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RESEARCHARTICLE

# ESTIMATION OF STATE-OF-CHARGE OF LITHIUM-ION CELL USING EXTENDED AND SIGMA-POINT KALMAN FILTERS FOR BATTERY MANAGEMENT SYSTEM

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ABSTRACT

Lithium-ion batteries are state-of-the-art energy storage technology. Instead of having remarkable features, a highly accurate, reliable, and cost-effective battery monitoring technology should continuously monitor the battery cell parameters and ensure the parameters are within the safe operating area recommended by the manufacturer. Direct measurement of the Li-ion cell's state of charge or SOC is impossible but can be estimated with reasonable accuracy with the help of state estimation algorithms. A precise estimation of SOC is always needed to enhance the cycle life and safety of lithium-ion batteries. Since the internal electrochemical kinetics of the Li-Ion cells are highly complex and non-linear, the non-linear variants of the Kalman filter, such as the Extended Kalman filter (EKF) and Sigma-Point Kalman filter (SPKF) perform exceptionally well in the presence of uncertainties in initial estimates and sensor measurements. This article evaluates the performance of EKF and SPKF for SOC estimation accuracy in terms of RMSE error. The experiment results show that SPKF slightly outperforms EKF. Both EKF and KF demonstrate strong robustness against current noise.



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# INTRODUCTION

Due to the global energy crisis, such as environmental pollution issues and climate change, electric vehicles (EVs) are being developed as alternatives to traditional internal combustion engine-powered vehicles. Due to the remarkable features of lithium-ion-based battery cells, such as high energy density, high cycle life, high nominal voltage, insignificant self-discharge rate, and no memory effect, LIBs are extensively used to manufactureEV battery packs. However, as a very temperature-dependent and loose life when operating at over-voltage and under-voltage conditions, close monitoring of the battery behaviour to estimate states such as state-of-charge is essential to maintaining safe and efficient operation, emphasising the importance of the battery management system (BMS). The BMS serves several purposes, including determining the remaining energy, estimating available power, voltage monitoring, cell balancing and lifetime predictions(1), (2), (3). The SOC is a dimensionless quantity usually represented as a percentage. It characterises the Li-Ion cell's available capacity and is mathematically represented as the ratio of available coulombs of charge to the total rated charge storing capacity in amp-hours.

$$SOC = \frac{Available\ amp{-}hours\ of\ cha}{Rated\ capacity\ (amp{-}hours)} \times 100\% \qquad (1)$$

SOC estimation is crucial for the BMS to perform tasks such as active or passive cell-balancing, avoid over- and under-charging conditions, and temperature detection to prevent the overheating of the Li-ion cells. Unfortunately, SOC cannot be measured directly by any high-end sensor technologies, so better and more robust algorithms with efficient sensors could estimate the SOC with high accuracy and precision. The researchers have proposed several state-of-charge estimation approaches, each with advantages and disadvantages. The methods are categorised based on the control strategies, such as open loop and closed loop and the choice of the Li-ion cell equivalent circuit model being implemented, such as Ampere-hour integration or coulomb counting, data-driven based and modal-based methods (2), (3), (4), (5). The Ampere-hour interaction method is open-loop and susceptible to error accumulation due to uncertainties in the sensor measurements and unknown estimates of the initial value of SOC and total capacity. The current sensor measurement contains biased components, sensor uncertainties, self-discharge components, etc. The Li-ion cell must be relaxed for a significant duration to achieve equilibrium of terminal voltage as open-circuit voltage tolookupthe corresponding SOC estimate. The process is infeasible for EV applications where recalibration is required when batteries are in dynamic conditions with frequent charging, discharging, and charge balancing processes. In contrast, other methods like black-box and model-based methods rely on large data sets of the Li-ion cell charging and discharging profile at different temperature conditions. However, the strategies are closed-loop with a relatively high estimation accuracy(3), (4), (5), (6). In a black-box-based approach, extensive training in the battery model, which is usually a neural network-based equivalent model of the cell, is required, which is a computationally burdensome and time-consuming process. In contrast to the black-box-basedmethod, the model-based approach is a trade-off between performance and time. The method requires an approximate battery model and knowledge of the sensor's covariance matrices and initial state uncertainties to predict the state reasonably. However, the method is a Minimum Mean Square Error Estimator (MMSE), which guarantees estimates convergent towards the actual value (2), (3), (4), (5), (6). Linear Kalman Filter is a model-based MMSE state observer that works on the recursive Bayesian estimation approach and is an optimal state estimator when the model of any system is linear with uncertainties assumed to be Gaussian and white. However, in practice, models of any complex dynamical systems are non-linear,and a linear Kalman filter cannot produce an estimate with acceptable accuracy. So, to tackle this challenge, non-linear variants of the Kalman filter can be implemented, such as Extended Kalman Filter and Unscented Kalman Filter (1), (7), (8). The Extended Kalman Filter is suitable for slightly non-linear models but is not exempted from the assumption that the uncertainties must be Gaussian and white. In the case of an Unscented Kalman filter, the model could be highly non-linear, and the method is exempted from the assumption of Gaussian uncertainties. However, implementing these algorithms comes with the expense of an intensive computational burden and high processing time. These methods produce reasonable estimates of SOC with a high convergence rate and narrower error bounds(1), (6).

