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RESEARCH ARTICLE

EFFECTS OF PARABOLIC AND INVERTED PARABOLIC TEMPERATURE GRADIENTS ON MAGNETO MARANGONI CONVECTION IN A COMPOSITE LAYER

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ABSTRACT

The problem of Magneto-Marangoni-convection is investigated in a two layer system comprising an incompressible electrically conducting fluid saturated porous layer over which lies a layer of the same fluid in the presence of a vertical magnetic field. The lower rigid surface of the porous layer and the upper free surface are considered to be insulating to temperature perturbations. At the upper free surface, the surface tension effects depending on temperature are considered. At the interface, the normal and tangential components of velocity, heat and heat flux are assumed to be continuous. The resulting eigenvalue problem is solved exactly for both parabolic and inverted parabolic temperature profiles and analytical expressions of the Thermal Marangoni Number are obtained. Effects of variation of different physical parameters on the Thermal Marangoni Number for both profiles are compared.

INTRODUCTION

The existence of fluid layer adjacent to a layer of a fluid saturated porous medium (composite layer) is a common occurrence in the natural, industrial environments and also, in some engineering problems like thermal energy storage system, solar collector with porous absorber, porous metal bearings(Morgan and Cameron 1957, Shir and Joseph 1966, Rhodes and Rouleau 1986), Porous rollers (Tao and Joseph 1962), porous layer insulation consisting of solid and pores (Masuoka 1974), in the study of blood flow in lungs (Fung and Sobin 1969, Fung 1974, Tang and Fung 1975) and in the study of synovial joints (Rudraiah et al 1998) and so on. In many situations particularly in Geophysics, Astrophysics, and in some industrial problems maintaining a uniform temperature gradient is limitation and non-uniform temperature gradient is a reality. In that case stability or instability of a fluid in the presence of a nonlinear temperature profile is of practical importance and has not been given much attention. Here we investigate the effect of Parabolic and inverted Parabolic temperature gradient on Marangoni convection in a composite layer in the presence of vertical magnetic field. However some literature is available on the effects of non uniform temperature gradients on Marangoni convection in single horizontal fluid and porous layers separately. Nanjundappa Rudraiah and Pradeep G Siddheshwar (2000) have investigated the effect of non-uniform basic temperature gradients on the onset of Marangoini convection in a horizontal layer of a Boussinesq fluid with suspended particles. It is observed that the fluid layer with suspended particles heated from below is more stable compared to the classical fluid layer without suspended particles. The problem has possible applications in microgravity situations. Shivakumara *et al.* (2002) have investigated the effect of different basic temperature gradients on the onset of ferroconvection driven by combined surface tension and buoyancy forces is studied. The results indicate that the stability of Rayleigh-Bernard-Marangoni ferroconvection is significantly affected by basic temperature gradients and the mechanism for suppressing or augmenting the same is discussed in detail. It is shown that the results obtained under the limiting conditions compare well with the existing ones. Melviana Johnson Fu *et al.* (2009) have studied the effect of six different non-uniform basic state temperature gradients on the onsets of Marangoni convection in a horizontal micropolar fluid layer bounded below by a rigid plate and above by non-deformable free surface subjected to a constant heat flux. They used Rayleigh Ritz technique to solve the resulting eigenvalue problem and discussed the influence of the various parameters on the onset of Marangoni convection. Siti Suzillian Putri Mohamed Isa *et al.* (2009) have investigated the effect of six different non-uniform basic temperature gradients on the onset of Marangoni convection in a horizontal layer with a free-slip bottom heated from below and cooled from above. They solved the resulting the eigenvalue problem using single-term Galerkin expansion procedure and have discussed the effect of the various parameters on the onset of Marangoni convection.

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Coming to the single porous layers, Shivakumara *et al.* (2012) have investigated the effect of different forms of basic temperature gradients on the criterion for the onset of convection in a layer of an incompressible couple stress fluid saturated porous medium is investigated. It is shown that the principle of exchange of stability is valid, and the eigen value problem is solved numerically using the Galerkin technique. The parabolic and inverted parabolic basic temperature profiles have the same effect on the onset of convection. The combined effects of vertical magnetic field and nonuniform temperature profiles on the onset of steady Marangoni convection in a horizontal layer of micropolar fluid are investigated by Mahmud *et al.* (2010). They obtained the closed-form expression for the Marangoni number M for the onset of convection, valid for polynomial-type basic temperature profiles upto a third order, is obtained by the use of the single-term Galerkin technique.

Formulation of the problem

Consider a horizontal two - component, electrically conducting fluid saturated isotropic sparsely packed porous layer of thickness d_m underlying a two component fluid layer of thickness d with an imposed magnetic field intensity H_0 in the vertical z – direction. The lower surface of the porous layer is rigid and the upper surface of the fluid layer is free with the surface tension effects depending on temperature. Both the boundaries are kept at different constant temperatures. A Cartesian coordinate system is chosen with the origin at the interface between porous and fluid layers and the z – axis, vertically upwards. The continuity, solenoidal property of the magnetic field, momentum energy, magnetic induction equations are,

$$\nabla\cdot\vec{q}=0 \qquad (1)$$

$$\nabla\cdot\vec{H}=0 \qquad (2)$$

$$°_0\left[\frac{\partial\vec{q}}{\partial t}+\left(\vec{q}\cdot\nabla\right)\vec{q}\right]=-\nabla P+«\nabla^2\vec{q}+«_p\left(\vec{H}\cdot\nabla\right)\vec{H} \qquad (3)$$

$$\frac{\partial T}{\partial t}+\left(\vec{q}\cdot\nabla\right)T=©\nabla^2 T \qquad (4)$$

$$\frac{\partial\vec{H}}{\partial t}=\nabla\times\vec{q}\times\vec{H}+¬_m\nabla^2\vec{H} \qquad (5)$$

For the porous layer,

$$\nabla_m\cdot\vec{q}_m=0 \qquad (6)$$

$$\nabla_m\cdot\vec{H}=0 \qquad (7)$$

$$°_0\left[\frac{1}{£}\frac{\partial\vec{q}_m}{\partial t}+\frac{1}{£^2}\left(\vec{q}_m\cdot\nabla_m\right)\vec{q}_m\right]=-\nabla_m P_m+«_m\nabla^2\vec{q}_m-\frac{«}{K}\vec{q}_m+«_p\left(\vec{H}\cdot\nabla_m\right)\vec{H} \qquad (8)$$

$$A\frac{\partial T_m}{\partial t}+\left(\vec{q}_m\cdot\nabla_m\right)T_m=©_m\nabla_m^2 T_m \qquad (9)$$

$$£\frac{\partial\vec{H}}{\partial t}=\nabla_m\times\vec{q}_m\times\vec{H}_m+¬_{em}\nabla_m^2\vec{H}_m \qquad (10)$$

Where the symbols in the above equations have the following meaning $\vec{q}=(u,v,w)$ is the velocity vector, $\vec{H}$ is the magnetic field, t is the time, $«$ is the fluid viscosity, $P=p+\frac{«_p H^2}{2}$ is the total pressure, $°_0$ is the fluid density, $«_p$ is the magnetic permeability, $A = \frac{(\circ_0 C_p)_m}{(\circ C_p)_f}$ is the ratio of heat capacities, C_p is the specific heat, K is the permeability of the porous medium, T is the temperature, © is the thermal diffusivity of the fluid, $\neg_m = \frac{1}{«_p \pm}$ is the magnetic viscosity, ± is the electrical conductivity, £ is the porosity, $\neg_{em} = \frac{\neg_m}{£}$ is the effective magnetic viscosity and the subscripts m and f refer to the porous medium and the fluid respectively.

