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RESEARCH ARTICLE

TO ANALYSIS OF GENERAL THEORY OF RELATIVITY IN REFERENCES OF STATIC CONFORMALLY FLAT CHARGED FLUID SPHERES AND SPHERICALLY SYMMETRIC CHARGED FLUIDS IN EINSTEIN – MAXWELL THEORY

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ABSTRACT

Static conformally flat charged fluid spheres have much attracted the relatives in recent years. In 1968, De and Raychaudhuri (9) have shown that in relativistic unit a pressure-less charged dust distribution in equilibrium will have the absolute value of the charge to mass ratio as unity. Many workers have already studied the charged fluid distribution in equilibrium. The Einstein-Maxwell field equations in the presence of matter and charge from a highly non-liner system of equations and so a small number of exact solution have been obtained. It is believed that exact solutions of the field equations in general relativity for extended charged distribution will prove useful in the study of quantum field theory in a Reimannin manifold as question of self-energy becomes answerable. Sphere of charged dust have been investigated by Papapetrou (23). Bonner and Wickramasuriya (5) and Raychaudhuri (24). It is believed that exact solutions of the field equations in general relativity for extended charged distribution will prove useful in the study of quantum field theory in a Reimannin manifold as question of self-energy becomes answerable. Sphere of charged dust have been investigated by Papapetrou (23). Bonner and Wickramasuriya (5) and Raychaudhuri (24). It is known that the pressure less charged distribution in equilibrium will have the absolute value of the charge to mass ratio as unity in relativistic units (De and Raychaudhuri (9)). Firstly, the solution does not reduce to the interior Schwarzschild solutions when tensor charge density equals zero. This is not surprising as the vanishing of σ_0 does not mean the absence of charge but only implies that the total charge in the sphere is zero. Secondly the gravitational self-energy contribution to the total gravitational mass inversely as the radius of the sphere and not inversely as the square of radius. It can be Mentioned that if one attempts to generalize Kyle and Martin assumption of taking

$$e^{\lambda 2}\sigma(r) \propto r^m, m \geq 0. \ Q(r) \propto r^{m+3},$$

The solution of Wilson can always be overlooked. Here $\sigma(r)$ is the proper charge density within the sphere λ is metric potential and Q(r) represent the total charge contained within the sphere of radius r.

$$Q(r) = 4\pi\int_0^r x^2 e^{\lambda 2}\sigma(x)dx$$

For a spherically symmetric charge distribution the unique exterior metric was obtained by Reissner (25) and Nordstorm (21).

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INTRODUCTION

A conformally flat spherically symmetric non-static internal solution was obtained by Singh and Abdussattar(21). Letter on Ray and Raj bali (19) found a general solution representing conformally flat perfect fluid distribution of spherical symmetry. They have also discussed various physical properties of the model. Gurses (12) has shown that the only static distribution of the fluid with positive density and pressure which would generate a conformally flate metric through the Einstein's equations without cosmological term is that described by the Schwarzschild interior solution. Burman (4) discussed the motion of the particles in conformally flate space-time. Singh and Abdussattar (21) has obtained a non-static generalization of the Schwarzschild interior solution which is conformal to flate space-time. They have also shown that the model admits of distribution of discrete particles and disorder variation. Zalcev (24) and Shikin (20) have obtained conformally flate non-static solution in general relativity theory and scalar tensor-theories of gravitation. Collinson (8) has shown that every conform ally flat ax symmetric stationary space-time is static, he has also Proved that if the source is perfect fluid the space-time is the interior Schwarzschild field. The Einstein-

Maxwell field equations in the presence of matter and charge from a highly non-liner system of equations and so a small number of exact solution have been obtained. It is believed that exact solutions of the field equations in general relativity for extended charged distribution will prove useful in the study of quantum field theory in a Reimannin manifold as question of self-energy becomes answerable. Sphere of charged dust have been investigated by Papapetrou (23). Bonner and Wickramasuriya (5) and Raychaudhuri (24). It is known that the pressure less charged distribution in equilibrium will have the absolute value of the charge to mass ratio as unity in relativistic units (De and Raychaudhuri (9)). Firstly, the solution does not reduce to the interior Schwarzschild solutions when tensor charge density equals zero. This is not surprising as the vanishing of σ_0 does not mean the absence of charge but only implies that the total charge in the sphere is zero. Secondly the gravitational self-energy contribution to the total gravitational mass inversely as the radius of the sphere and not inversely as the square of radius. It can be

Mentioned that if one attempts to generalize Kyle and Martin assumption of taking

$$e^{\lambda 2}\sigma(r) \propto r^m, m \geq 0.$$

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The solution of Wilson can always be overlooked. Here$\sigma(r)$ is the proper charge density within the sphere λ is metric potential and Q(r) represent the total charge contained within the sphere of radius r.

$$Q(r) = 4\,\pi\int_0^r x^2\, e^{\lambda 2}\sigma(x)dx$$

For a spherically symmetric charge distribution the unique exterior metric was obtained by Reissner (25) and Nordstorm (21).

