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RESEARCH ARTICLE

THE INFLUENCE OF THE RAYLEIGH NUMBER ON THE FLOW OF NATURAL CONVECTION IN A DIFFERENTIALLY HEATED CAVITY

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ABSTRACT

In this work, we present a two-dimensional numerical study in a square cavity filled with air. The flow of natural convection is laminar in unsteady regime. The two vertical sides are subject to an isothermal temperature difference, while the other two horizontal walls are adiabatic. The objective of our study is to highlight the influence of Rayleigh numbers on the structure of the flow, the temperature distribution in the cavity and on heat transfer. The equations of conservation of mass, momentum and energy were solved by the COMSOL multiphysics computer code. The numerical method used is that of finite elements implemented in the COMSOL calculation code. A validation study in a differentially heated square cavity was carried out in order to compare our results to those of the literature. We studied pure natural convection by varying the Rayleigh number from 1 to 10^7. Results are presented as isotherms, pressure, streamlines and temperature fields. It emerges from this study that for a Rayleigh number less than 1000, the fluid presents a vertical thermal stratification due to heat transfer only by conduction which does not generate any convective flow. But for Ra greater than 1000, we see distortions due to the influence of the Rayleigh number on natural convection. The fluid is Newtonian for a Prandtl number $P_r = 0{,}71$.

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INTRODUCTION

The study of natural convection in cavities has been the subject of several research studies in recent years because of its importance in scientific applications: boilers, heating of a house (case of a radiator) and energy conservation …Etc. This method of transfer has a very wide scope of application. Several authors have looked at this study with different cavities. The authors studied experimentally and numerically in laminar and unsteady flow by varying the Rayleigh number (Ra) which is a constant of natural convection. This is how, between the authors [1] [2], had studied free convection in a differentially heated square cavity using the finite difference method for the resolution of the equations. Authors such as [3] have numerically studied the circulation of air in natural convection in laminar regime in an inclined square enclosure. The so-called passive horizontal walls are adiabatic, and the vertical walls are isothermal. The equations governing the phenomenon are solved by the finite volume method in Fortran language and Fluent as calculation code. The authors also studied the influence of certain parameters such as the Rayleigh number, the Nusselt number and the inclinationα of the cavity on the velocity and temperature profiles. The analysis of the results of the numerical simulation shows us that from the Rayleigh number$R_a = 10^4$, the convective movement begins and for $R_a = 10^6$ the convection is dominant. The authors concluded that the heat exchange increases in the enclosure when R_a increases therefore $R_a$has a great influence on the thermal convection in the cavity. [4] carried out studies of natural convection in a 2D square enclosure filled with air whose horizontal walls are adiabatic. The left vertical wall is heated to spatial sinusoidal temperature variables relative to a constant average value which is higher than the temperature of the right-side wall. To show that the parameter which governs natural convection in a differentially heated cavity is the Rayleigh number, [5]studied in a cavity, the evolution of isotherms for different values of the Rayleigh number. They showed that for $R_a = 10^3$, the structures of the isotherms remained parallel to the vertical wall, which means that the heat transfer is pure conduction in this zone. When they increase the Rayleigh number to$R_a = 10^6$,they see that the isotherms are more curved so this shows that there is a significant temperature gradient. The circulation of air in a habitat which is a closed cavity whose walls are subjected to isothermal temperatures, has been the subject of several experimental researches for decades by [6] [7]. These authors have shown in the literature through experimental results and modeling the presence of two types of flows which are generated by buoyancy forces. The different configurations were used to define the types of flow in the cavities. The authors varied the Rayleigh number R_a to study its behavior on the flow in this closed cavity and the studies showed that for$R_a \leq 10^3$, heat transfer by conduction dominates. ForR_a between 10^3 and 10^9, this variation generates an intensification of the looped air circulation in the boundary layer near the vertical walls. In this interval of$10^3 \leq R_a \leq 10^9$, the flow is laminar.

When we go to$R_a > 10^9$ the flow regime is turbulent. A validation study in a differentially heated square cavity was carried out to compare our results with those in the literature (Hong Wang, 2006). In the case of our two-dimensional numerical study, the cavity studied has the shape of a square whose vertical walls are isothermal, and the horizontal walls are adiabatic. The flow of natural convection is laminar in unsteady regime. We study the influence of Rayleigh numbers on flow structure, temperature distribution and heat transfer using the finite element method implemented in the COMSOL Multiphysic computer code. The results will be obtained in the form of temperature fields, velocity isovalues and vorticity. the analysis of the behavior of the Rayleigh number on the flow would be pronounced.

