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RESEARCH ARTICLE

INVERSE CLOSED DOMINATION IN THE CORONA OF GRAPHS

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ABSTRACT

In this paper, we investigate the inverse closed domination in the corona of graphs. Some interesting relationships are known between closed domination, independent domination and inverse closed domination.

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INTRODUCTION

Throughout this study, we only consider graphs which are finite, simple and undirected. The symbol $V(G)$ denotes the vertex set and $E(G)$ denotes the edge set of G. The order of G refers to the cardinality of $V(G)$ and the size of G refers to the cardinality of $E(G)$. The symbol $|V(G)|$ denotes the *order* of G and $|E(G)|$ denotes the *size* of G. If $|E(G)| = 0$, then G is an *empty graph*. An empty graph of order n is denoted by K_n. If $V(G)$ is singleton, G is called a trivial graph

Any graph H is a *subgraph* of G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. For a non-empty $S \subseteq V(G)$, $\langle S \rangle$ denotes the subgraph H of G for which $|E(H)|$ is the maximum size of a subgraph of G with vertex set S. An edge e of G is said to be incident to vertex v whenever $e = uv$ for some $u \in V(G)$. The symbol $G - v$ denotes the resulting subgraph of G after removing v from G and all edges in G incident to v. If $u, v \in V(G)$, the symbol $G + uv$ denotes the graph obtained from G by adjoining to G the edge uv.

The *closed neighborhood* $N_G[v]$ of a vertex v of G is the set consisting of v and every neighbor of v in G. Any $S \subseteq V(G)$ is a *dominating set* in G if $\cup_{v \in S} N_G[v] = V(G)$. A dominating set in G is also called a γ-set in G. The minimum cardinality $\gamma(G)$ of a γ-set in G is the *domination number* of G. Any γ-set in G of cardinality $\gamma(G)$ is referred to as the minimum γ-set in G.

A dominating set is an *independent dominating set* if $uv \notin E(G)$ for all $u, v \in S$. The minimum cardinality of an independent dominating set is called the *independence domination number* of G, denoted by $\gamma_i(G)$. Any independent dominating set in G of cardinality $\gamma_i(G)$ is referred to as a *γ_i-set* in G. Two distinct vertices u and v of G are *neighbors* in G if $uv \in E(G)$.

A dominating set is called a *closed dominating set* if given a graph G, choose $v_1 \in V(G)$ and put $S_1 = \{v_1\}$. if $N_G[S_1] \neq V(G)$, choose $v_2 \in V(G) \setminus S_1$ and put $S_2 = \{v_1, v_2\}$. Where possible, $k \geq 3$, choose $v_k \in V(G) \setminus N_G[S_{k-1}]$ and put $S_k = \{v_1, v_2, ..., v_k\}$. There exists a positive k such that $N_G[S_k] = V(G)$. The smallest cardinality of a closed dominating set is called the *closed domination number* of G, and denoted by $\gamma(G)$. A closed dominating set of cardinality $\gamma(G)$ is called *γ-set* of G. A closed dominating set S is said to be in its canonical form if it is written as $S = \{v_1, v_2, . . . , v_k\}$, where the vertices v_j satisfy the properties given above.
Therefore,

Let D be a minimum dominating set in G. The dominating set $S \subseteq V(G) \setminus D$ is called an *inverse dominating set* with respect to D. The minimum cardinality of inverse dominating set is called an *inverse domination number* of G and is denoted by $\gamma^{-1}(G)$. An inverse dominating set of cardinality $\gamma^{-1}(G)$ is called *γ^{-1}-set* of G. Motivated by the definition of inverse domination in a graph, we define a new domination parameter.

Let C be a minimum closed dominating set in G. The closed dominating set $S \subseteq V(G) \setminus C$ is called an *inverse closed dominating set* with respect to C. The minimum cardinality of an inverse closed dominating set is called an *inverse closed domination number* of G and is denoted by γ-1 (G). An inverse closed dominating set of cardinality γ-1 (G) is called γ-1-*set* of G.