The basic steady state is assumed to the quiescent and we consider the solution of the form,

$$\left[u, v, w, P, T, \vec{H}\right] = \left[0, 0, 0, P_b(z), T_b(z), H_0(z)\right] \quad (11)$$

in the fluid layer and in the porous layer

$$\left[u_m, v_m, w_m, P_m, T_m\right] = \left[0, 0, 0, P_{mb}(z_m), T_{mb}(z_m)\right] \quad (12)$$

Where the subscript 'b' denotes the basic state. The temperature distributions $T_b(z)$, $T_{mb}(z_m)$, are found to be

$$-\frac{\partial T_b}{\partial z} = \frac{(T_0 - T_u)}{d} h(z) \text{ in } 0 \le z \le d \quad (13)$$

$$-\frac{\partial T_{mb}}{\partial z_m} = \frac{(T_l - T_0)}{d_m} h_m(z_m) \text{ in } 0 \le z_m \le d_m \quad (14)$$

Where $T_0 = \frac{© d_m T_u + ©_m d T_l}{© d_m + ©_m d}$ is the interface temperature , $h(z)$ and $h_m(z_m)$ are temparature gradients in fluid and porous layer respectively such that $\int_0^d h(z)\, dz = d$ and $\int_0^{d_m} h_m(z_m)\, dz_m = d_m$.

In order to investigate the stability of the basic solution, infinitesimal disturbances are introduced in the form,

$$\left[\vec{q}, P, T, \vec{H}\right] = \left[0, P_b(z), T_b(z), H_0(z)\right] + \left[\vec{q}', P', {}^-, \vec{H}'\right] \quad (15)$$

and

$$\left[\vec{q}_m, P_m, T_m, \vec{H}\right] = \left[0, P_{mb}(z_m), T_{mb}(z_m), H_0(z_m)\right] + \left[\vec{q}'_m, P'_m, {}^-{}_m, \vec{H}'\right] \quad (16)$$

Where the primed quantities are the perturbed ones over their equilibrium counterparts. Now Equations (15) and (16) are substituted into the Equations (1) to (10) and are linearised in the usual manner. Next, the pressure term is eliminated from (3) and (8) by taking curl twice on these two equations and only the vertical component is retained. The variables are then non-dimensionalised using d, $\frac{d^2}{©}$, $\frac{©}{d}$, $T_0 - T_u$ and H_0 as the units of length, time velocity, temperature, and the magnetic field in the fluid layer and d_m, $\frac{d_m^2}{©_m}$, $\frac{©_m}{d_m}$, $T_l - T_0$ as the corresponding characteristic quantities in the porous layer. Note that the separate length scales are chosen for the two layers so that each layer is of unit depth.

In this manner the detailed flow fields in both the fluid and porous layers can be clearly obtained for all the depth ratios $\hat{d} = \frac{d_m}{d}$.

The dimensionless equations for the perturbed variables are given by,

$$\frac{1}{\Pr}\frac{\partial \nabla^2 w}{\partial t} = \nabla^4 w + Q2_{fm}\frac{\partial \nabla^2 H_z}{\partial z} \qquad (17)$$

$$\frac{\partial -}{\partial t} = w\, h(z) + \nabla^2 - \qquad (18)$$

$$\frac{\partial H_z}{\partial t} = \frac{\partial w}{\partial z} + 2_{fm}\nabla^2 H_z \qquad (19)$$

$$\frac{2}{\Pr_m}\frac{\partial \nabla_m^2 w_m}{\partial t} = \hat{«}\ {}^2\nabla_m^4 w_m - \nabla_m^2 w_m + {}^2 Q_m 2_{mm}\frac{\partial \nabla_m^2 H_{zm}}{\partial z_m} \qquad (20)$$

$$A\frac{\partial -_m}{\partial t} = w_m\, h_m(z_m) + \nabla_m^2 -_m \qquad (21)$$

$$£\frac{\partial H_{zm}}{\partial t} = \frac{\partial w_m}{\partial z} + 2_{mm}\nabla_m^2 H_{zm} \qquad (22)$$

For the fluid layer $\Pr = \frac{¬}{©}$ is the Prandtl number, $Q = \frac{«_p H_0^2 d^2}{«©2_{fm}}$ is the Chandrasekhar number, $2_{fm} = \frac{¬_{mv}}{©}$ is the diffusivity ratio. For the porous layer, $\Pr_m = \frac{£¬_m}{©_m}$ is the Prandtl number, ${}^2 = \frac{K}{d_m^2} = Da$ is the Darcy number, $\hat{«} = \frac{¬_m}{¬}$ is the viscosity ratio, $Q_m = \frac{«_p H_0^2 d_m^2}{«©_m 2_{mm}} = Q£\hat{d}^2$ is the Chandrasekhar number $2_{mm} = \frac{¬_{em}}{©_m}$, in the porous layer.

We make the normal mode expansion and seek solutions for the dependent variables in the fluid and porous layers according to

$$\begin{bmatrix} w \\ - \\ H \end{bmatrix} = \begin{bmatrix} W(z) \\ \Theta(z) \\ H(z) \end{bmatrix} f(x,y)e^{nt} \qquad (23)$$

and

$$\begin{bmatrix} w_m \\ -_m \\ H_m \end{bmatrix} = \begin{bmatrix} W_m(z_m) \\ \Theta_m(z_m) \\ H_m(z_m) \end{bmatrix} f(x_m,y_m)e^{n_m t} \qquad (24)$$

With $\nabla_2^2 f + a^2 f = 0$ and $\nabla_{2m}^2 f_m + a_m^2 f_m = 0$, where a and a_m are the non-dimensional horizontal wave numbers, n and n_m are the frequencies. Since the dimensional horizontal wave numbers must be the same for the fluid and porous layers, we must have $\frac{a}{d} = \frac{a_m}{d_m}$ and hence $a_m = \hat{d}a$. Substituting Equations (23) and (24) into the Equations (17) to (22) and denoting the differential operator $\frac{\partial}{\partial z}$ and $\frac{\partial}{\partial z_m}$ by D and D_m respectively, an Eigen value problem consisting of the following ordinary differential equations is obtained,

In $0 \le z \le 1$,

$$\left(D^2 - a^2 + \frac{n}{\Pr}\right)\left(D^2 - a^2\right)W = -Q{}^2{}_{fm} D\left(D^2 - a^2\right)H \qquad (25)$$

$$\left(D^2 - a^2 + n\right)\Theta + W\,h(z) = 0 \qquad (26)$$

$$\left[{}^2{}_{fm}\left(D^2 - a^2\right) + n\right]H + DW = 0 \qquad (27)$$

In $0 \le z_m \le 1$,

$$\left[\left(D_m^2 - a_m^2\right)\hat{«}\;{}^2 + \frac{n_m}{\Pr_m}{}^2 - 1\right]\left(D_m^2 - a_m^2\right)W_m = -Q_m{}^2{}_{mm}\;{}^2 D_m\left(D_m^2 - a_m^2\right)H_m \qquad (28)$$

$$\left(D_m^2 - a_m^2 + An_m\right)\Theta_m + W_m h_m(z_m) = 0 \qquad (29)$$

$$\left[{}^2{}_{mm}\left(D_m^2 - a_m^2\right) + n_m £\right]H_{mz} + DW_m = 0 \qquad (30)$$