Physical Model and Mathematical formulation

Physical Model: [8] is very concrete and clearly illustrates the nature of the problem. Let's imagine a room whose floor and ceiling are supposed to be adiabatic. One of the vertical walls has a bay window; the one facing it is equipped with a wall radiator; it is cold outside, and the radiator is heated (Figure 1). The surface temperature of the bay window is lower than the ambient temperature in the windowed room. An air flow will therefore be established downwards. On the opposite wall, the radiator is hotter than the ambient air and an upward flow will take place. Such a phenomenon is called convection. In the case of our study, we take the cavity as being a dwelling with a surface area of 1m*1m. The figure represents the differentially heated cavity whose two vertical sides are at$T_C = 293{,}5K$ and $R_a = 288{,}5K$. the horizontal sides and the front and rear faces are adiabatic. The numerical study of natural convection in our case would be carried out in two dimensions by the finite element method for this, we pose some..The hypotheses adopted for this simulation are: No internal heat sources, The mass transfer is negligible The heat transfer by radiation is negligible, The heat released by viscous friction is negligible compared to any other phenomenon, the medium is assumed to be transparent. The fluid is Newtonian, incompressible and obeys the Boussinesq approximation.No volumetric heat production. Before the initial instant, we assume the air is at rest in the enclosure at an average temperature $T_o = (T_C + T_f)/2$ With $T_C > T_f$ respectively the temperatures of the hot wall and the cold wall.

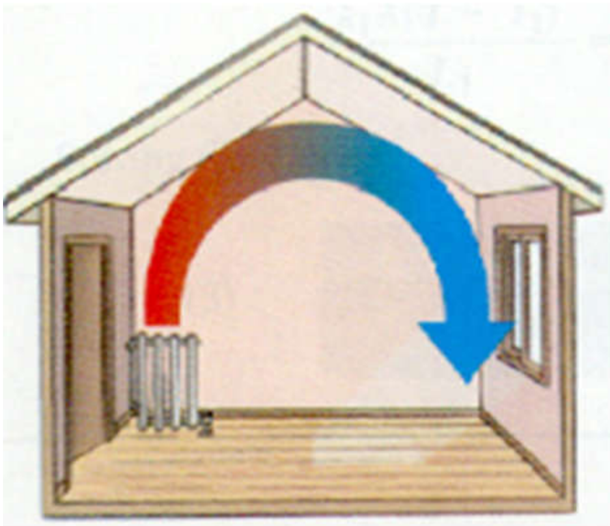


Figure 1. Convection phenomenon illustrated by the rise of hot air (in red) generated by the radiator and cools on contact with the ceiling, then descends (in blue) [8]

Mathematical Formulation

For the numerical simulation of natural convection, let us first present the equations which can model this natural convection. This expresses the laws of conservation of mass (continuity), momentum (Navier-stores) and energy. We apply the equations in the case of a plane cavity taking into account the discussion of the numerical method used for the resolution of these equations

When the vertical walls of a cavity filled with air are subjected to a constant temperature difference, the flow generated depends on several parameters, the main ones of which are:

-The Rayleigh number $R_a = \frac{g.\beta.\Delta T.H^3}{\alpha\nu}$ - (1)

-The Prandtl number $P_r = \frac{\nu}{\alpha}$

Dimensionless equations

Equation dimensionality consists of making equations dimensionless using reduced variables. . Let's bring these equations into dimensionless form, to do this let's define the characteristic quantities by designating by:

- H a characteristic linear dimension of the flow,
- V0 the reference speed
- t0 a reference time,
- 5t0 being the reference temperature difference
- P0 a reference pressure:

$V_0 = \sqrt{g\beta H\Delta T_0}$; $t_0 = \frac{H}{V_0}$ $p_o = \rho_o V_o^2$By designating these characteristic quantities, we will obtain the following reduced variables:

$x = \frac{X}{H}$;$y = \frac{Y}{H}$;$u = \frac{U}{V_0}$;$v = \frac{V}{V_0}$;$\tau = \frac{t}{t_0}$; $\theta = \frac{T-T_0}{T_C-T_f}$;$\acute{P} = \frac{P}{P_0}$The characteristic scales used for dimensionless equations analogous to those of the references (A.ELKASMIL, 1999) (Cheikh, 2007) [9] [10]

-the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4)$$

-momentum equations

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_0}\sqrt{\frac{P_r}{R_a}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial P}{\partial x}\frac{\partial v}{\partial \tau} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \ddot{\Theta} + \frac{1}{\rho_0}\sqrt{\frac{P_r}{R_a}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial P}{\partial y} \quad (5)$$