THE FIELD EQUATIONS

We use here the static spherically symmetric line element in the form

$$ds^2 = e^3 dt^2 - e^r\, dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

Whereα and βare function of r only. The Einstein-Maxwell equation for the charged perfect fluid distribution in general relativity is

$$R_{ij} - 1/2Rg_{ij} = -8\pi T_{ij} \quad (2)$$

$$((-g)^{1/2}F^{ij})_j = J^i(-8)^{1/2} \quad (3)$$

$$F_{(ij;k)} = 0 \quad (4)$$

Where $T_{ij}$is the energy momentum tensor, J^i is the current four vector, R_{ij} is the Ricci tensor and R the curvature scalar. For the system under study the energy momentum tensor T^i_j splits up into two parts viz. M^i_j and E^i_j for matter and charges respectively i.e.

$$M^i_j = ((p+P)u^i u_j - p\delta^i_j) \quad (5)$$

With

$u^i u_j = 1(6)$

The non-vanishing components of M^i_j are

$$M^1_1 = M^2_2 = M^3_3 = -\rho,\ M^4_4 = \rho$$

Thus the Einstein-Maxwell field equation are

$$e^{-\propto}\left(\frac{1}{r^2} + \frac{\alpha'}{r}\right) - \frac{1}{r^2} = -8\pi\rho - E^2 \quad (6)$$

$$\frac{1}{r^2}e^{-\propto}\left(\frac{1}{r^2} + \frac{\beta'}{r}\right) - \frac{1}{r^2} = -8\pi\rho - E^2 \quad (7)$$

$$e^{-\propto}\left(\frac{1}{4}\beta'\alpha' - \frac{1}{4}\beta'^2 - \frac{1}{2}\beta'' - \frac{1}{2}\left(\frac{\beta' - \alpha'}{r}\right)\right) = -8\pi\rho - E^2 \quad (8)$$

Where p is the interior pressure and ρ is the pure gravitational mass density. To solve (5), we get

$$E^2 = \frac{Q^2(r)}{r^4} \tag{9}$$

Where Q(r) represents the total charge contained within the sphere of radius r, we have

$$Q(r) = 4\pi \int \rho_e r^2 dr \tag{10}$$

Where ρ_e is the charge density.

3.3 SOLUTION OF THE FIELD EQUATIONS

We have five equation (1) – (4) and (6) in six variables $\alpha, \beta, Q(r), Ej, \rho$ and ρ. Hence the system is indeterminate. To make the system determinate we require one more relation. For this we choose uniform mass density in the form

$$\rho = A+Br^2, (B<0) \tag{11}$$

Where A and B are constant. Now integrating equation (6) we get

$$e^{-\alpha} = 1+\eta r^2+\xi(r) \tag{12}$$

Where η is integration constant and ξ(r) is

$$\xi(r) = 2r^2 \int \frac{e^2}{r} dr \tag{13}$$

From (11), (12) and (13) we get

$$\xi'/r+\xi/r^2 = -8\pi(A+Br^2) - E^2 - 3\eta \tag{14}$$

Now differentiating (4) we get

$$\frac{dE^2}{dr} + 2\frac{E^2}{r} = -\frac{16\pi}{3} Br \tag{15}$$

Solution of (5) we get

$$E^2 = \frac{\bar{A}+b^2r^2}{r^2} \tag{16}$$

$$Q^2(r) = \bar{A}r^2+b^2r^6 \tag{17}$$

Where $\bar{A}$ is the integration constant and

$$B^2 = \frac{-4\pi}{3} B, (B < 0) \tag{18}$$

Case 1: when $\bar{A} \neq 0$

Using (16) into (13) we find

$$\xi(r) = -\bar{A}+b^2r^2 \tag{19}$$

From (19) and (12) we get

$$e^{-\alpha} = 1-\bar{A}+b^2r^4+\eta r^2 \tag{20}$$

Inserting (20) and (12) into (12) we can prove

$$\eta = -\frac{8\pi}{3}A = -\frac{1}{R^2} \tag{21}$$

Where R^2 is different from R_0^2 which is dependent on the pure gravitational mass density ρ; so that

$$e^{-\alpha} = C-\frac{r^2}{R^2}+b^2r^4 \tag{22}$$

Where $C=1-\bar{A}$

Using equation (16) and (22) into p eliminate of (21) and (22) we get

$$\beta''+\frac{1}{2}\beta'^2-\left(\frac{1}{2}-\frac{\frac{-r}{R^2}+2b^2r^3}{c-\frac{r^2}{R^2}+b^2r^4}\right), \beta = \frac{2(c-1)+2b^2r^4}{r^2\left(c-\frac{r^2}{R^2}+b^2r^2\right)} \tag{23}$$