It is known that the principle of exchange of instabilities holds for magneto convection in both fluid and porous layers separately for certain choice of parameters. Therefore, we assume that the principle of exchange of instabilities holds even for the composite layers. In other words, it is assumed that the onset of convection is in the form of steady convection and accordingly we take $n = n_m = 0$. And eliminating the magnetic field in Equations (25) and (28) from Equations (27) and (30) we get,

In $0 \le z \le 1$,

$$\left(D^2 - a^2\right)^2 W(z) = QD^2 W(z) \qquad (31)$$

$$\left(D^2 - a^2\right)\Theta(z) + W(z)\,h(z) = 0 \qquad (32)$$

In $0 \le z_m \le 1$,

$$\left[\left(D_m^2 - a_m^2\right)\hat{«}\;{}^2 - 1\right]\left(D_m^2 - a_m^2\right)W_m(z_m) = \;{}^2 Q_m D_m^2 W_m(z_m) \qquad (33)$$

$$\left(D_m^2 - a_m^2\right)\Theta_m(z_m) + W_m(z_m)\,h_m(z_m) = 0 \qquad (34)$$

Where $h(z)$, $h_m(z_m)$ are the non-dimensional temperature gradients with $\int_0^1 h(z)\,dz = 1$ and $\int_0^1 h_m(z_m)\,dz = 1$. Thus we note that, in total we have a 12th order ordinary differential equation and we need 12 boundary conditions to solve them.

Boundary conditions

The bottom boundary is assumed to be rigid and insulating to temperature so that at $z_m = 0$

$$w_m = 0, \frac{\partial w_m}{\partial z_m} = 0, \frac{\partial T_m}{\partial z_m} = 0 \qquad (35)$$

The upper boundary is assumed to be free, insulating temperature so the appropriate boundary conditions at $z = d$ are,

$$w = 0, \quad \hat{«}\frac{\partial^2 w}{\partial z^2} = -\frac{\partial ±_t}{\partial T}\left[\nabla_2^2 T\right], \quad \frac{\partial T}{\partial z} = 0$$

Where $\pm_t = \pm_0 - \pm_T T$ is the surface tension, here $\pm_T = -\left(\frac{\partial \pm_t}{\partial T}\right)_{T=T_0}$

At the interface (i.e., at $z = 0, z_m = d_m$), the normal component of velocity, tangential velocity, temperature, heat flux are continuous and respectively yield following Nield (1977),

$$w = w_m, \frac{\partial w}{\partial z} = \frac{\partial w_m}{\partial z_m}, T = T_m, © \frac{\partial T}{\partial z} = ©_m \frac{\partial T_m}{\partial z_m} \qquad (36)$$

We note that two more velocity conditions are required at $z = 0$ Since we have used the Darcy-Brinkman equations of motion for the flow through the porous medium, the physically feasible boundary conditions on velocity are the following, at $z = 0$ and

$z_m = d_m$

$$P_m - 2«_m \frac{\partial w_m}{\partial z_m} = P - 2« \frac{\partial w}{\partial z} \qquad (37)$$

which will reduce to

$$«\left(3\nabla_2^2 + \frac{\partial^2}{\partial z^2}\right)\frac{\partial w}{\partial z} = -\frac{«_m}{K}\frac{\partial w_m}{\partial z_m} + «_m \ {}^2\left(3\nabla_{2m}^2 + \frac{\partial^2}{\partial z_m^2}\right)\frac{\partial w_m}{\partial z_m}$$

The other appropriate velocity boundary condition at the interface $z = 0, z_m = d_m$ can be,

$$«\left(-\frac{\partial^2 w}{\partial z^2} + \nabla_2^2 w\right) = «_m\left(-\frac{\partial^2 w_m}{\partial z_m^2} + \nabla_{2m}^2 w_m\right) \qquad (38)$$

All the Twelve boundary conditions (35) to (38) are non-dimensionalised and are subjected to Normal mode expansion and are given by

$$W(1) = 0,\ D^2W(1) + M\, a^2\Theta(1) = 0,\ \ D\Theta(1) = 0,$$

$$\hat{T}W(0) = W_m(1), \quad \hat{T}\hat{d}DW(0) = D_m W_m(1),$$

$$\hat{T}\hat{d}^2\left(D^2 + a^2\right)W(0) = \hat{«}\left(D_m^2 + a_m^2\right)W_m(1)$$

$$\hat{T}\hat{d}^3 \ {}^2\left(D^3W(0) - 3a^2DW(0)\right) = -D_m W_m(1) + \hat{«} \ {}^2\left(D_m^3 W_m(1) - 3a_m^2 D_m W_m(1)\right)$$

$$\Theta(0) = \hat{T}\Theta_m(1), \quad D\Theta(0) = D_m\Theta_m(1),$$

$$w_m(0) = 0, \quad D_m w_m(0) = 0,\ D_m\Theta_m(0) = 0 \qquad (39)$$

The Equations (31) to (34) are to be solved with respect to the boundary conditions (39).

Exact Solution

The solutions of the Equations (31) and (33) are independent of Θ and Θ_m can be solved and expressions for W and W_m can be obtained as,

$$W(z) = A_1 Cosh(\text{¢} z) + A_2 Sinh(\text{¢} z) + A_3 Cosh(\P z) + A_4 Sinh(\P z) \quad (40)$$

$$W_m(z_m) = A_5 Cosh(C_4 z_m) + A_6 Sinh(C_4 z_m) + A_7 Cosh(C_5 z_m) + A_8 Sinh(C_5 z_m) \quad (41)$$

Where $\text{¢} = \frac{\sqrt{Q} - \sqrt{Q + 4a^2}}{2}$, $\P = \frac{\sqrt{Q} + \sqrt{Q + 4a^2}}{2}$ and $C_4 = \sqrt{\frac{C_1 + C_3}{2}}, C_5 = \sqrt{\frac{C_1 - C_3}{2}}$ and A_1, A_2, A_3, A_4 and A_5, A_6, A_7, A_8 are constants to be determined using the velocity boundary conditions of (39) , and the expressions for $W(z)$ and $W_m(z)$ are

$$W(z) = A_1 \left[Cosh(\text{¢} z) + a_1 Sinh(\text{¢} z) + a_2 Cosh(\P z) + a_3 Sinh(\P z) \right] \quad (42)$$

$$W_m(z_m) = A_1 \left[a_4 Cosh(C_4 z_m) + a_5 Sinh(C_4 z_m) + a_6 Cosh(C_5 z_m) + a_7 Sinh(C_5 z_m) \right] \quad (43)$$

Parabolic Temperature profile

Following Sparrow *et al*. (1964), we consider a parabolic temperature profile of the form

$$h(z) = 2z \quad and \quad h_m(z_m) = 2z_m \quad (44)$$

Substituting eq. (44) into the heat equations (32) and (34), the expressions for Θ and Θ_m are obtained as

$$\Theta(z) = A_1 \left[a_8\, Coshaz + a_9 Sinhaz + f_1(z) \right] \quad (45)$$

$$\Theta_m(z_m) = A_1 \left[a_{10}\, Cosha_m z_m + a_{11} Sinha_m z_m + f_{m1}(z_m) \right] \quad (46)$$