-The energy equation

$$\frac{\partial \Theta}{\partial \tau} + u\frac{\partial \Theta}{\partial x} + v\frac{\partial \Theta}{\partial y} = \frac{\Delta T}{\sqrt{R_a P_r}}\left(\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2}\right) \quad (6)$$

• Vorticity-current function formalism.
The advantage of using this formalism is to reduce the number of equations and to bring out dominant variables [11] In the case of a two-dimensional Cartesian system defines:
Vorticity:

$$\Omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial x} (7)$$

The current function Ψ defined by

$$:\begin{cases} \frac{\partial \Psi}{\partial y} = u \\ \frac{\partial \Psi}{\partial x} = -v \end{cases} (8)$$

Alors $\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \Omega = 0 (9)$

$$\frac{\partial \Omega}{\partial \tau} + u\frac{\partial \Omega}{\partial x} + v\frac{\partial \Omega}{\partial y} = \frac{1}{\rho_0}\sqrt{\frac{P_r}{R_a}}\left(\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2}\right) - \frac{\partial \theta}{\partial x} (10)$$

dimensionality of boundary conditions
-We impose a non-slip condition on the fluid particles on the walls of the rigid and impermeable enclosure, so that: $U = V = 0$ on these boundaries -At the initial instant, τ=0 $\Psi(x, y) = 0$; $\Omega(x, y, 0)$; $\theta(x, y, 0) = 0$

- for the temperature on the boundaries of the physical domain: :$\begin{cases} \theta = 1 \quad pour x = 0 ; 0 \le y \le H \\ \theta = 1 \quad pour x = 0 ; 0 \le y \le H \end{cases}$
- while for adiabatic walls: $\frac{\partial \theta}{\partial y} /_{y=0,1} = 0$.

The theoretical analysis undertaken made it possible to reduce the number of variables to three (the temperature, the current function Ψ and the vorticity(Ω), and to demonstrate that the system to be solved is a function of two main quantities: the Prandtl number P_r and the Rayleigh number R_a
Concerning the reference length H used in the work of [12]

$R_a = \frac{g.\beta.\Delta T.H^3}{\nu\alpha}$, R_a=1,10310^9 H^3(15)

- The following thermo-physical properties of air are taken at 20°C: Thermal conductivity λ; Stefan-Boltzmann constant σ ; Kinematic viscosity V; Thermal expansion coefficient β; heat capacity at pressure notedCp; Density ρ;

RESULTS AND DISCUSSION

Validation: In order to verify the accuracy of our numerical work, a validation of the numerical code is carried out by taking into account certain numerical and experimental studies which exist in the literature. Indeed, we will validate our work with the results of (Hong Wang, 2006) [12] in the case of a square cavity containing air whose two sides, ceiling and floor, are assumed to be adiabatic but at uniform temperatures. The left and right vertical faces are differentially heated to ΔT=10K. The validation curves are represented following the study.

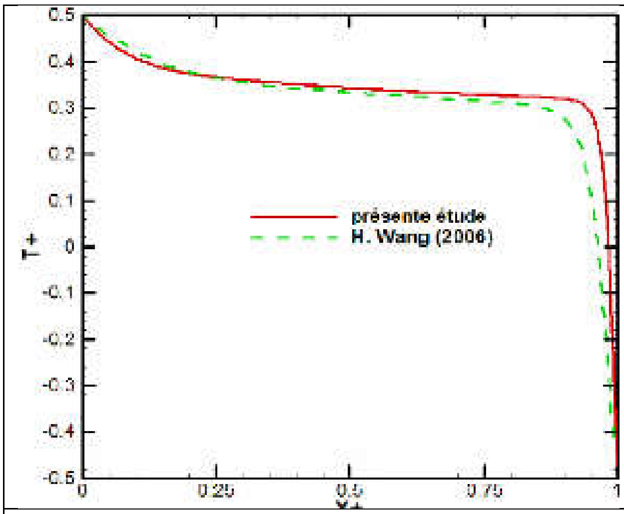


Figure a: comparisons of high wall temperature profiles[12]