## BATTERY MODELLING

***State-of-Charge Definition:*** SOC is the ratio of the remaining coulombs of charge (amp-hours) in a Li-ion cell to its maximal charge-storing capacity (amp-hours). The SOC can be calculated if the initial SOC, total capacity, and charging or discharging current are known for the duration and can be mathematically described in Eq. (2).

In the continuous-time domain,

$$SOC_t = SOC_{t_0} - \frac{1}{Q}\int_{\tau=t_0}^{t}(\eta(\tau)\, i_{cell}(\tau))d\tau \quad (2)$$

Where η(t) is a coulombic efficiency, In the discrete-time domain, Eq. (2)can be represented as,

$$SOC[k+1] = SOC[k] - {\eta[k]\, i_{cell}[k]\Delta t}/{Q} \quad (3)$$

$i_{cell}$ is a charging or discharging current, which I considered positive and negative while discharging and charging. 'Q' is a recent Li-ion capacity since the cell loses capacity due to a rise in the cell's equivalent series resistance (ESR) after several charge-discharge cycles, called 'capacity fade'.

***Equivalent Circuit Model (ECM) of Li-Ion Cell:*** Li-ion cells incorporate different complex electrochemical kinetics inside, such as mass transfer, migration of ions between electrodes, side reactions and current collector reactions. Thus, battery models with high reliability and accuracy are crucial for the model-based estimation approach(1), (2), (3).The Li-ion battery cell model can be categorised as an Empirical Model (EM), Electrochemical Model (ECM), Electrical Equivalent Circuit Model (EECM), Electrochemical Impedance Model (ECIM), and Data-Driven Model (DDM). EECM is usually implemented for state-of-charge estimation because of its simplicity, lower computational burden, and high compatibility with embedded system applications. The Electrical ECM can be categorised into the simple Rint, Randles, and nth-order RC (nRC) models(1), (2), (3).In this article, I have implemented the algorithm to estimate the SOC for the general nRC model, including the hysteresis component in series with the OCV source. The mathematical model for the single RC EECM model using Kirchoff's law is shown in Fig. 1.

$$\frac{dz(t)}{dt} = \dot{z}(t) = -\frac{\eta(t)i_{cell}(t)}{Q_{cell}} \quad (3)$$

Where z(t) is the SOC at time 't'. We have assumed that the cell current is negative while charging, and the time rate of change of SOC increases. Similarly, the cell current is positive while discharging, and the time rate of change of SOC decreases.

$$\dot{v_p}(t) = \frac{dv_P(t)}{dt} = -\frac{1}{R_1C_1}v_p(t) + \frac{1}{C_1}i_{cell}(t) \quad (4)$$

$$\dot{i_R}(t) = \frac{di_{R_1}(t)}{dt} = -\frac{1}{\tau_1}i_{R_1}(t) + \frac{1}{\tau_1}i_{cell}(t) \quad (5)$$

$$\tau = R_1C_1$$

Where'τ' is a time constant representing the slow diffusion process towards equilibrium OCV due to the Hybrid Pulse Power Characterization (HPPC) pulse charging or discharging test.