By use of transformation

$$r^2 = \chi \tag{24}$$

And

$$\chi^{-1/2}e^{\beta/2} = Z \tag{25}$$

Equation (3.3.12) is transformed to

$$\frac{d^2z}{d\chi^2}+\left[\chi^{-1}\frac{1}{2}\left(\frac{1-2b^2R^2\chi}{cR^2-\chi+b^2R^2\chi2}\right)\frac{dz}{d\chi}\right]$$

$$=\frac{2(c-1)R^2}{4\chi2(cR^2-\chi+b^2R^2\chi2)^2} \tag{26}$$

Again using the transformation

$$u = \frac{D}{R}\frac{1}{(c)^{1/2}}\ln\left[\frac{\left(c-\frac{\chi}{R^2}+b^2\chi2\right)^{1/2}}{\chi}+\frac{c}{\chi}-\frac{1}{2\sqrt{c}}R^2\right] \tag{27}$$

Where D is constant, equation (3.3.15) is changed into

$$D^2\frac{d^2z}{du^2}-\frac{(2c-1)R^2}{4}Z = 0 \tag{27}$$

The final solution of equation (3.3.12) may be written as

$$e^{\beta} = r^2\left(k_1\sinh\left\{\frac{1}{2}\left(2-\frac{1}{c}\right)^{1/2}\ln\left[\left(\frac{c-\frac{r^2}{R^2}+b^2r^4}{r^2}\right)^{\frac{1}{2}}+\frac{c}{r^2}-\frac{1}{2\sqrt{c}R}\right]\right\}+k_2\cosh\left\{\frac{1}{2}\left(2-\frac{1}{c}\right)^{1/2}\ln\left[\left(\frac{c-\frac{r^2}{R^2}+h^2r^4}{r^2}\right)^{1/2}+\frac{c}{r^2}\frac{1}{2\sqrt{c}R^2}\right]\right\}\right)^2 \tag{28}$$

where k_1 and k_2 are the integration constant. Now using equation (28) and (7) the pressure ρ is given by

$$8\pi\rho = 4b^2r^2-\frac{3}{R^2}+\frac{2e}{r^2}-\left(\frac{e}{r^2}+\frac{1}{R^2}+r^2b^2\right)\beta' \tag{29}$$

$$8\pi\rho = \frac{2c}{r^2}-\frac{3}{R^2}+4b^2r^2-\left(c-\frac{r^2}{R^2}+r^2b^4\right)$$

$$X\frac{\frac{2}{r}-(2-\frac{1}{c})^{1/2}\left[\frac{\frac{r^2}{R^2}-2c}{r^2\left(1-\frac{r^2}{R^2}+b^2r^4\right)^{1/2}}-\frac{2c}{r^2}\right]}{\left(c_2-\frac{r^2}{R^2}+b^2r^4\right)+c-\frac{r^2}{2\sqrt{cR^2}}}$$

$$X\frac{k_1+k_2\tanh\left\{\frac{1}{2}\left(2-\frac{1}{c}\right)^{1/2}\ln\left[\left(c-\frac{r^2}{R^2}+b^2r^4\right)^{1/2}+\frac{c}{r^2}-\frac{1}{2\sqrt{cR^3}}\right]\right\}}{k_2+k_1\tanh\left\{\frac{1}{2}\left(2-\frac{1}{c}\right)^{1/2}\ln\left[\frac{\left(c-\frac{r^2}{R^2}+b^2r^4\right)^{1/2}}{r^2}+\frac{c}{r^2}-\frac{1}{2\sqrt{cR^2}}\right]\right\}} \quad (30)$$

Case II. Uniform charge density sphere

When A =0 then from (3.7)

$$Q^2(r) = b^2r^6 \quad (31)$$

This is the case of uniform charge density distribution or the case of uniform charge density sphere with the surface of charge spherical thin shell. From (9), we find

$$b^2 = \frac{16\pi^2}{9}\rho_c^2 \quad (\rho_c = \text{constant}) \text{ Or } \quad b = \left(\frac{4\pi}{3}\right)\rho_c \quad (32)$$