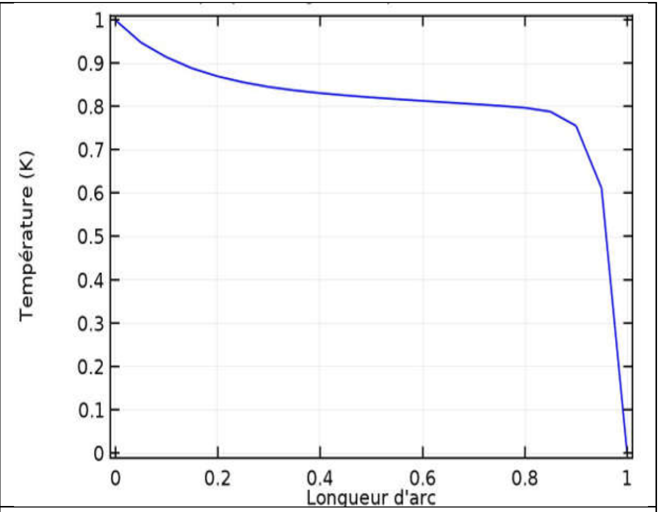


Figure b: upper wall temperature (the present work)

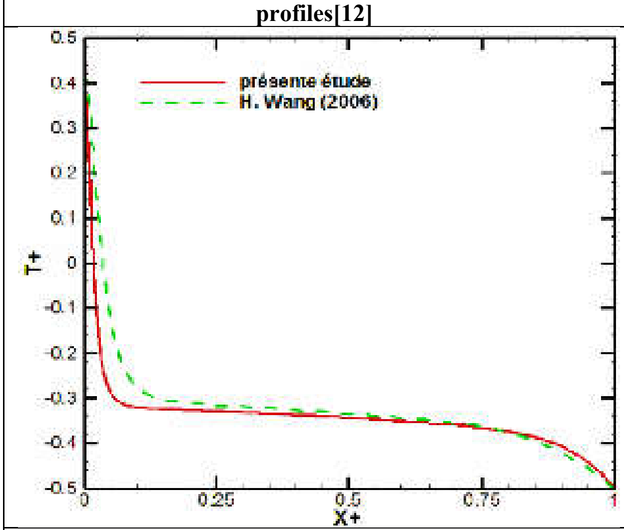


Figure c: comparisons of temperature profiles on the lower wall [12]

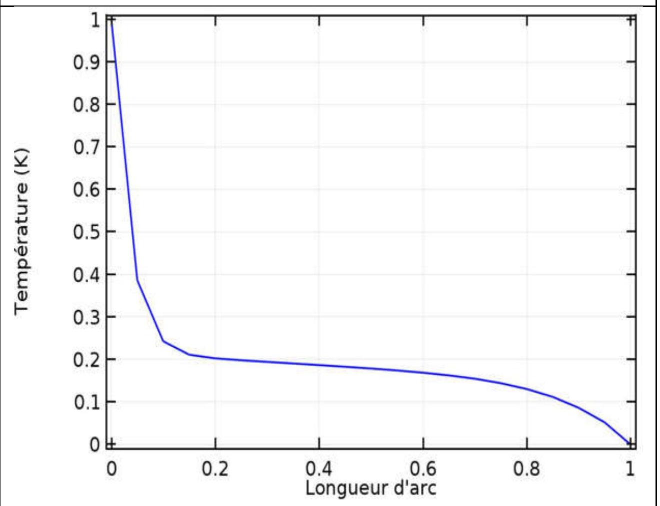


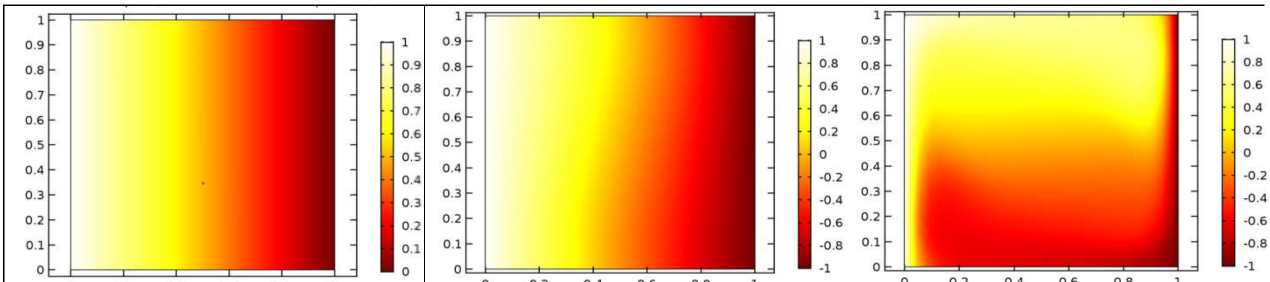
Figure d: temperature of the lower wall (the present work)
Figure d: temperature of the lower wall (the present work)

Figure 3. validation curves

Figure 3 shows the temperatures near the top wall and the bottom wall. From the first contact of the fluid with the hot wall Figure (a), we notice an upward movement of the hot air. There is a strong heat exchange and gradually as the fluid moves along the upper adiabatic wall it cools. Figure (b) shows that the air cools as it descends towards the lower adiabatic wall.