$$v(t) = OCV(z(t)) - R_0 i_{cell}(t) - v_p(t) \quad (6)$$

### *State-space Model of the Li-Ion Cell*

In this paper, we have considered the hysteresis effect in the OCV-SOV characteristics, which is described as hysteresis voltage component $v_H(t)$ in the EECM in series with OCV(z(t)). The hysteresis state equation is expressed mathematically in Eq. (7),

$$\dot{h}(t) = A_H(t)\, h(t) + B_H(t)\, M(z(t),\ \dot{z}(t)) \quad (7)$$

$$A_H(t) = \left|\frac{\eta(t)\, i_{cell}(t)\, \gamma}{Q_{cell}}\right|, and\ B_H(t) = -A_H(t)$$

In the discrete-time domain, all states can be described as follows,

$$z[k+1] = z[k] - {\eta[k]\, i_{cell}[k]\Delta t}/{Q_{cell}} \quad (8)$$

$$i_R[k+1] = A_{RC}\, i_R[k] + B_{RC}\, i_{cell}(k) \quad (9)$$

$$A_{RC} = \exp\left(\frac{-\Delta t}{R\, C}\right), and\ B_{RC} = 1 - A_{RC}$$

$$h[k+1] = A_h[k]\, h[k] + B_h[k]\, sgn(i_{cell}(k)) \quad (10)$$

$$A_h[k] = \exp\left(-\left|\frac{\eta[k]\, i_{cell}[k]\, \gamma\, \Delta t}{Q_{cell}}\right|\right), and$$

$$B_h[k] = A_h[k] - 1, where\ \exp(.) = e^{(.)}$$

The hysteresis voltage term comprises instantaneous and dynamic hysteresis. The instantaneous hysteresis changes when the sign of the current $sgn(i_{cell}(k))$ changes(9), (10).

Overall hysteresis voltage is given in Eq. (11)

$$v_{hyst}[k] = M\, h[k] + Ns(k)\ (11)$$

$$where, s[k] = \begin{cases} sgn(i_{cell}[k]), & |i_{cell}(k)| > 0 \\ s[k-1], & otherwise \end{cases} \quad (11)$$

The state equation for the Li-Ion Cell Model is given by Eq. (12)

$$X[k+1] = A(k)\, X[k] + B(k)\, U(k)$$

$$X[k] = \begin{bmatrix} z(k) \\ i_R(k) \\ h(k) \end{bmatrix}, (i_R(k))^T = [i_{R_1}(k) \quad i_{R_2}(k)]$$

$$A[k] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & A_{RC} & 0 \\ 0 & 0 & A_h[k] \end{bmatrix}$$

$$B(k) = \begin{bmatrix} {-\eta[k]\Delta t}/{Q_{cell}} & 0 \\ & \\ B_{RC} & 0 \\ 0 & A_h[k] - 1 \end{bmatrix}$$

$$A_{RC} = \begin{bmatrix} A_{RC} & 0 \\ 0 & A_{RC2} \end{bmatrix}, A_{RCk} = \exp\left(-\frac{\Delta t}{R_k C_k}\right)$$

$$B_{RC} = \begin{bmatrix} B_{RC} \\ B_{RC} \end{bmatrix}, B_{RCk} = 1 - A_{RCk}$$

$$U[k] = \begin{bmatrix} i_{cell}(k) \\ sgn(i_{cell}(k)) \end{bmatrix} \quad (12)$$

Where, X(k) is a state vector, U(k) is a input vector Now, the output equation is given by Eq. (13),

$$v[k] = OCV(z[k]) + v_{hyst}[k] - v_R(k) - v_{RC}(k) \tag{13}$$

$$v_R[k] = R_0\, i_{cell}[k], v_{RC}[k] = \sum R_j\, i_{R_j}(k)$$

For the 1RC ECM Model,

$$v[k] = OCV(z[k]) + v_{hyst}[k] - R_0 i_{cell}[k] - R_1 i_{R_1}(k)$$

The complete state-space model of Li-Ion cell is given by Eq. (14) and Eq. (15),

$$X[k+1] = A[k]\, X[k] + B[k]\, U(k) \tag{14}$$

$$v[k] = f(X[k],\ U(k)) \tag{15}$$

***Experiments:*** The data was collected from the lithium-ion cells after carefully defined laboratory procedures and following the manufacturer's recommended limits. The experiments were conducted by discharging and charging the lithium-ion cell within the minimum and maximum voltage limits.The experiment was conducted to obtain data for the OCV vs SOC relationship by performing experimental procedures, and using those experimental data to perform another laboratory procedure was performed to get the model's unknown parameters by performing a dynamic model fit using an optimisation procedure.