Then from (3.3.1) we have

$$\rho = A - \frac{3}{4\pi}b^2r^2 \quad (32)$$

Where A is the total mass density and the $(\frac{3}{4\pi})b^2r^2$ is the electromagnetic self-density. Equation (3.3.23) implies that the pure gravitational mass density A is inhomogeneous but the total mass density A is homogeneous. Also we have from (3.3.11).

$$e^{-\alpha} = 1-\frac{r^2}{R^2} + b^2R^4 \quad (32)$$

In this case eq^n.(3.3.18) can be transformed and gives

$$e^{\beta} = \left(\frac{k_2-k_1}{4}\right)^2\left[\left(1-\frac{r^2}{R^2}+b^2R^4\right)^{\frac{1}{2}}+1-\frac{r^2}{R^2}\right]$$

$$+\frac{(k_2^2-k^2)r^2}{2}+\left(\frac{k_2+k_1}{4}\right)^2r^2\left[\left(1-\frac{r^2}{R^2}+b^2R^4\right)^{1/2}+1-\frac{r^2}{R^2}\right]^{-1} \quad (33)$$

Also pressure is given by

$$8\pi\rho = 4b^2r^2 - \frac{3}{R^2}+\frac{2}{r^2}-\left(\frac{1}{r^2}-\frac{1}{R^2}+br\right)\beta' \quad (34)$$

Case III: when B=0(or $b^2 = 0$)

Then from (3.3.6) and (3.3.7) we get

$$E^2 = \frac{\bar{A}}{r^2} \text{ and } Q^2(r) = \bar{A}r^2 \quad (35)$$

Also

$$\rho = A \quad (36)$$

Hence this is the case of uniform mass density

Using (35), (12) & (13) we get

$$e^{-\alpha} = 1-\bar{A}+\eta r^2 = c-\frac{r^2}{R_0^2} \tag{37}$$

Where c= 1-$\bar{A}$ and $\frac{1}{R_0^2} = \frac{8\pi\rho}{3}$ is the Schwarzeild radius. Using equation

(35), (37) into ρ eliminate of (21) and (22) we get

$$\beta'' + \frac{\beta'^2}{2} + \frac{r}{r^2-eR_0^2} - \frac{1}{r}\beta' = 2R_0^2\frac{(e-1)}{r^2(r^2-eR_0^2)} \tag{38}$$

After suitable substitution and transformations the final solutions of (3.3.30) is (for $r^2 \geq R_0^2$)

$$e^{\beta} = r^2\left\{c_1 \sin\left[\frac{1}{\sqrt{e}}\tan^{-1}\left(\frac{r^2}{cR_0^2}-1\right)^{\frac{1}{2}}\right] + c_2\cos\left[\frac{1}{\sqrt{e}}\tan^{-1}\left(\frac{r^2}{cR_0^2}-1\right)^{1/2}\right]\right\}^2 \tag{39}$$

Where c_1 and c_2 are integration constant. Also pressure is given by

$$8\pi\rho = \frac{2c}{r^2} - \frac{3}{R_0^2} - \frac{2}{\sqrt{c}}\left(\frac{c}{r^2}-\frac{1}{R_0^2}\right)\left(\frac{r^2}{cR_0^2}-1\right)^{-1/2}$$

$$X\frac{c_1 - c_2\tan\left[\frac{1}{\sqrt{c}}\tan^{-1}\left(\frac{r^2}{cR_0^2}-1\right)^{1/2}\right]}{c_1\tan\left(\frac{1}{\sqrt{c}}\tan^{-1}\left(\frac{r^2}{cR_0^2}-1\right)^{1/2}\right)+c_2} \tag{40}$$

For $r^2 < cR_0^2$, the final solution of (38) is

$$c^{\beta} = r^2\left\{c_1\sinh\left[\frac{1}{\sqrt{c}}\tanh^{-1}\left(1-\frac{r^2}{cR_0^2}\right)^{1/2}\right] + c_2\cosh\left[\frac{1}{\sqrt{c}}\tanh^{-1}\left(1-\frac{r^2}{cR_0^2}\right)^{1/2}\right]\right\}^2 .. \tag{41}$$

And also

$$8\pi\rho = \frac{2c}{r^2} - \frac{3}{R_0^2} - \frac{2}{\sqrt{c}}\left(\frac{c}{r^2}-\frac{1}{R_0^2}\right)\left(1-\frac{r^2}{cR_0^2}\right)^{-1/2}$$

$$X\frac{c_1 + c_2\tanh\left[\frac{1}{\sqrt{c}}\tan_h^{-1}\left(1-\frac{r^2}{cR_0^2}\right)^{1/2}\right]}{c_1\tanh\left(\frac{1}{\sqrt{c}}\tan_h^{-1}\left(1-\frac{r^2}{cR_0^2}\right)^{1/2}\right)+c_2} \tag{42}$$

If $\bar{A} \neq 0$ and $c \neq 0$(35) and (42) are not regular, they will diverge, we cannot get a physically reasonable solution for charged percent fluid with constant mass density. The constant appearing in the Rocessner-Nondsrom metric outside the boundary.