RESULTS AND DISCUSSION

As part of our study, the numerical simulation was carried out for Rayleigh numbers ranging from$R_a = 1$ to $R_a = 10^7$



$R_a = 10^3$	$R_a = 10^4$	$R_a = 10^7$

Figure 4 above represents the temperature field in the differentially heated cavity whose two horizontal sides are adiabatic and the other two verticals are at$T_C = 293,5K$ and $T_f = 288,5K$. We notice a high temperature close to the hot wall and gradually, there is distribution of this temperature towards the middle of the cavity, but it is weak at the edge of the cold wall.

The more the Rayleigh number increases, the more the heat flow moves towards the middle of the cavity. We observe that at$R_a = 10^7$ this flow reaches the cold wall and causes the cold heat to move towards the floor.

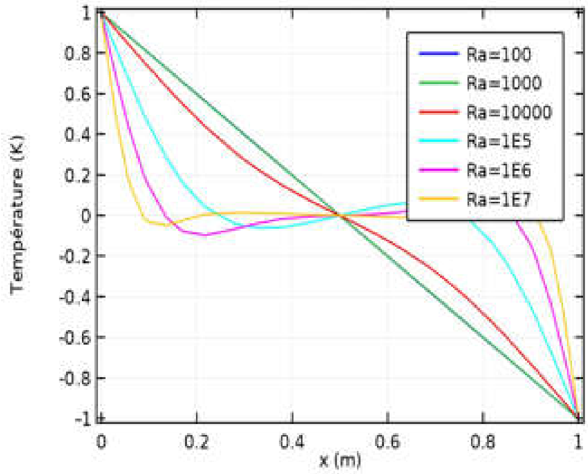
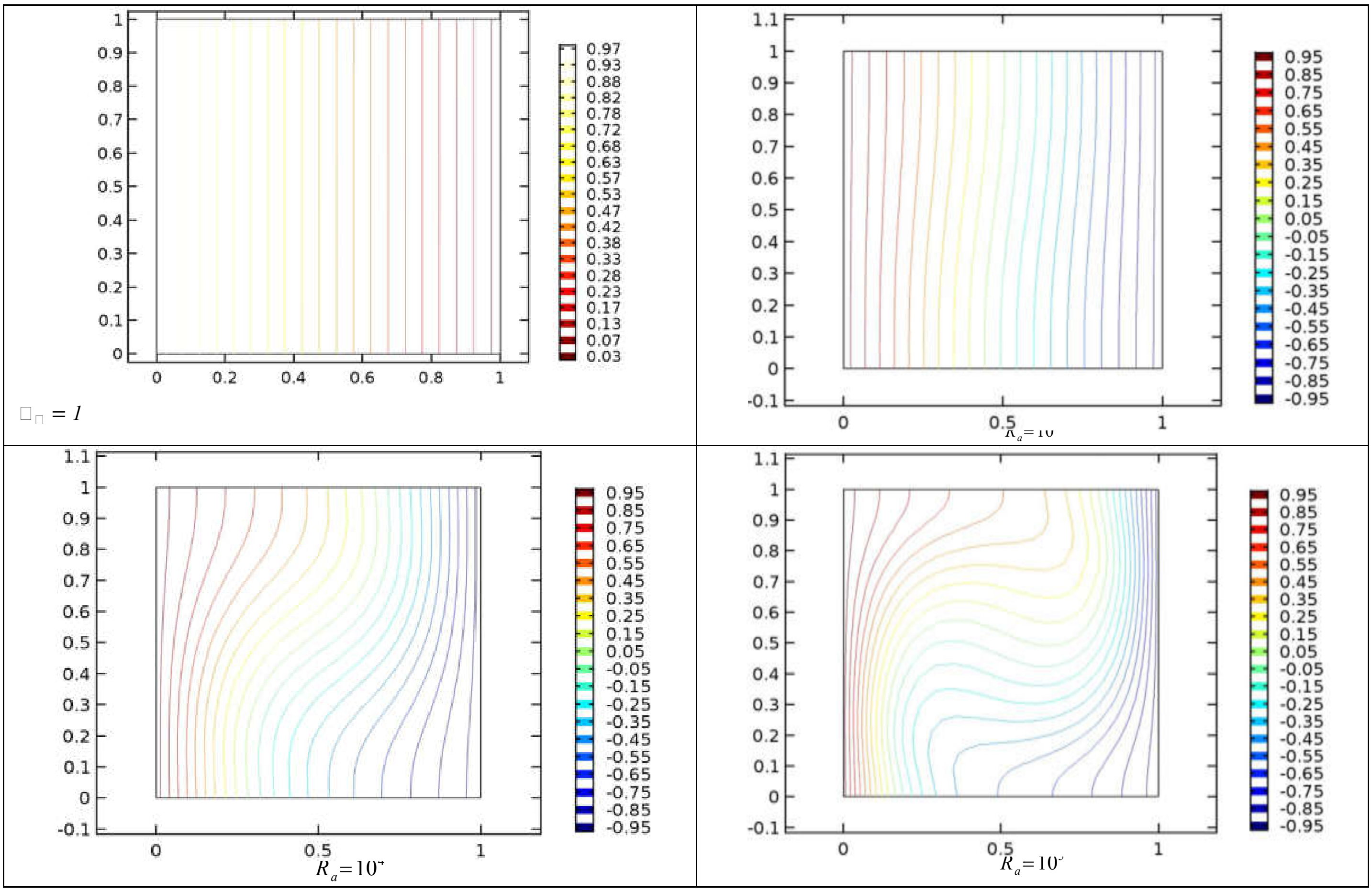