***Lithium-Ion Battery Test bench:*** I implemented the code in the Octave Software to perform the experimental procedures to get the OCV and SOC relationship. Since the characteristics contain a hysteresis effect due to the mismatch between the true OCV and experimental OCV at different SOC. In the code, the option of hysteresis is also included, which can be used to refine the OCV estimate to high accuracy.

# RESULTS

In Fig. (4) and Fig. (5), the filer performance is robust and exercised in the UDDS Drive cycles scenario. The EKF filter starts with an assumed initial SOC of 90%. Still, the main task is to find the sensor covariance matrix value to make the filter capable of operating if the initial estimate of the SOC has an error. The final results show that the rms value of the SOC estimation error is 0.17012%, and the SOC estimation error bound of 0.22558%, whereas in the case of SPKF, the rms value of the SOC estimation error is 0.0978267% and SOC estimation error bounds 0.225504%.

**STATE-OF-CHARGE ESTIMATION AND ANALYSIS**

Kalman filter (KF) is a minimum mean square error estimator that works on the principle of the recursive Bayesian estimation approach. KF estimates the optimal solution if the model of the dynamical system is linear and all the uncertainties, such as sensor and state uncertainties, are Gaussian, which is not a realistic situation. However, KF works pretty well if the model is merely nonlinear. KF finds wide applications such as Guidance, Navigation, and Control (GNC), target tracking, robot positioning, and signal processing, to name a few. Since KF is infeasible for non-linear models, two non-linear variants, such as EKF and SPKF, are extensively applied tostate estimation of nonlinear systems. However, each has advantages and drawbacks(1), (2). The EKF algorithm is well suited for slightly non-linear models, and the assumptions regarding the probability distribution of the uncertaintiesmust be white and Gaussian. The method is computationally very demandingbecause of the estimation of the Jacobian matrices for both state and output function models. In contrast, the SPKF algorithm applies to the highly non-linear models, anduncertaintiesdon't need to be Gaussian. However, the method is conceptually complicated andstarts with the collection of sigma points, which best represent the priori statistic, to generate the collection of sigma points for the posterior statistic. The principle of Kalman Filters is to estimate the state with the minimum variance possible by performing a two-step process. KF does this by first predicting thestate or priori estimate by using past output sensor measurements data and the state equation of the system's state-space model and then correcting the predicted estimate by adding the additional correction term, which is the product of the innovationterm and the Gain term called Kalman Gain to estimate the improved or posterior estimate. The innovation is the difference between the actualoutput sensor data and the predicted output based on the expected state estimate or priori(1), (5).

***Extended Kalman Filter (EKF):*** The extended Kalman filter (EKF) algorithm is the most preferred method for the battery parameter/state estimationand is a nonlinear version of the LKF. EKF works on the principle of linearisation of the nonlinear function at the operating point for every sampling instant. The algorithm computes the partial derivatives of the non-linear function by using first-order Taylor series expansion. The computation of the Jacobian matrix is to be performed at every sampling instant, which is a computationally very expensive task. A limitation of the EKF algorithm is that only first-order accuracy can be achieved using first-order Taylor expansion in the linearization process. The accuracy of the EKF algorithm depends on the ECM model of the battery and the prior knowledge of covariance matrices of the sensor uncertainties(1), (2), (4), (5), (6), (9), (10).