THE FIELD EQUATION

We consider the line element in the form

$$ds^2 = e^v dt^2 - e^{\lambda} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{43}$$

Where λ and v are function of r only. The Einstein-Maxwell field equation for the charged perfect fluid distribution in general relativity are (Adler(1))

$$R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = -8\pi T_{\alpha\beta} \tag{44}$$

$$F;^{\alpha\beta}\beta = 4\pi J^{\alpha} = 4\pi\sigma u^{\alpha} \tag{45}$$

$$F_{[\alpha\beta;\gamma]} = 0 \tag{46}$$

Where $T_{\alpha\beta}$ is the energy momentum tensor, J^{α} is the charged current four vector, $R_{\alpha\beta}$ is the Ricci tensor and R the scalar of curative tensor.

For the system under study the energy momentum tensor T^{α}_{β}splits up into two parts viz. $\overline{T}^{\alpha}_{\beta}$ And E^{α}_{β} respectivly.

$$T^{\alpha}_{\beta} = \overline{T}^{\alpha}_{\beta} + E^{\alpha}_{\beta} \tag{47}$$

Where

$$\overline{T}^{\alpha}_{\beta} = \left[(\rho + P)u^{\alpha}u_{\beta} - p8^{\alpha}_{\alpha}\right] \tag{48}$$

With

$$u^{\alpha}u_{\alpha} = 1 \tag{49}$$

The non-vanishing component of T^{α}_{β} are

$$\overline{T}^{1}_{1} = \overline{T}^{2}_{2} = \overline{T}^{3}_{3} = -p \text{ and } \overline{T}^{4}_{4} = p \tag{50}$$

Here p is internal pressure, p and σare densities of matter and charges respectively, u^{α} is the velocity vector of the matter. The static condition is given by

$$u^{1} = u^{2} = u^{3} = 0 \text{ and } u4 = (g_{44}) -\tfrac{1}{2} \tag{51}$$

i.e. $u^{4} = e^{\frac{-v}{2}}$

The electromagnetic energy momentum tensor $E^{\alpha}_{\beta}$is given by

$$E^{\alpha}_{\beta} = - F_{\beta\gamma}F^{\alpha\gamma} + \frac{1}{4}8^{\alpha}_{\beta}F_{lm}F^{lm} \tag{52}$$

We assumed the field to be purely electronic i.e. $F_{\alpha\gamma}$ =0 and $F_{4\gamma} = \phi \quad \gamma = \phi\lambda$ where ϕ is the electrostatic potential.

Thus the Einstein-Maxwell field equation are reduces into the form

$$e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda}{r}\right) - \frac{1}{r^2} = -8\pi\rho - E \tag{53}$$

$$\frac{1}{r^2} - e^{-\lambda}\left(\frac{1}{r^2} - \frac{v'}{r}\right) = -8\pi\rho + E \tag{54}$$

$$e^{-\lambda}\left[\frac{1}{4}v'\lambda' - \frac{1}{4}v'^2 - \frac{1}{2}v'' - \frac{1}{2}\left(\frac{v' - \lambda'}{r}\right)\right] = -8\pi\rho - E \tag{55}$$

Where

$$E = -F^{41}F_{41} \tag{56}$$

And

$$4\pi\sigma = \left(\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \frac{\lambda' + v'}{2}F^{41}\right)e^{\frac{v}{2}} \tag{57}$$

By the use of equation (53) – (55), we get the expression

For ρ, p and E as

$$8\pi p = \frac{e^{-\lambda}}{2}\left(\frac{3v'}{2r} + \frac{v''}{2} - \frac{\lambda'v'}{4} + \frac{v'^2}{4} - \frac{\lambda'}{2r} + \frac{1}{r^2}\right) - \frac{1}{2r^2} \tag{58}$$

$$8\pi\rho = e^{-\lambda}\left(\frac{5\lambda'}{4r} + \frac{v''}{4} - \frac{\lambda'v'}{8} + \frac{v'^2}{8} + \frac{v'}{4r} - \frac{1}{2r^2}\right) + \frac{1}{2r^2} \tag{59}$$

$$2E = e^{-\lambda}\left(\frac{v''}{2} - \frac{\lambda' v'}{4} + \frac{v'^2}{4} - \frac{v'}{2r} - \frac{\lambda'}{2r} - \frac{1}{r^2}\right) + \frac{1}{r^2} \quad (60)$$