Where

$$f_1(z) = -\begin{bmatrix} \frac{2z}{\text{¢}^2 - a^2}\left(a_1 Sinh\text{¢} z + Cosh\text{¢} z\right) - \frac{4\text{¢}}{\left(\text{¢}^2 - a^2\right)^2}\left(Sinh\text{¢} z + a_1 Cosh\text{¢} z\right) + \\ \frac{2z}{\P^2 - a^2}\left(a_3 Sinh\P z + a_2\, Cosh\P z\right) - \frac{4\P}{\left(\P^2 - a^2\right)^2}\left(a_2 Sinh\P z + a_3\, Cosh\P z\right) \end{bmatrix}$$

$$f_{m1}(z_m) = -\begin{bmatrix} \frac{2z_m}{c_4^2 - a_m{}^2}\left(a_4 Coshc_4 z_m + a_5\, Sinhc_4 z_m\right) - \frac{4c_4}{\left(c_4^2 - a_m{}^2\right)^2}\left(a_5 Coshc_4 z_m + a_4\, Sinhc_4 z_m\right) + \\ \frac{2z_m}{c_5^2 - a_m{}^2}\left(a_6 Coshc_5 z_m + a_7\, Sinhc_5 z_m\right) - \frac{4c_5}{\left(c_5^2 - a_m{}^2\right)^2}\left(a_7 Coshc_5 z_m + a_6\, Sinhc_5 z_m\right) \end{bmatrix}$$

Thermal Marangoni number

The expressions of $\Theta(1)$ and $W(1)$ are substituted in $(39)^2$ and an expression for M_1 is obtained as

$$M_1 = -\frac{\left[¢^2 Cosh¢ + a_1 ¢^2 Sinh¢ + a_2 ¶^2 Cosh¶ + a_3 ¶^2 Sinh¶ \right]}{a^2 \left[\begin{array}{l} a_8 Cosha + a_9 Sinha - \dfrac{2\left(Cosh¢ + a_1 Sinh¢\right)}{(¢^2 - a^2)} + \dfrac{4¢\left(a_1 Cosh¢ + Sinh¢\right)}{\left(¢^2 - a^2\right)^2} \\ - \dfrac{2\left(a_2 Cosh¶ + a_3 Sinh¶\right)}{(¶^2 - a^2)} + \dfrac{4¶\left(a_3 Cosh¶ + a_2 Sinh¶\right)}{(¶^2 - a^2)^2} \end{array} \right]} \tag{47}$$

The Inverted Parabolic temperature profile

For the inverted parabolic temperature profile we have

$$h(z) = 2(1-z) \quad and \quad h_m(z_m) = 2(1-z_m) \tag{48}$$

Substituting eq. (48) into the heat equations (32) and (34), the expressions for Θ and Θ_m are obtained as

$$\Theta(z) = A_1 \left[a_{12}\, Coshaz + a_{13}\, Sinhaz + f_2(z) \right] \tag{49}$$

$$\Theta_m(z_m) = A_1 \left[a_{14}\, Cosha_m z_m + a_{15}\, Sinha_m z_m + f_{m2}(z_m) \right] \tag{50}$$

Where

$$f_2(z) = -\left[\begin{array}{l} \dfrac{2(1-z)}{¢^2 - a^2}\left(a_1 Sinh¢\, z + Cosh¢\, z\right) + \dfrac{4¢}{\left(¢^2 - a^2\right)^2}\left(Sinh¢\, z + a_1 Cosh¢\, z\right) + \\ \dfrac{2(1-z)}{¶^2 - a^2}\left(a_3 Sinh¶\, z + a_2\, Cosh¶\, z\right) + \dfrac{4¶}{\left(¶^2 - a^2\right)^2}\left(a_2 Sinh¶\, z + a_3\, Cosh¶\, z\right) \end{array} \right]$$

$$f_{m2}(z_m) = -\left[\begin{array}{l} \dfrac{2(1-z_m)}{c_4^2 - a_m^{\ 2}}\left(a_4 Coshc_4 z_m + a_5\, Sinhc_4 z_m\right) + \dfrac{4c_4}{\left(c_4^2 - a_m^{\ 2}\right)^2}\left(a_4\, Sinhc_4 z_m + a_5 Coshc_4 z_m\right) + \\ \dfrac{2(1-z_m)}{c_5^2 - a_m^{\ 2}}\left(a_6 Coshc_5 z_m + a_7\, Sinhc_5 z_m\right) + \dfrac{4c_5}{\left(c_5^2 - a_m^{\ 2}\right)^2}\left(a_6\, Sinhc_5 z_m + a_7 Coshc_5 z_m\right) \end{array} \right]$$

Thermal Marangoni Number

The expressions of $\Theta(1)$ and $W(1)$ are substituted in $(39)^2$ and an expression for M_2 is obtained as

$$M_2 = -\frac{\left[¢^2 Cosh¢ + a_1 ¢^2 Sinh¢ + a_2 ¶^2 Cosh¶ + a_3 ¶^2 Sinh¶ \right]}{a^2 \left[a_{12} Cosha + a_{13}\, Sinha - \dfrac{4¢\left(Sinh¢ + a_1 Cosh¢\right)}{\left(¢^2 - a^2\right)^2} - \dfrac{4¶\left(a_2 Sinh¶ + a_3 Cosh¶\right)}{(¶^2 - a^2)^2} \right]} \tag{51}$$

Where

$$a_1 = \frac{\Delta_{33}}{\Delta_{32}}$$

$$a_2 = -\frac{1}{\Delta_{29}}\left[a_1\,\Delta_{30} + \Delta_{31}\right]$$

$$a_3 = \frac{a_2\,\Delta_{22} + \Delta_{23} - a_1\,\Delta_{21}}{\Delta_{20}}$$

$$a_4 = \frac{a_2\,\Delta_{18} + \Delta_{19}}{\Delta_{17}}$$

$$a_5 = \frac{\hat{T}\left(1 + a_2\right) - a_4\Delta_1}{\Delta_2}$$

$$a_6 = -a_4$$

$$a_7 = -\frac{a_5\,c_4}{c_5}$$

$$a_8 = \hat{T}a_{10}\cosh a_m + \hat{T}a_{11}\sinh a_m + \Delta_{35}$$

$$a_9 = \frac{a_{10}a_m\sinh a_m + a_{11}\,a_m\cosh a_m + \Delta_{36}}{a}$$

$$a_{10} = \frac{\Delta_{39}}{\Delta_{38}}$$

$$a_{11} = -\frac{\Delta_{37}}{a_m}$$

$$a_{12} = \hat{T}a_{14}\cosh a_m + \hat{T}a_{15}\sinh a_m + \Delta_{41}$$

$$a_{13} = \frac{a_{14}\;a_m\sinh a_m + a_{15}\,a_m\cosh a_m + \Delta_{42}}{a}$$

$$a_{14} = \frac{\Delta_{45}}{\Delta_{44}}$$

$$a_{15} = -\frac{\Delta_{43}}{a_m}$$

$$\Delta_1 = Coshc_4 - Coshc_5$$

$$\Delta_2 = Sinhc_4 - \frac{c_4}{c_5}Sinhc_5$$

$$\Delta_3 = c_4\,Sinhc_4 - c_5\,Sinhc_5$$

$$\Delta_4 = c_4\,Coshc_4 - c_4\,Coshc_5$$

$$\Delta_5 = \text{«}\left[\left(c_4^2 + a_m^2\right)Coshc_4 - \left(c_5^2 + a_m^2\right)Coshc_5\right]$$

