swell shows that the sea from the Gulf of Guinea to Benin is rarely calm. It is characterized by a small proportion of wavelets and sheep and by a strong presence of very strong swells and their significant height  $H_s$  varies between  $0,8m$  and  $1,3m$  with a peak period  $T_p \approx 12s$ . The curves of figures 4a, 4b, 4c and 4d, reveal that the wave heights in Benin vary sinusoidally over the course of a day. These heights take their maximum values  $H_{max} \approx 1,3m$  and  $H_{max} \approx 1,1m$  respectively around 05h and 18h GMT while that they regain their minimum values  $H_{min} \approx 0,6m$  and  $H_{min} \approx 0,5m$  respectively around 00h and 12h. Their evolution also shows that the swells are regular in Benin. The variations of these heights during any typical day show that the swells in the Gulf of Guinea are regular in Benin and the stable average value of their height varies between  $0.5m$  and  $1.3m$ . Over the course of a year, the periods of large swells extend from June to November. The diagrams in Figure 5 show that the annual distribution of swell heights makes it possible to distinguish two seasons: a strong wave season in September and a weak swell season from December to January. The annual distribution of the different swells reveals a seasonal variation in the swell climate. Energetic or strong swells ( $H_s > 1,8m$ ) are observed in September. Average swells ( $1,0m < H_s < 1,8m$ ), are observed throughout the year. Weak swells ( $H_s < 1m$ ) cover the months of December and January. The curves in Figures 6 which reflect the influence of wind speed on the frequency spectrum and the directional spectrum of swells in the study area. The curves depend on parameters representing the speed of the wind which are at the origin of the waves. We see on these curves that in the zone of action of the wind, the waves gradually increase in amplitude, while evolving towards lower frequencies, which corresponds to longer waves. Note that wind speed increases with wave height. In other words, in both cases the peak of the spectrum is an increasing function of time and wind speed. The diagrams in Figures 7a, 7b, 7c, 7d show the statistical distribution of wave propagation directions in the coastal zone of Benin. These diagrams show that the dominant direction of swell propagation on this coast is South-South-West (SSW). However, there is a spread between the South-West (SW) and South directions on either side of the South-South-West (SSW) direction. In the Gulf of Guinea at Cotonou, the main direction of swell propagation is South-South-West (SSW). This direction makes approximately an angle  $60^\circ \leq \theta \leq 70^\circ$  relative to the West-East direction. Thus the obliquity of the swells on this coast is worth  $\alpha = \theta - \theta_0$  such as  $40^\circ \leq \alpha \leq 50^\circ$ .

All the curves in Figure 8 show that all the wave parameters in deep waters, outside of any obstacle, are almost constant. *Lescourbes*  $B = \frac{H}{H_0}$  and  $E = \frac{T}{T_0}$  represent the variations of height and period respectively in deep water and in the shoaling zone. They are almost constant in deep (offshore) waters. ( $B \approx 1$  and  $E \approx 1$ ) and strictly increase in the shoaling zone ( $1 \leq B \leq 3$ ) and this amplification stops at the breaking point. The crest-to-trough height of the swell is therefore constant in deep waters (offshore) and strictly increasing in the shoaling zone. This height, which increases strictly when the local water depth decreases. Curve A decreases in the shoaling zone. In the coastal zone of the Gulf of Guinea at Cotonou, the wave length decreases when the local water depth decreases and remains proportional to the square root of the latter ( $L \sim \sqrt{d}$ ). The evolution of curve D shows that the group speed increases when  $\mu$  varies from 0,5 at 0,125 before starting to decrease to the breaking point. As for curve D, it shows that this speed decreases strictly in the shoaling zone. This result justifies the fact that the speed of a system, which is opposed by resistance to move forward, cannot increase. The variations of C show that the phase velocity decreases but this decrease is more accentuated in the shoaling zone. The phase  $C_\phi$  and group  $C_g$  swell velocities in the Cotonou shoaling zone decrease when the local water depth decreases. But in this zone, the group speed is always greater than the phase speed.