State-space model of the Li-Ion Cell

$$X[k+1] = f(X[k], U[k], w(k)) \tag{16}$$

$$(X[k])^T = [z(k) \quad (i_R(k))^T \quad h(k)]$$

$U[k]^T = [i_{cell}(k) \quad sgn(i_{cell}(k))]$

$$v[k] = h(X[k], U[k], r(k)) \tag{17}$$

Where, $w[k] \sim N(0, Q[k]), and\ r(k) \sim N(0, R(k))$

EKF Algorithm utilises the Taylor series expansion of the non-linear function at the operating estimates, $\hat{X}^+[k]\ and\ \overline{w}(k)$, However, the noise is white and Gaussian, but for the sake of generality, the mean of process noise we consider as $\overline{w}(k)$(1), (9), (10).

$$\hat{X}^-[k] = E(X(k)/\{v[0], v[1], \ldots\ldots, v(k-1)\}) \tag{18}$$

$$\hat{X}^+[k] = E(X(k)/\{v[0], v[1], \ldots\ldots, v(k)\}) \tag{19}$$

$$\hat{X}^-[k+1] = f(\hat{X}^+[k], U[k], \overline{w}(k)) \tag{20}$$

Using the Taylor series expansion method,

$$\tilde{X}^-[k] = f(X[k], U[k], w[k]) - \hat{X}^-(k) \tag{21}$$

$$\tilde{X}^-[k] = \hat{A}[k]\tilde{X}^+[k] + \hat{B}[k]\widetilde{w}(k) \tag{22}$$

$$\hat{A}[k] = \left.\frac{\partial f(X[k], U[k], w[k])}{\partial X(k)}\right|_{X[k]=\hat{X}^+(k)}, and$$

$$\hat{B}[k] = \left.\frac{\partial f(X[k], U[k], w(k))}{\partial w(k)}\right|_{w[k]=\overline{w}(k)}$$

$$Var(\tilde{X}^-[k]) = E[\tilde{X}^-[k] \times (\tilde{X}^-[k])^T] \tag{23}$$

$$Var(\tilde{X}^-[k]) = \hat{A}[k]Var(\tilde{X}^+[k])\hat{A}(k)^T + \hat{B}(k)Var(\widetilde{w}(k))\hat{B}(k)^T \tag{24}$$

Similarly, for the output equation,

$$\hat{v}[k] = E(v(k)/\{v[0], v[1], \ldots, v(k-1)\}) \tag{25}$$

$$\hat{v}[k] = h(\hat{X}^-[k], U[k], \bar{r}(k)) \tag{26}$$

Using the Taylor Series Expansion,

$$\tilde{v}[k] = h(X[k], U[k], r(k)) - \hat{v}(k) \tag{27}$$

$$\tilde{v}[k] = \hat{C}[k]\tilde{X}^-[k] + \widehat{D}(k)\tilde{r}(k) \tag{28}$$

$$\hat{C}[k] = \left.\frac{\partial h(X[k], U[k], r(k))}{\partial X(k)}\right|_{X[k]=\hat{X}^-[k]}$$

$$\widehat{D}[k] = \left.\frac{\partial h(X[k], U[k], r(k))}{\partial r(k)}\right|_{w[k]=\overline{w}[k]}$$

$$Var(\tilde{v}[k]) = E(\tilde{v}[k](\tilde{v}[k])^T)$$

$$Var(\tilde{v}[k]) = \hat{C}(k)Var(\tilde{X}^-[k])\hat{C}[k]^T + \widehat{D}(k)Var(\tilde{r}[k])\widehat{D}(k)^T \tag{29}$$

Kalman Gain K(k),

$$K[k] = Var(\tilde{X}^-[k])\hat{C}(k)^T(\hat{C}(k)Var(\tilde{X}^-[k])\hat{C}[k]^T + \widehat{D}(k)Var(\tilde{r}[k])\widehat{D}(k)^T)^{-1} \tag{30}$$

Posterior Estimates of State and Error Covariance Matrix

$$\hat{X}^{+}[k] = \hat{X}^{-}[k] + K[k]\tilde{v}(k) \tag{31}$$

$$Var(\tilde{X}^{+}[k]) = (I - K(k)\hat{C}(k))Var(\tilde{X}^{-}[k])(I - K[k]\hat{C}[k])^{T} + K(k)Var(\tilde{v}(k))K(k)^{T} \tag{32}$$

Where, I is an identity matrix.

In Fig. (6), the SOC estimation error RMS value of 0.339114% is achieved.