$$\Delta_6 = \text{«}\left[\left(c_4^2 + a_m^2\right)Sinhc_4 - \left(c_5^2 + a_m^2\right)\frac{c_4}{c_5}Sinhc_5\right]$$

$$\Delta_7 = \hat{T}\quad{}^2\hat{d}^3\left(\text{¢}^3 - 3a^2\text{¢}\right)$$

$$\Delta_8 = \hat{T}\quad{}^2\hat{d}^3\left(\text{¶}^3 - 3a^2\text{¶}\right)$$

$$\Delta_9 = \left(-1 - 3a_m^2\text{«}\quad{}^2\right)\left(c_4\,Sinhc_4 - c_5\,Sinhc_5\right)$$

$$\Delta_{10} = \left(-1 - 3a_m^2\text{«}\quad{}^2\right)\left(c_4\,Coshc_4 - c_4\,Coshc_5\right)$$

$$\Delta_{11} = \hat{\text{«}}^{\,2}\left(c_4^{\,3}\, Sinhc_4 - c_5^{\,3}\, Sinhc_5\right)$$

$$\Delta_{12} = \hat{\text{«}}^{\,2}\left(c_4^{\,3}\, Coshc_4 - c_4\, c_5^{\,2}\, Coshc_5\right)$$

$$\Delta_{13} = \Delta_9 + \Delta_{11}$$

$$\Delta_{14} = \Delta_{10} + \Delta_{12}$$

$$\Delta_{15} = \hat{T}\,\hat{d}^2\left(\text{¢}^{\,2} + a^2\right)$$

$$\Delta_{16} = \hat{T}\,\hat{d}^2\left(\text{¶}^{\,2} + a^2\right)$$

$$\Delta_{17} = \Delta_5 - \frac{\Delta_1\,\Delta_6}{\Delta_2}$$

$$\Delta_{18} = \Delta_{16} - \hat{T}\,\frac{\Delta_6}{\Delta_2}$$

$$\Delta_{19} = \Delta_{15} - \hat{T}\,\frac{\Delta_6}{\Delta_2}$$

$$\Delta_{20} = \hat{T}\hat{d}\,\text{¶}$$

$$\Delta_{21} = \hat{T}\hat{d}\,\text{¢}$$

$$\Delta_{22} = \frac{\Delta_{18}\,\Delta_3}{\Delta_{17}} + \hat{T}\,\frac{\Delta_4}{\Delta_2} - \frac{\Delta_{18}\,\Delta_1\Delta_4}{\Delta_2\,\Delta_{17}}$$

$$\Delta_{23} = \frac{\Delta_{19}\,\Delta_3}{\Delta_{17}} + \hat{T}\,\frac{\Delta_4}{\Delta_2} - \frac{\Delta_{19}\,\Delta_1\Delta_4}{\Delta_2\,\Delta_{17}}$$

$$\Delta_{24} = \frac{\Delta_{18}\,\Delta_{13}}{\Delta_{17}} + \hat{T}\,\frac{\Delta_{14}}{\Delta_2} - \frac{\Delta_{18}\,\Delta_1\Delta_{14}}{\Delta_2\,\Delta_{17}}$$

$$\Delta_{25} = \frac{\Delta_{19}\,\Delta_{13}}{\Delta_{17}} + \hat{T}\,\frac{\Delta_{14}}{\Delta_2} - \frac{\Delta_{19}\,\Delta_1\Delta_{14}}{\Delta_2\,\Delta_{17}}$$

$$\Delta_{26} = Cosh\text{¶} + \frac{\Delta_{22}}{\Delta_{20}}\, Sinh\text{¶}$$

$$\Delta_{27} = Sinh\text{¢} - \frac{\Delta_{21}}{\Delta_{20}}\, Sinh\text{¶}$$

$$\Delta_{28} = Cosh\text{¢} + \frac{\Delta_{23}}{\Delta_{20}}\, Sinh\text{¶}$$

$$\Delta_{29} = \frac{\Delta_8\,\Delta_{22}}{\Delta_{20}} - \Delta_{24}$$

$$\Delta_{30} = \Delta_7 - \frac{\Delta_8\,\Delta_{21}}{\Delta_{20}}$$

$$\Delta_{31} = \frac{\Delta_8\,\Delta_{23}}{\Delta_{20}} - \Delta_{25}$$

$$\Delta_{32} = \Delta_{27} - \frac{\Delta_{30}\,\Delta_{26}}{\Delta_{29}}$$

$$\Delta_{33} = \frac{\Delta_{26}\,\Delta_{31}}{\Delta_{29}} - \Delta_{28}$$

$$\Delta_{34} = \left[\frac{4¢^{2}}{\left(¢^{2}-a^{2}\right)^{2}} - \frac{2}{¢^{2}-a^{2}}\right]\left(a_{1}\,Sinh¢ + Cosh¢\right) - \frac{2¢}{¢^{2}-a^{2}}\left(Sinh¢ + a_{1}Cosh¢\right)$$

$$-\frac{2}{¶^{2}-a^{2}}\left(\left(¶a_{2}+a_{3}\right)Sinh¶ + \left(¶a_{3}+a_{2}\right)Cosh¶\right) + \frac{4¶^{2}}{\left(¶^{2}-a^{2}\right)^{2}}\left(a_{3}Sinh¶ + a_{2}\,Cosh¶\right)$$

$$\Delta_{35} = \hat{T}\left(\frac{4c_{4}\left(a_{4}\,Sinhc_{4} + a_{5}\,Coshc_{4}\right)}{\left(c_{4}^{2}-a_{m}^{2}\right)^{2}} + \frac{4c_{5}\left(a_{6}\,Sinhc_{5} + a_{7}\,Coshc_{5}\right)}{\left(c_{5}^{2}-a_{m}^{2}\right)^{2}}\right)$$

$$-\frac{4¢\,a_{1}}{\left(¢^{2}-a^{2}\right)^{2}} - \frac{4¶\,a_{3}}{\left(¶^{2}-a^{2}\right)^{2}} - 2\hat{T}\left(\frac{\left(a_{5}\,Sinhc_{4} + a_{4}\,Coshc_{4}\right)}{\left(c_{4}^{2}-a_{m}^{2}\right)} + \frac{\left(a_{7}\,Sinhc_{5} + a_{6}\,Coshc_{5}\right)}{\left(c_{5}^{2}-a_{m}^{2}\right)}\right)$$

$$\Delta_{36} = \frac{2}{\left(¢^{2}-a^{2}\right)} - \frac{4¢^{2}}{\left(¢^{2}-a^{2}\right)^{2}} - \left(\frac{4¶^{2}}{\left(¶^{2}-a^{2}\right)^{2}} - \frac{2}{\left(¶^{2}-a^{2}\right)}\right)a_{2} - \frac{2c_{4}}{\left(c_{4}^{2}-a_{m}^{2}\right)}\left(a_{4}\,Sinhc_{4} + a_{5}\,Coshc_{4}\right)$$