The exterior metric is taken as usual Reissner-Nordstrom line element given by

$$ds^2 = \left(1 - \frac{2\bar{M}}{r} + \frac{Q_0^2}{r^2}\right) dt^2 - \left(1 - \frac{2\bar{M}}{r} + \frac{Q_0^2}{r^2}\right) dr^2 - r^2\,(d\theta^2 + \sin^2\theta d\phi^2) \ \ldots\ldots\ldots \quad (61)$$

Where $Q_0 = Q\,(r_0)$ and $\bar{M}$ is the total mass of the sphere. The total mass, as measured by an external observer, inside the fluid sphere of radius r_0 is given by

$$\bar{M} = 4\pi\int_0^{r_0} p(r).r^2 dr (62)$$

SOLUTION OF THE FEILD EQUATIONS: We have three equation (4.2.16)-(4.2.18) in five variables (i, p, E, λ, v) and thus the system is indeterminate. In order to make the system determinate, we require two more equation or relation. For this we choose λ and v as two free fluid variables as

(4.3.1) $$e^{\lambda} = \frac{Lr^4+Mr^2+N}{r^2+N} \quad (63)$$

$$e = \frac{Ar^4+Cr+N}{4B}; \quad (64)$$

where A,B,C,L,M,N are arbitaray constant Use of equation (63), (64) in (57), (58)-(60) yields

$$16\pi p = \frac{r^4+N}{Lr^4+Mr^2+N}\left[\frac{72A^2r^5+108ACr^5+48NAr^2+5C^2+6NCr}{4(Ar^4+Cr+N)^2} - \frac{2r^4N(L-1)(4Ar^6+3Cr^3+2Nr^2)}{(r^4+N)(Lr^4+Mr^2+N)(Ar^4+Cr+N)} + \frac{1}{r^2}\right] - \frac{1}{2r^2} \quad (65)$$

$$16\pi\rho = \frac{r^4+N}{Lr^4+Mr^2+N}\left[\frac{2r^5N(L-1)(12Ar^4+11Cr+10N)}{(r^4+N)(Lr^4+Mr^2+N)(Ar^4+Cr+N)} + \frac{4A^2r^6+6ACr^5+3C^2r^2+2NCr}{4(Ar^4+Cr+N)^2} - \frac{1}{r^2}\right] + \frac{1}{2r^2} \quad (66)$$

$$2E = \frac{r^4+N}{Lr^4+Mr^2+N}\left[+\frac{4A^2r^6+6ACr^5+3C^2r^2+2NCr}{2(Ar^4+Cr+N)^2} - \frac{4r^4N(L-1)(Ar^6+Cr^3+2Ar^3+Cr)}{(r^4+N)(Lr^4+Mr^2+N)(Ar^4+Cr+N)} - \frac{1}{r^2}\right] + \frac{1}{2r^2} \quad (67)$$

$$4\pi\sigma = \left[\frac{\partial F}{\partial r} + \frac{2}{r}F^{41} + \frac{\frac{4r^3(NL-MR^2-N)+4r^5M+2NMr)}{(r^4+N)(Lr^4+Mr^2+N)} + \frac{4r^3A+c}{Ar^4+Cr+N}}{2}.F^{41} \times \left(\frac{Ar^4+Cr+N}{4B}\right)^{\frac{1}{2}}\right] \quad (68)$$

Now using the boundary condition at $r = r_0$ the constant appearing in the solution are found to be

$$\frac{r_0^4+N}{Lr_0^4+Mr_0^2+N} = 1 - \frac{2\bar{M}}{r_0} + \frac{Q_0^2}{r_0^2} \quad (69)$$

$$\frac{Ar_0^4+Cr_0+N}{4B} = 1 - \frac{2\bar{M}}{r_0} + \frac{Q_0^2}{r_0^2} \quad (70)$$

$$\frac{4Ar_0^3+C}{8B} = \frac{\bar{M}}{r_0^2} - \frac{Q_0^2}{r_0^3} \quad (71)$$

Now we consider the following different cases:

Case: 1 when M =0 then we have

$$e^{\lambda} = \frac{Lr^4+N}{r^4+N} \quad (72)$$

$$e^{v} = \frac{Ar^4+Cr+N}{4B} \quad (73)$$

Using equation (4.3.10) - (4.3.11) in (57), (58)- (60) we get

$$16\pi p = \frac{r^4+N}{Lr^4+N}\left[\frac{72A^2r^6+108ACr^5+48NAr^2+5C^2r^2+6NCr}{4\left(Ar^4+Cr+N\right)^2} - \frac{2r^4N(L-1)(4Ar^6+3Cr^3+2Nr^2)}{(r^4+N)\left(Lr^4+N\right)\left(Ar^4+Cr+N\right)} + \frac{1}{r^2}\right] - \frac{1}{2r^2}\ldots \quad (74)$$

$$16\pi\rho = \frac{r^4+N}{Lr^4+N}\left[\frac{2r^6N(L-1)(12Ar^4 11Cr+10N)}{(r^4+N)\left(Lr^4+N\right)\left(Ar^4+Cr+N\right)}\right] + \left[\frac{4A^2r^6+6ACr^5+3C^2r^2+2NCr}{4\left(Ar^4+Cr+N\right)^2} - \frac{1}{r^2}\right] + \frac{1}{2r^2}\ldots \quad (75)$$

$$2E = \frac{r^4+N}{Lr^4+N}\left[\frac{4A^2r^6+6ACr^5+3C^2r^2+2NCr}{2\left(Ar^4+Cr+N\right)^2} - \frac{4r^4N(L-1)(Ar^6+Cr^3 2Ar^3+Cr^2)}{(r^4+N)\left(Lr^4+N\right)\left(Ar^4+Cr+N\right)} - \frac{1}{r^2}\right] + \frac{1}{2r^2}\ldots \quad (76)$$

$$4\pi\sigma = \left[\frac{\partial F}{\partial r} + \frac{2}{r}F^{41} + \frac{\frac{4r^3N(L-1)}{(r^4+N)(Lr^4+N)} + \frac{4r^3A+c}{Ar^4+Cr+N}}{2}.F^{41} \times \left(\frac{Ar^4+Cr+N}{4B}\right)^{\frac{1}{2}}\right]\ldots \quad (77)$$

Now using the boundary condition at $r = r_0$ we have

$$\frac{r_0^4+N}{Lr_0^4+N} = 1 - \frac{2\bar{M}}{r_0} + \frac{Q_0^2}{r_0^2} \quad (78)$$

$$\frac{Ar_0^4+Cr_0+N}{4B} = 1 - \frac{2\bar{M}}{r_0} + \frac{Q_0^2}{r_0^2} \quad (79)$$

$$\frac{4Ar_0^3+C}{8B} = \frac{\bar{M}}{r_0^2} + \frac{Q_0^2}{r_0^3} \quad (80)$$

Case-II when C =0, then we have

$$e^{\alpha} = \frac{Lr^4+Mr^2+N}{r^4+N} \quad (81)$$

$e^{\beta} = \frac{Ar^{-4}+N}{4B}$ (81-a) and the spin density K is given by Using equation (81) and (81-a) in equation (57), (58)-(60) we get

$$16\pi p = \frac{r^4+N}{Lr^4+Mr^4+N}\left[\frac{6Ar^2(3Ar^3+2N)}{\left(Ar^4+N\right)} - \frac{4r^5N(L-1)(2Ar^4+N)}{(r^4+N)\left(Lr^4+Mr^2+N\right)(Ar^4+N)} + \frac{1}{r^2}\right] - \frac{1}{2r^2} \quad (82)$$

$$16\pi\rho = \frac{r^4+N}{Lr^4+Mr^2+N}\left[\frac{4r^6N(L-1)(6Ar^4+5N)}{(r^4+N)\left(Lr^4+Mr^2+N\right)(Ar^4+N)}\right] + \left[\frac{A^2r^6}{\left(Ar^4+N\right)^2} - \frac{1}{r^2}\right] + \frac{1}{2r^2} \quad (83)$$

$$2E = \frac{r^4+N}{Lr^4+Mr^2+N}\left[\frac{2Ar^2r^3}{\left(Ar^4+N\right)} - \frac{4Ar^2N(L-1)(2r^2+1)}{(r^4+N)\left(Lr^4+Mr^2+N\right)(Ar^4+N)} - \frac{1}{r^2}\right] + \frac{1}{2r^2} \quad (84)$$

$$4\pi\sigma = \left[\frac{\partial F}{\partial r} + \frac{2}{r}F^{41} + \frac{\frac{4r^3\left(NL-Mr^2-N\right)+2Mr(2r^4+N)}{(r^4+N)(Lr^4+Mr^2+N)} + \frac{4r^3A}{Ar^4+N}}{2}.F^{41} \times \left(\frac{Ar^4+N}{4B}\right)^{\frac{1}{2}}\right] \quad (85)$$