## Unscented Kalman Filter (SPKF)

The unscented transformation (UT) was developed to address the deficiencies of linearisation by providing a more direct and explicit mechanism for transforming mean and covariance information. In the EKF, the state distribution is approximated by a Gaussian random variable, which is then propagated analytically through the first-order linearization of the nonlinear system. This can introduce significant errors in the true posterior mean and covariance of the transformed Gaussian Random Variable (GRV), which may lead to sub-optimal performance and sometimes filter divergence(1), (7), (8), (11), (12). The SPKF addresses this problem by using a deterministic sampling approach. A GRV again approximates the state distribution but is now represented using a minimal set of carefully chosen sample points (sigma points).The nonlinear function is applied to each sigma point to yield a collection of allthe transformed points. The statistics of the transformed points can then be calculated to estimate the nonlinearly transformed mean and covariance.These sample points capture entirely the true mean and covariance of the GRV and, when propagated through the actual non-linear system, capture the posterior mean and covariance accurately to the 3rd order (Taylor series expansion) for any nonlinearity. The EKF, in contrast, only achieves first-order accuracy. Remarkably, the computational complexity of the SPKF is in the same order as that of the EKF(1), (7), (8), (11), (12).The Algorithm starts with a set of sample points. Researchers call them sigma points, and the number of such points depends on the dimension of the state vector. If the size of the state vector is L, the number of points will be $N = 2 \times l + 1$(11), (12).The concept of the Sigma Point Kalman Filter, as discussed below,

I represented the $\chi$ be a set of sigma points,

$$\chi = \left\{\bar{X}, \bar{X} + \sqrt{Var(\tilde{X})}, \bar{X} - \sqrt{Var(\tilde{X})}\right\}$$

Where $X$ is a state vector, and $Var(\tilde{X})$ is anerror covariance matrix of X, $Var(\tilde{X}) = E((X - \bar{X})(X - \bar{X})^{T})$

$$\bar{X} = \sum_{i=0}^{N} \alpha_i^{(m)} \chi_i \,, Var(\tilde{X}) = \sum_{i=0}^{N} \alpha_i^{(c)} (\chi_i - \bar{X})(\chi_i - \bar{X})^{T}$$

$$\alpha_i^{(m)}, and\ \alpha_i^{(c)} are\ real\ scalars,$$

$$where, \sum_{i=0}^{N} \alpha_i^{(m)} = 1, and \sum_{i=0}^{N} \alpha_i^{(c)} = 1$$

I represented the output set of sample points by a symbol $\mathcal{Y}$

$$\mathcal{Y} = f(\chi) \tag{34}$$

$$\bar{y} = \sum_{i=0}^{N} \alpha_i^{(m)} \mathcal{Y}_i \tag{35}$$

$$Var(\tilde{y}) = \sum_{i=0}^{N} \alpha_i^{(c)} (\mathcal{Y}_i - \bar{y})(\mathcal{Y}_i - \bar{y})^{T} \tag{36}$$

Where, $\bar{y}, and\ Var(\tilde{y})$ are mean and covariance of output sigma points.Sigma Point Kalman Filter (SPKF) state estimation procedure for Li-Ion Cell is described below,

$$\hat{X}_{aug}^{+}[k] = [\hat{X}^{+}[k]^{T} \quad \bar{w}[k] \quad \bar{r}[k]] \tag{37}$$

$$Var\left(\tilde{X}_{aug}^{+}[k]\right) = diag\left(Var\left(\hat{X}^{+}[k]\right), Var(\tilde{w}), Var(\tilde{r})\right) \tag{38}$$

Where, $\hat{X}_{aug}^{+}[k]$ and $Var\left(\tilde{X}_{aug}^{+}[k]\right)$ are the prior estimates of the Augmented State and Error covariance matrix.

$$\chi_{aug}^{+}(k) = \{\hat{X}_{aug}^{+}[k], \hat{X}_{aug}^{+}[k] \pm \gamma\sqrt{Var(\tilde{X}_{aug}^{+}[k])}\} \tag{39}$$

$\chi_{aug}^{+}[k]$,is the augmented set of sigma points which can be split into the state estimate vector $\chi_{X}^{+}(k)$, process noise vector $\chi_{w}^{+}(k)$, and observation noise vector $\chi_{r}^{+}(k)$.

$$\chi_{X}^{-}[k+1] = f(\chi_{X}^{+}[k], U[k], \chi_{w}^{+}(k)) \tag{40}$$

$\chi_{X}^{-}(k)$, is a set of sigma points for the predicted (*priori*) state estimate.