Domination is one of the most well-studied concepts in graph theory. The reader is referred to (Canoy *et al*, Chartrand *et al* 2012, Cockayne *et al* 1977, Go *et al* 2011) for the fundamental concepts and recent developments of the domination theory, and to (Berge 1962, Haynes *et al* 2002, Walikar *et al* 1979) for its various applications. Inverse domination is studied further in (Jude Annie Cynthia *et al* 2018, Kiunisala *et al* 2014, Tamizh Chelvan *et al* 2013).

RESULTS

A classical result in the domination theory which was introduced by Ore in 1962 is the following theorem:

Theorem 2.1 *Let G be a graph with no isolated vertex and S a minimum dominating set. Then* $V(G) \setminus S$ *is a dominating set in G.*

This motivates a new domination parameter, the inverse closed domination in graphs. Theorem 2.1 guarantees the existence of a γ-1-*set* in a graph G. Since the inverse closed dominating set of any graph G of order n cannot be $V(G)$, it follows that γ-1 $(G) \neq n$ and hence γ-1 $(G) < n$. Since γ-1 (G) does not always exist in a connected nontrivial graph G, we denote by G- c 1 the family of all graphs with an inverse closed dominating set. Thus, for the purpose of this study, it is assumed that all connected nontrivial graphs considered belong to the family G- c 1. From the definitions, the following result is immediate.

Remark 2.2 *Let G be a connected graph of order* $n \geq 2$. *Then*

(*i*) $1 \leq \gamma$-1 $(G) < n$;
(*ii*) $\gamma(G) \leq \gamma(G) \leq \gamma$-1 $(G) \leq \gamma$-1 (G).

Consider, for example, the graph G in Figure 1. We have the set {f,g,h} as a minimum dominating set, thus $\gamma(G) = 3$. The set *{a, i, g, h}* is the minimum closed dominating set, thus $\gamma(G) = 4$. The set *{b, c, d, e, f, j, k, l, m}* is the minimum inverse closed dominating set, thus γ-1 $(G) = 9$ and the set *{a, b, c, d, e, i, j, k, l, m}* is the minimum inverse dominating set, thus γ-1 $(G) = 10$.

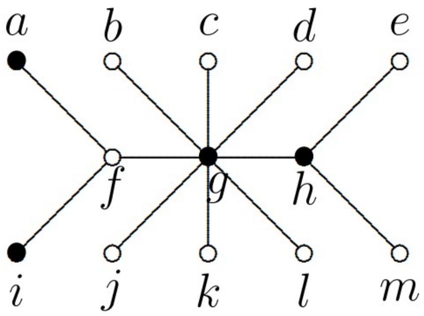

Figure 1. Graph *G* where $\gamma(G) \leq \gamma(G) \leq \gamma$-1 $(G) \leq \gamma$-1 (G)

Since any independent dominating set is a closed dominating set, it follows that the inequality $\gamma(G) \leq \gamma(G) \leq \gamma i(G)$ holds. From Figure 1, *{a, e, g, i, m}* is the minimum independent dominating set, thus $\gamma i(G) = 5$. The following remark holds.

Remark 2.3 *Let G be a connected nontrivial graph of order* $n \geq 2$. *Then* $\gamma i(G) \leq \gamma$-1 (G).

Recall that by an independent set we mean any $S \subseteq V(G)$ such that $uv \notin E(G)$ for all $u, v \in S$, and the symbol $\beta(G)$ denotes the maximum cardinality of an independent set in G. An independent set S with $|S| = \beta(G)$ is called a β-set.

CORONA OF GRAPHS

The *corona* of two graphs G and H is the graph $G \circ H$ obtained by taking one copy of G and $|V(G)|$ copies of H, and then joining the *ith* vertex of G to every vertex in the *ith* copy of H. We denote by Hv that copy of H whose vertices are adjoined with the vertex v of G. In effect, $G \circ H$ is composed of the subgraphs $Hv + v = Hv + \langle\{v\}\rangle$ joined together by the edges of G.