$$+\left(\frac{4c_{4}^{\;2}}{\left(c_{4}^{2}-a_{m}^{2}\right)^{2}} - \frac{2}{\left(c_{4}^{2}-a_{m}^{2}\right)}\right)\left(a_{5}\,Sinhc_{4} + a_{4}\,Coshc_{4}\right) - \frac{2c_{5}}{\left(c_{5}^{2}-a_{m}^{2}\right)}\left(a_{6}\,Sinhc_{5} + a_{7}\,Coshc_{5}\right)$$

$$+\left(\frac{4c_{5}^{\;2}}{\left(c_{5}^{2}-a_{m}^{2}\right)^{2}} - \frac{2}{\left(c_{5}^{2}-a_{m}^{2}\right)}\right)\left(a_{7}\,Sinhc_{5} + a_{6}\,Coshc_{5}\right)$$

$$\Delta_{37} = \left(\frac{4c_{5}^{\;2}}{\left(c_{5}^{2}-a_{m}^{2}\right)^{2}} - \frac{2}{\left(c_{5}^{2}-a_{m}^{2}\right)}\right)a_{6} + \left(\frac{4c_{4}^{\;2}}{\left(c_{4}^{2}-a_{m}^{2}\right)^{2}} - \frac{2}{\left(c_{4}^{2}-a_{m}^{2}\right)}\right)a_{4}$$

$$\Delta_{38} = \hat{T}a\,Sinha\;Cosha_{m} + a_{m}\,Cosha\,Sinha_{m}$$

$$\Delta_{39} = \frac{\Delta_{37}}{a_{m}}\left(\hat{T}a\,Sinha\;Sinha_{m} + a_{m}\,Cosha\;Cosha_{m}\right)$$

$$-\left(a\,Sinha\,\Delta_{35} + \Delta_{36}\,Cosha + \Delta_{34}\right)$$

$$\Delta_{40} = \left[\frac{2}{¢^{2}-a^{2}} - \frac{4¢^{2}}{\left(¢^{2}-a^{2}\right)^{2}}\right]\left(a_{1}\,Sinh¢ + Cosh¢\right)$$

$$+\left[\frac{2}{¶^{2}-a^{2}} - \frac{4¶^{2}}{\left(¶^{2}-a^{2}\right)^{2}}\right]\left(a_{3}Sinh¶ + a_{2}\,Cosh¶\right)$$

$$\Delta_{41} = \frac{2}{\left(¢^{2}-a^{2}\right)} + \frac{4¢\,a_{1}}{\left(¢^{2}-a^{2}\right)^{2}} + \frac{2a_{2}}{\left(¶^{2}-a^{2}\right)} + \frac{4¶\,a_{3}}{\left(¶^{2}-a^{2}\right)^{2}}$$

$$-\frac{4c_{4}\hat{T}}{\left(c_{4}^{2}-a_{m}^{2}\right)^{2}}\left(a_{4}\,Sinhc_{4} + a_{5}\,Coshc_{4}\right)$$

$$-\frac{4c_{5}\hat{T}}{\left(c_{5}^{2}-a_{m}^{2}\right)^{2}}\left(a_{6}\,Sinhc_{5} + a_{7}\,Coshc_{5}\right)$$

$$\Delta_{42}=\frac{2¢\,a_1}{\left(¢^2-a^2\right)}+\frac{4¢^2}{\left(¢^2-a^2\right)^2}-\frac{2}{\left(¢^2-a^2\right)}$$
$$+\frac{2a_3¶}{\left(¶^2-a^2\right)}-\left(\frac{2}{\left(¶^2-a^2\right)}-\frac{4¶^2}{\left(¶^2-a^2\right)^2}\right)a_2$$
$$+\left(\frac{2}{\left(c_4^2-a_m^2\right)}-\frac{4c_4^{\;2}}{\left(c_4^2-a_m^2\right)^2}\right)\left(a_5\,Sinhc_4+a_4\,Coshc_4\right)$$
$$+\left(\frac{2}{\left(c_5^2-a_m^2\right)}-\frac{4c_5^{\;2}}{\left(c_5^2-a_m^2\right)^2}\right)\left(a_7\,Sinhc_5+a_6\,Coshc_5\right)$$
$$\Delta_{43}=-\frac{2c_4a_5}{\left(c_4^2-a_m^2\right)}-\frac{2c_5a_7}{\left(c_5^2-a_m^2\right)}$$
$$+\left(\frac{2}{\left(c_4^2-a_m^2\right)}-\frac{4c_4^{\;2}}{\left(c_4^2-a_m^2\right)^2}\right)a_4$$
$$+\left(\frac{2}{\left(c_5^2-a_m^2\right)}-\frac{4c_5^{\;2}}{\left(c_5^2-a_m^2\right)^2}\right)a_6$$
$$\Delta_{44}=\hat{T}a\,Sinha\;Cosha_m+a_m\,Cosha\,Sinha_m$$
$$\Delta_{45}=\frac{\Delta_{43}}{a_m}\left(\hat{T}a\,Sinha\;Sinha_m+a_m\,Cosha\;Cosha_m\right)$$
$$-\left(a\,Sinha\,\Delta_{41}+\Delta_{42}\,Cosha+\Delta_{40}\right)$$

RESULTS AND DISCUSSION

The Thermal Marangoni numbers M_1 and M_2 obtained as a functions of the parameters are drawn versus the depth ratio $\hat{d}$ and the results are represented graphically showing the effects of the variation of one physical quantity, fixing the other parameters. The fixed values of the parameters are $\hat{T}=1$, ¤ $=0.7$, $=0.2$ and « $=2.0$ The effects of the parameters a, , Q, $\hat{«}$ and ¤ on the Thermal Marangoni number are obtained and portrayed in the figures 1 to 5 respectively.

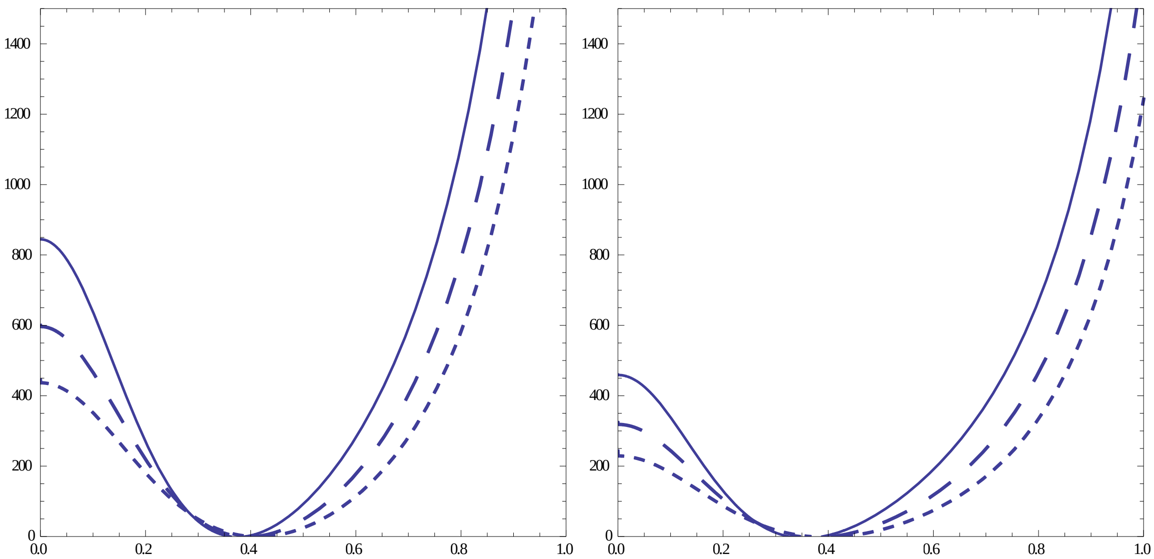

Fig. 1a&1b. The effects of a, horizontal wave number on the Thermal Marangoni numbers M for