Now, Estimating the Mean and error covariance of the predictedor priori set of sigma points, I represented the mean as $\hat{X}_{X}^{-}(k)$, and Covariance as $Var(\tilde{X}_{X}^{-}(k))$,

$$\hat{X}_{X}^{-}[k+1] = E(f(X[k], U[k], w(k))/\{v[0], v[1], \ldots, v(k)\}) \tag{41}$$

$$\hat{X}_{X}^{-}[k] = \sum_{i=0}^{N} \alpha_{i}^{(m)} \chi_{Xi}^{-}[k] \tag{42}$$

$$\chi_{Xi}^{-}[k] = f(\chi_{Xi}^{+}[k-1], U[k-1], \chi_{wi}^{+}(k-1))$$

$$Var\left(\tilde{X}_{X}^{-}[k]\right) = \sum_{i=0}^{N} \alpha_{i}^{(c)} (\chi_{Xi}^{-}[k] - \hat{X}_{X}^{-}[k])(\chi_{Xi}^{-}[k] - \hat{X}_{X}^{-}[k])^{T} \tag{43}$$

$\mathcal{Y}[k]$ is a set of sigma points for output estimate,

$$\mathcal{Y}[k] = h(\chi_{X}^{-}[k], U[k], \chi_{r}^{+}[k]) \tag{44}$$

Now, Estimating the Mean and error covariance of the output estimate set of sample points, I represented the mean $\hat{v}(k)$, and error covariance matrix as $Var(\tilde{v}(k))$,

$$\hat{v}[k] = E(h(X[k], U[k], r(k))/\{v[0], v[0], \ldots, v(k-1)\}) \tag{45}$$

$$\hat{v}[k] = \sum_{i=0}^{N} \alpha_{i}^{(m)} \mathcal{Y}_{i}(k) \tag{46}$$

$$\mathcal{Y}_{i}[k] = h(\chi_{Xi}^{-}[k], U[k], \chi_{ri}^{+}[k])$$

Now, the Estimator Gain matrix or Kalman Gain Matrix,

$$K[k] = Cov(\left(\chi_{X}^{-}[k] - \hat{X}_{X}^{-}[k]\right), (\mathcal{Y}[k] - \hat{v}[k])) \times Var(\mathcal{Y}[k] - \hat{v}[k])^{-1} \tag{47}$$

$$Cov\left[\left(\chi_{X}^{-}[k] - \hat{X}^{-}[k]\right), (\mathcal{Y}[k] - \hat{v}[k])\right] = \sum_{i=0}^{N} \alpha_{i}^{(c)} \left(\chi_{Xi}^{-}[k] - \hat{X}^{-}[k]\right) \times (\mathcal{Y}_{i}[k] - \hat{v}[k])^{T}$$

$$Var(\mathcal{Y}[k] - \hat{v}[k]) = \sum_{i=0}^{N} \alpha_{i}^{(c)} (\mathcal{Y}_{i}[k] - \hat{v}[k])(\mathcal{Y}_{i}[k] - \hat{v}[k])^{T}$$

Posterior estimates of the state and error covariance,

$$\hat{X}_X^+[k] = \hat{X}_X^-[k] + K(k) \times (v[k] - \hat{v}(k)) \quad (48)$$

$$Var(\tilde{X}_X^+[k]) = Var(\tilde{X}_X^-[k]) - K(k)Var(\tilde{v}(k))K(k)^T \quad (49)$$

In Fig. (7), the SOC estimation error RMS value of 0.415591% is achieved.

# CONCLUSION

In this article, the performance of both the Extended Kalman filter and the Unscented Kalman filter is analysed to estimate the state of charge of a Li-Ion battery cell in the presence of uncertainties in the initial SOC and sensor measurements. Since the accuracy of the model-based algorithms depends on the approximate model of the Li-Ion cell, we implemented Thevenin's RC ECM with the hysteresis component, which is the best approximate representative of the Li-Ion cell.The simulation resultsshow that the SPKF method outperforms the EKF method regarding SOC estimation convergence rate with narrower bounds in the presence of both the process and sensor uncertainties. However, both methods perform satisfactorily and exhibit robustness against sensor noise regardless of the same battery model.

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