Figure 5. Graph of temperatures for different values of Ra

Figure 5 shows the temperature profiles as a function of x for different values of R_a. The temperature distribution is linear for$R_a \leq 10^3$, and a strong temperature gradient in the vicinity of the isothermal faces is observed when Ra increases up to 10^7. Figure 6 represents the temperature isovalues for different values of the Rayleigh number, it shows that for a low value of the Rayleigh number, the fluid (air) presents a vertical thermal stratification due to heat transfer by conduction which does generate no convective flow. From the result presented in this figure, we can see that the energy transfer for is one-dimensional. This is explained by the fact that the isotherms are perpendicular to the main direction of heat transfer, direction ox. For $R_a \succ 10^3$ there is distortion of the isotherms and their contraction at the vertical interfaces is mainly due to the Rayleigh number, which is relatively high, and to the significant temperature gradient in this interface.



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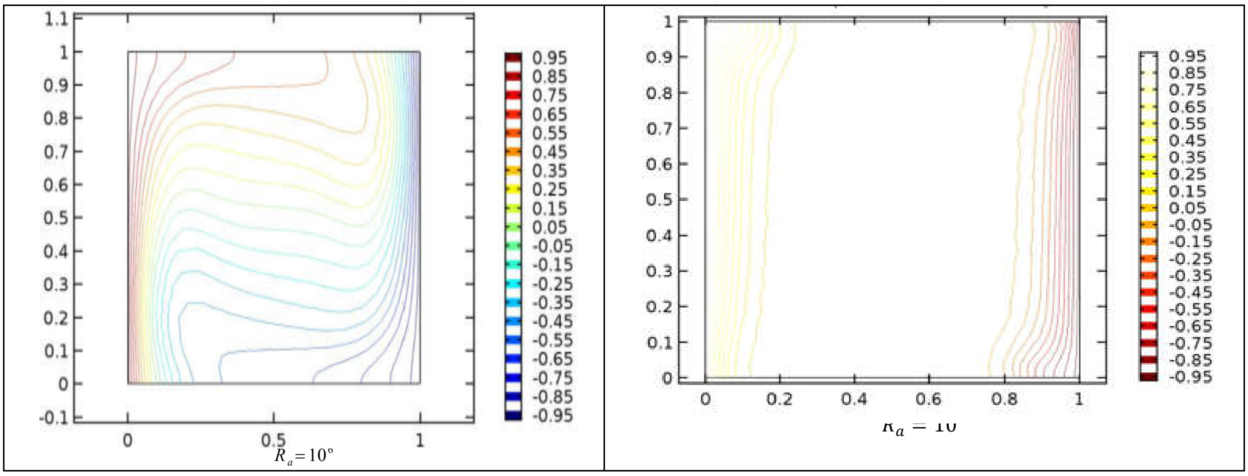