Theorem 3.1 (Canoy, S.R. Jr. and M.A. Labendia) *Let G be a connected graph and H be any graph. Then* $C \subseteq V(G \circ H)$ *is an independent dominating set in* $G \circ H$ *if and only if* $C \cap V(G)$ *is an independent set in G and* $C \cap V(Hv + v)$ *is an independent dominating set in* $Hv + v$ *for every* $v \in V(G)$.

Theorem 3.2 (Canoy, S.R. Jr. and M.A. Labendia) *Let G be a connected graph of order n and H be any graph with* $\gamma(H) \neq 1$. *If* $C \subseteq V(G \circ H)$ *is a* γi*-set in* $G \circ H$*, then* $C \cap V(G)$ *is a* β*-set in G.*

Theorem 3.3 (Go C.E. and S.R. Canoy Jr. 2011) *Let G be a connected graph and H be any graph. Then* $C \subseteq V(G \circ H)$ *is a dominating set in* $G \circ H$ *if and only if* $C \cap V(Hv + v)$ *is a dominating set in* $Hv + v$ *for every* $v \in V(G)$.

Theorem 3.4 (Tacbobo, T.L. and F.P. Jamil 2012) *Let G and H be any two graphs, with* $\gamma(H) \geq 2$, *and let* $S \subseteq V(G \circ H)$. *If S is a minimum closed dominating set in* $G \circ H$, *then* $S \cap V(G)$ *is a closed dominating set in G. In this case, if* $S \cap V(G)$ *is nonindependent (resp. independent) dominating in G, then* $S \cap V(G)$ *is a maximum non-independent (resp. independent) closed dominating set in G.*

In particular, Theorem 3.1, Theorem 3.2 and Theorem 3.4 yield the following corollary.

Corollary 3.5 *Let G and H be connected graphs with* $\gamma(H) \geq 1$, *and let* $S \subseteq V(G \circ H)$. *Then S is a* γ*-set in* $G \circ H$ *if and only if* $S \cap V(G)$ *is a* β*-set in G and* $S \cap V(Hv)$ *is a* γ*-set in Hv for all* $v \in V(G) \setminus S$

Proposition 3.6 *Let G and H be any graphs. Then*

γ-1 $(G \circ H) = (\gamma(H) - 1)\beta(G) + |V(G)|$.

Proof: Verify that the claims are true for $|V(G)| = 1$ or $\gamma(H) = 1$. Now we assume that $|V(G)| > 1$ and $\gamma(H) > 1$. Let $S \subseteq V(G \circ H)$ be a γ-1-*set* in $G \circ H$, and let $D \subseteq V(G \circ H) \setminus S$ be a γi-set in $G \circ H$. By Theorem 3.2, $D \cap V(G)$ is a β-set in G and for each $v \in V(G \circ H) \setminus D$, $D \cap V(Hv)$ is a γ-*set* in Hv. For each $v \in D \cap V(G)$, choose a γ-set Sv in Hv, and define $S* = (\cup_{v \in D \cap V(G)} Sv) \cup (V(G) \setminus D)$. By Theorem 3.3, $S*$ is a dominating set in $G \circ H$. Since $S* \cap D = \emptyset$, $S*$ is an inverse close dominating set in $G \circ H$. Thus, $|S| \leq |S*|$. On the other hand, for each $v \in D \cap V(G)$, $v \notin S$ so that $S \cap V(Hv + v) = S \cap V(Hv)$. By Theorem 3.1, $S \cap V(Hv)$ is a dominating set in Hv, and thus, $|Sv| \leq |S \cap V(Hv + v)|$ for all $v \in D \cap V(G)$. Thus,

$$|S| \geq \sum_{v \in D \cap V(G)} |S_v| + |V(G) \setminus D| = |S^*|.$$

Therefore, $|S| = \gamma(H)\beta(G) + |V(G)| - \beta(G) = (\gamma(H) - 1)\beta(G) + |V(G)|$.

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