$$Q = 50, \Phi = 0.7, « = 2.0, \quad = 0.2$$

The effects of a, horizontal wave number on the Thermal Marangoni numbers for both the parabolic and inverted parabolic profiles M_1, M_2 are shown in Fig. 1a and 1b respectively for a = 1.0, 1.1 and 1.2. The line curve is for a = 1.0, the thick dotted curve for a = 1.1 and the thin dotted curve for a = 1.3. From the figures it is clear that the Thermal Marangoni number for the parabolic profile is more than that for the inverted parabolic profile. At the value of $\hat{d} = 0.4$, the effect of both the profiles are neutral and no effect of the horizontal wavenumber a on the thermal Marangoni number. The curves for the three wavenumbers for both the profiles are converging upto the value of $\hat{d} = 0.4$, where as the three curves are diverging for the values of the depth ratio $\hat{d} \geq 0.4$. For both the profiles, when the value of a, the horizontal wave number is increased, the Thermal Marangoni numbers decrease and its effect is to destabilize the system. That is, its effect is to advance surface tension driven convection.

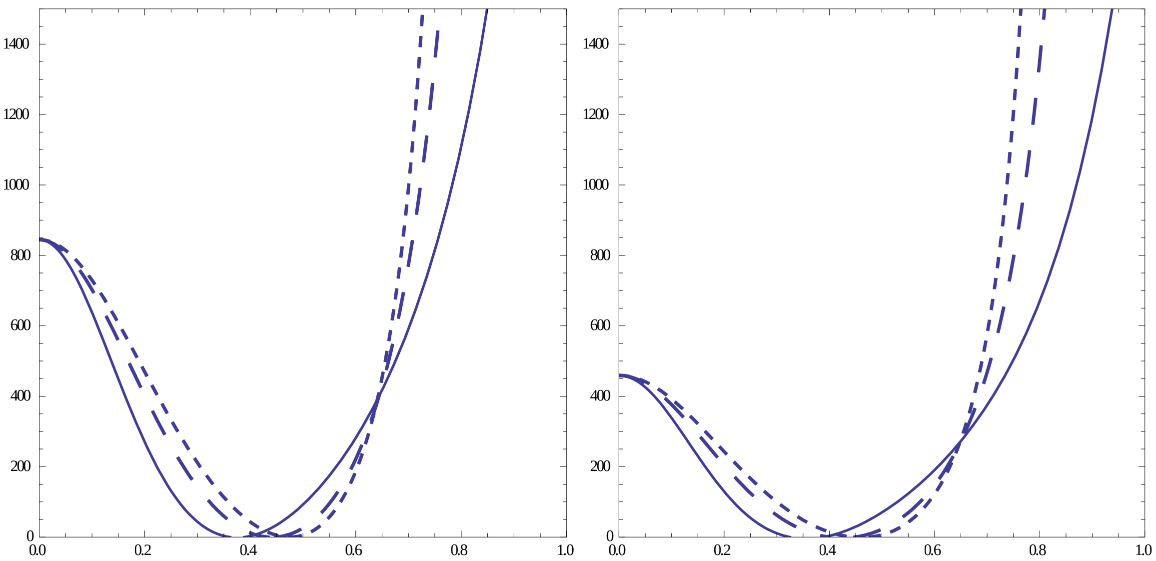

Fig.2a & 2b.The effects of on the Thermal Marangoni number M *for*

$$Q = 50, \Phi = 0.7, « = 2.0, \ a = 1.0$$

The effects of the porous parameter $= \sqrt{\frac{K}{d_m^2}}$ on the thermal Marangoni numbers for the both the profiles are exhibited in the Figs.2a and 2b. The curves are for $= 0.2, 0.3, 0.4$ The line curve is for = 0.2, the thick dotted curve for = 0.3 and the thin dotted curve for = 0.4. The curves diverge for smaller values of the depth ratio, converge near $\hat{d} = 0.4$ and again diverge and converge at $\hat{d} = 0.65$ and, as the depth ratio is further increased the curves diverge. For smaller values of depth ratio, increase in the value in the value of the porous parameter increases the thermal Marangoni number, where as for values of the depth ratio $0.4 \leq \hat{d} \leq 0.65$, the increase in the value of the porous parameter is to decrease the thermal Marangoni number and again for values of $\hat{d} \geq 0.65$ the behavior again reverses. So, the onset of surface tension driven convection can either be made faster or delayed by choosing an appropriate value of the porous parameter depending on the depth ratio. In other words increasing the permeability of the porous matrix one can destabilize and also stabilize the fluid layer system, this may be due to the presence of magnetic field.

Figure 3 exhibits the effects of the magnetic field on the onset of convection by the Chandrasekhar number $Q = \frac{«_p H_0^2 d^2}{«©^2_{fm}}$. The line curve is for Q = 50, the thick dotted curve for Q = 55 and the thin dotted curve for Q = 60. From the figures it is clear that the Thermal Marangoni number for the parabolic profile is more than that for the inverted parabolic profile for a fixed value of depth ratio. At the values of $\hat{d} = 0.3\ to\ 0.4$, the effect of both the profiles are neutral and there is no effect of the Q on the thermal Marangoni number. The curves for the three Chandrasekhar numbers for both the profiles are converging upto the value of $\hat{d} = 0.3$, where as the three curves diverge for for the values of the depth ratio $\hat{d} \geq 0.4$. For both the profiles, when the value of the Chandrasekhar number is increased, the Thermal Marangoni numbers increase and hence stabilize the system. That is the Marangoni convection is delayed for the smaller values of $\hat{d}$ that is for values of $\hat{d} \leq 0.3$ and $\hat{d} \geq 0.4$.