Now applying the boundary conditions at $r = r_0$ we have

$$\frac{r_0^4+N}{\left(Lr^4+Mr_0^2+N\right)} = 1 - \frac{2\bar{M}}{r_0} + \frac{Q_0^2}{r_0^2} \quad (86)$$

$$\frac{Ar_0^4+N}{4B} = 1 - \frac{2\bar{M}}{r_0} + \frac{Q_0^2}{r_0^2} \quad (87)$$

$$\frac{Ar_0^3}{2B} = \frac{\bar{M}}{r_0^2} + \frac{Q_0^2}{r_0^3}; \quad (88)$$

Also the constant ξ is

Case-III: M=0, C=0 then we have

$$e^{\alpha} = \frac{Lr^4+N}{r^4+N} \quad (89)$$

$e^{\beta} = \frac{Ar^4+N}{4B}$; (89-a)and the spin density K is

Using equation (89) - (89-a) in equation (57), (58)-(60) we get

$$16\pi p = \frac{r^4+N}{Lr^4+N}\left[\frac{18A^2r^6+12NAr^2}{(Ar^4+N)^2} - \frac{4r^4N(L-1)(2Ar^6+Nr^2)}{(r^4+N)(Lr^4+N)(Ar^4+N)} + \frac{1}{r^2}\right] - \frac{1}{2r^2} \quad (90)$$

$$16\pi\rho = \frac{r^4+N}{Lr^4+N}\left[\frac{4r^5N(L-1)(6Ar^6+5Nr^2)}{r(r^4+N)(Lr^4+N)(Ar^4+N)} + \frac{A^2r^6}{(Ar^4+N)^2} - \frac{1}{r^2}\right] \quad (91)$$

$$2E = \frac{r^4+N}{Lr^4+N}\left[\frac{2A^2r^6}{(Ar^4+N)^2} - \frac{4r^4N(L-1)(Ar^6+2Ar^3)}{(r^4+N)(Lr^4+N)(Ar^4+N)} - \frac{1}{r^2}\right] + \frac{1}{2r^2} \quad (92)$$

$$4\pi\sigma = \left[\frac{\partial F}{\partial r} + \frac{2}{r}F^{41} + \frac{\frac{4r^3(N(L-1)}{(r^4+N)(Lr^4+N)} + \frac{4r^3A}{Ar^4+N}}{2}.F^{41} \times \left(\frac{Ar^4+N}{4B}\right)^{\frac{1}{2}}\right] \quad (93)$$

Now applying the boundary conditions at r =r_0 we have

$$\frac{r_0^4+N}{(Lr_0^4+N)} = 1 - \frac{2\bar{M}}{r_0} + \frac{Q_0^2}{r_0^2} \quad (94)$$

$$\frac{Ar_0^4+N}{4B} = 1 - \frac{2\bar{M}}{r_0} + \frac{Q_0^2}{r_0^2} \quad (95)$$

$$\frac{Ar_0^3}{2B} = \frac{\bar{M}}{r_0^2} + \frac{Q_0^2}{r_0^3}; \quad (96)$$

CONCLUSION

The solution of Einstein-Maxwell field equation for Static conform ally flat that charged perfect fluid sphere by using a suitable form of mass density. The result gives uniform charge density and uniform mass density distribution also. Various physical parameters can be calculated by using different boundary conditions. In spherical symmetric metric, we have solved Einstein-Maxwell field equation by taking a suitable form of matter density and charge density hence various parameters can also be calculated by putting varies conditions. Solutions of Electromagnetic equation and scalar field for cylindrically symmetric metric (Satcher metric) in two different cases (i) directly solved in terms of F_{ij} components (ii) in terms of two potentials θ_2 and θ_3. The thesis shows that starting from any solution to the electro vacuum field equation it is possible to generate a whole class of solution to the Einstein-Maxwell mass less field equation by a suitable redefinition of one of the metric coefficients. Considering all these facts this thesis is very-very useful for finding different parameters using different boundary conditions. Almost a century later, the General Theory of Relativity remains the single most influential theory in modern physics, and one of the few that almost everyone, from all walks of life, has heard of (even if they may be a little hazy about the details). Einstein's General Theory predicted the existence of black holes many years before any evidence of such phenomena, even indirect evidence, was obtained, and was highly suggestive of an origin of the universe beginning with a Big Bang type event, although Einstein himself was highly suspicious of both of those possibilities.

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