Figure 6. Temperature isovalues for different Rayleigh values

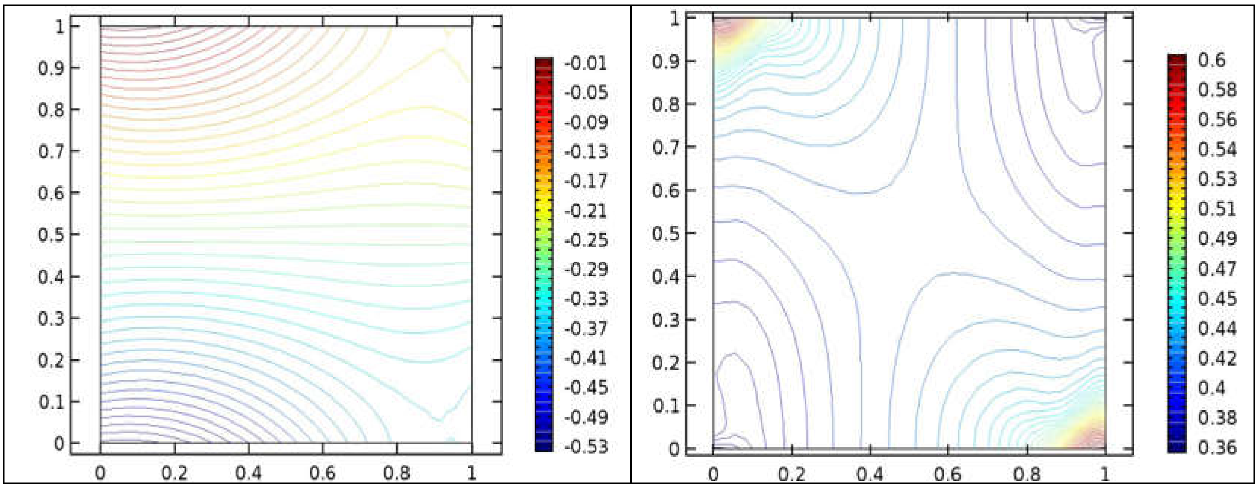

Figure 7a : $R_a = 1$	Figure 7b : $R_a = 10^7$

Figure 7. Pressure isovalues different values of R_a

The Rayleigh number modifies the temperature isotherms Figure 7a and 7b represent the pressure isovalues in the enclosure. For $R_a = 1$, the flow pressure is very low throughout the enclosure space. For $R_a = 10^7$ we observe an increase in pressure at the two opposite corners but an absence of pressure in the center of the cavity. We can say that when we increase the Rayleigh number, the flow presents a strong pressure towards the walls and at the corners of the cavity.

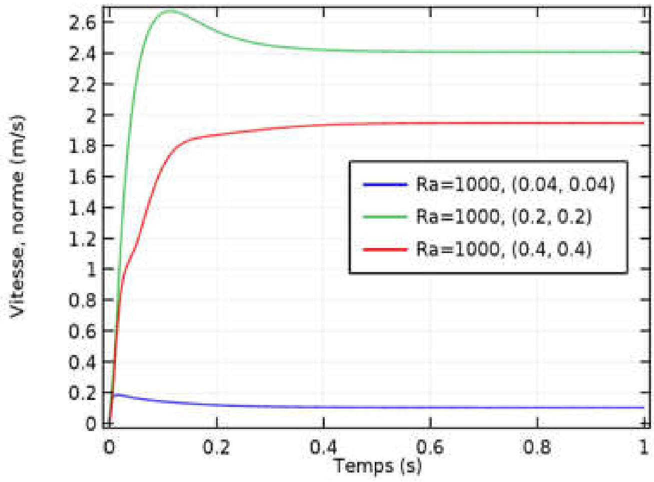

Figure 8. Point graph of velocity at different position for $R_a = 10^3$

This figure 8 reflects the evolution of the temperature at different positions for$R_a = 10^3$. We notice that close to the hot wall, the fluid particles have a low speed at the point (x=0.04; y=0.04) but gradually, the fluids have a high speed then this speed stabilizes and takes a horizontal direction.