## CONCLUSION

The four years of data made it possible to describe the evolution of the different swell heights. Offshore swells in the Gulf of Guinea are moderate big, very strong, agitated and have mean significant heights  $H_s$  between 0.45 m and 1.40 m and periods of 8 to 16 s. The swells of gust of wind (Very big) have mean significant heights between 2.10 and 2.20 m with periods that oscillate between 10 and 18 s. On the whole, the swells generally come from the South and South-South-West (SSW) directions. However, there is a spread between the South-West (SW) and South directions on either side of the South-South-West (SSW) direction. In the Gulf of Guinea at Cotonou, the main direction of swell propagation is that of South-South-West (SSW) which induces the erosion observed towards the East. The statistical study of the annual distribution of swell heights reveals that its significant value varies between 0.5 m and 1.8 m with a frequent average value of 1.1 m. The results show that significant coastal swell heights are less than 2 m. They are the main driver of the morpho-dynamic evolution of the coast. Their annual distribution makes it possible to distinguish two swell seasons: a strong swell season in September and a weak swell season from December to January. The annual distribution of the different swells reveals a seasonal variation in the swell climate. Energetic or strong swells ( $H_s > 1,8m$ ) are observed in September. Average swells ( $1,0m < H_s < 1,8m$ ) are observed all year round. Weak swells ( $H_s < 1m$ ) cover the months of December and January. The swells of storm have average significant heights of between 2.10 and 2.48 m. The peak periods oscillate between 9 and 14 s. These swells are the main driver of the profound degradation of the coast in the Gulf of Guinea. In the Gulf of Guinea at Cotonou, the swells are regular and have a constant average height  $H_{om} = 0,8m$  and an average period  $T_m = 11s$  in deep waters (offshore). It is important to note that the amplitude of the velocity decreases exponentially with a coefficient proportional to the wave number  $k$ . Therefore, the orbital velocity of waves with a longer period (lower wave number) will be more present at the bottom than that of waves with shorter periods. The propagation speed of the swells depends on their wavelength, their amplitude and when approaching the coast, the water depth. Waves are therefore characterized by energy spectra, which show characteristic quantities, with for example a significant height  $H_s$ , a significant period  $T_s$ , etc....

### Scientific Notation Tables

$v$  : Stable average wind speed in the Gulf of Guinea at Cotonou, during one day (m/s);

$\rho$  : Density of sea water ( $kg/m^3$ );

$\eta$  : Vertical elevation of the water level relative to the free surface (m);

$\beta$  : Inclination of the seabed relative to the horizontal (rad);

$g$  : Gravity acceleration ( $m/s^2$ );

$H$  : Swell crest-to-trough height (m);

$H_0$  : Height crest to trough of the offshore swell (m);

$H_s$  : Significant crest-to-trough wave height (m);

$H_{max}$  : Maximum crest-to-trough swell height (m);

$H_{min}$  : Minimum crest-to-trough wave height (m);

$L$  : Wavelength of the swell (m);

$L_0$  : Offshore swell wavelength (m);

$T$  : Swell period (s);

$T_0$  : Offshore swell period (s);

$d$  : Distance free surface - seabed (m);

$\mu = \frac{d}{L_0}$  : Distance free surface - seabed relative offshore;

$\vec{r}$  : Vector position of a point located on the free surface;

$C_g$  : Swell group speed (m/s);

$C_\phi$  : Swell phase speed (m/s);

$C_{\phi_0}$  : Offshore swell phase speed (m/s);

$C_{g_0}$  : Offshore swell group speed (m/s);

$A = \frac{L}{L_0}$  : Coefficient of variations of the wavelength;

$B = \frac{H}{H_0}$  : Height variation coefficient crest to trough of the swell;

$C = \frac{C_\phi}{C_{\phi_0}}$  : Phase speed variation coefficient;

$D = \frac{C_g}{C_{g_0}}$  : Group speed variation coefficient;

$E = \frac{T}{T_0}$  : Period variation coefficient;

$S_f(\omega)$  : Pierson-Moskowitz swell frequency spectrum;

$S_D(\omega, \theta)$  : Directional spectra of Pierson-Moskowitz swells;

MCA : Millennium Challenge Account.

## Authors Contributions

All authors have conceived the study. The main author led the drafting and coordination of the work (manuscript). All co-authors contributed to the draft and gave final approval for publication.

**Conflict of Interest:** The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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