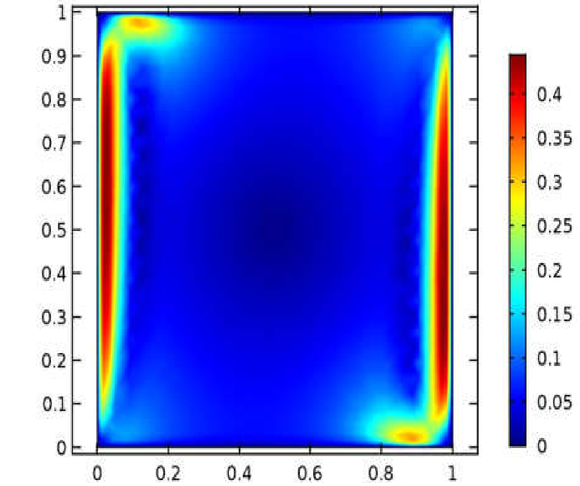

Figure 9: velocity field for$R_a = 10^7$

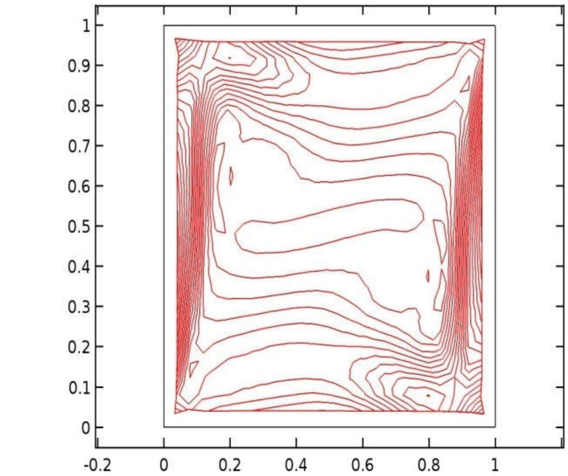
Figure 10: velocity isovalues for $R_a = 10^6$

A simple and very concrete example clearly illustrates the nature of the problem. Let's imagine a room whose floor and ceiling are supposed to be thermally insulated. One of the vertical walls has a bay window; the one facing it is equipped with a wall radiator; it is cold outside and the radiator is heated (figure I). The surface temperature of the bay window is lower than the ambient temperature T_∞ in the windowed room. An air flow will therefore be established downwards. On the opposite wall, the radiator is hotter than the ambient air and an upward flow will take place [8]. Figure 9 and figure 10 show that near the hot wall (T_C), the fluid subjected to a high temperature becomes light following its low density and therefore carries out an upward movement up to the ceiling while, near the cold wall (T_f) the fluid close to this wall and therefore heavy descends towards the lower adiabatic wall (plank). This movement is done until the fluid reaches thermal equilibrium.

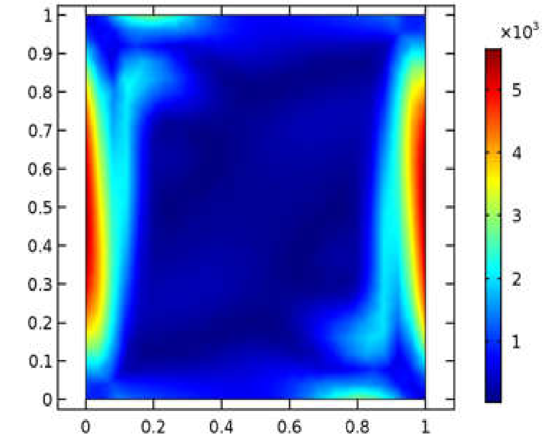

Figure 11. vorticity (1/s) For $R_a = 1$

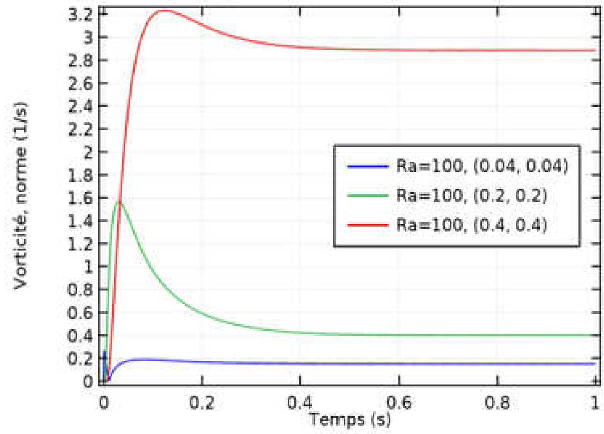

Figure 12. Evolution of vorticity (1/s) for $R_a = 10^2$

We notice that the fluid located near the hot wall undergoes, for $R_a = 10^2$ and in different positions, a significant thermal exchange from the first contact. Then the heat exchange gradually decreases to reach an asymptotic limit. The opposite phenomenon is observed at point (0.2;0.2) by comparison with Figure 8 because the vorticity is the inverse of the flow speed.

CONCLUSION

So far we have not said where the moving fluid comes from and where it goes. Indeed, in a closed enclosure where the mass of the fluid is constant, any local movement will have repercussions more or less markedly over the entire domain of the fluid.

In natural convection, such a movement will be generated by temperature gradients and it can give rise to circulation of the fluid contained in the enclosure. we note that the hot fluid located close to the hot wall rises to the level of the ceiling (high wall), it descends along the cold wall then returns to the hot wall via the floor. we see that the temperature distribution is linear for low Ra, and a strong temperature gradient near the isothermal faces when Ra increases. Following this study, we plan to move on to coupling by applying thermal radiation to the walls of the cavity, to see the influence of emissivity on natural convection.

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