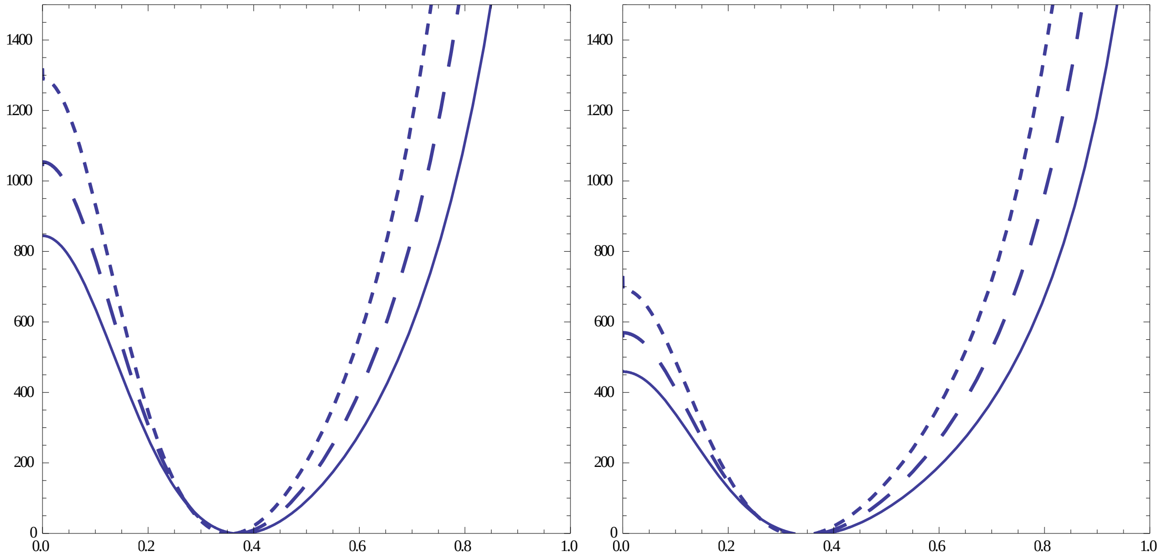

Fig.3a & 3b.The effects of Q on the Thermal Marangoni number M *for*

$$\Phi = 0.7, \hat{«} = 2.0,\ a = 1, \quad = 0.2$$

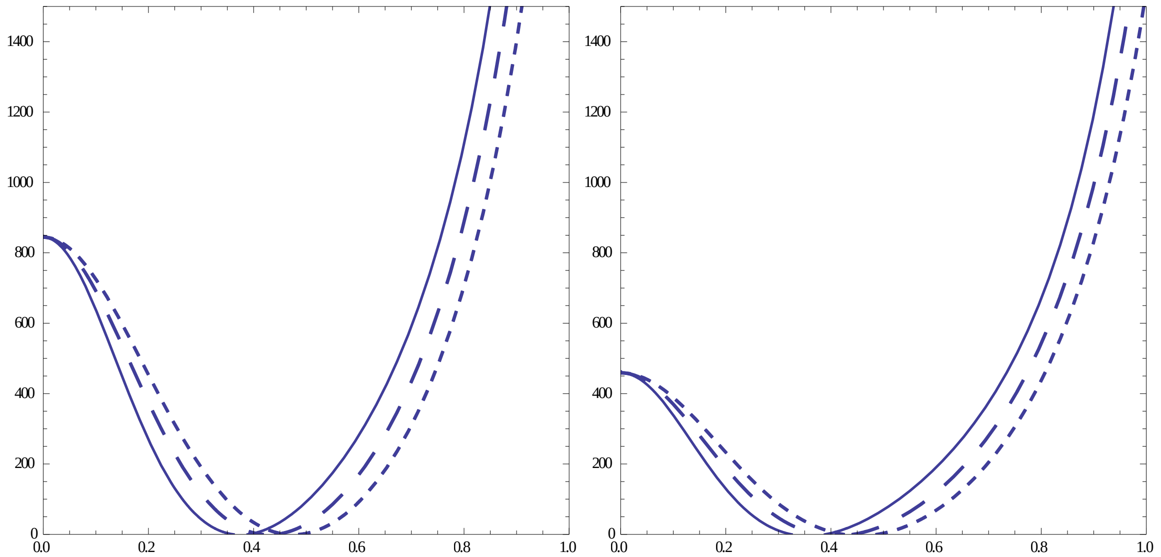

Fig. 4a & 4b. The effects of $\hat{«}$ on the Thermal Marangoni number M *for*

$$Q = 50, \Phi = 0.7, a = 1, \quad = 0.2$$

The effects of the viscosity ratio $\hat{«} = \frac{«_m}{«}$, which is the ratio of the effective viscosity of the porous matrix to the fluid viscosity are displayed in Fig. 4a and 4b. The line curve is for $\hat{«} = 2.0$, the thick dotted curve for $\hat{«} = 2.5$ and the thin dotted curve for $\hat{«} = 3.0$. The curves diverge and again converge between the values of depth ratio $0 \leq \hat{d} \leq 0.4$. The curves are diverging for the values of the depth ratio $\hat{d} \geq 0.4$ for both the profiles and the behavior of the change in the viscosity ratio reverses. Increase in the value of the viscosity ratio increases the thermal Marangoni number for the values of depth ratio $0 \leq \hat{d} \leq 0.4$. Whereas the same decreases the thermal Marangoni number for $\hat{d} \geq 0.4$. The effect of the viscosity ratio is to stabilize the system for smaller values of the depth ratio, while the effect of the same is to destabilize the system for later values of the depth ratio.

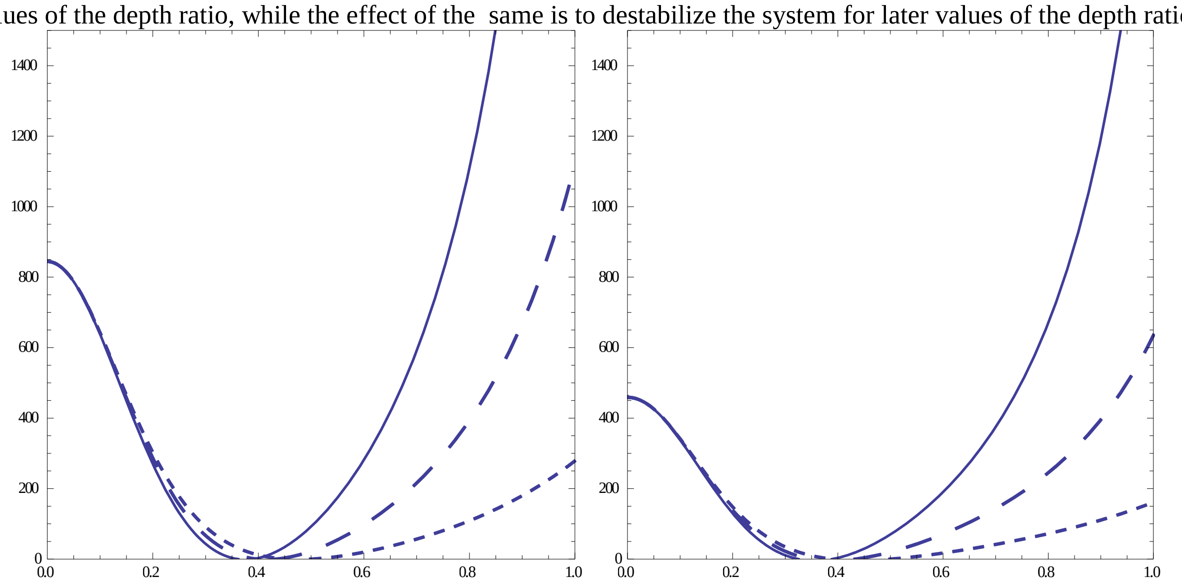

Fig.5a & 5b.The effects of ¤ on the Thermal Marangoni number M *for*

$$Q = 50, \hat{«} = 2.0,\ a = 1, \quad = 0.2$$

Figures 5a and 5b depict the effects of ¤ , the porosity, on the Thermal Marangoni numbers M_1, M_2 for the parabolic and inverted parabolic profiles respectively. The line curve is for ¤ $= 0.7$, the thick dotted curve for ¤ $= 0.8$ and the thin dotted curve for ¤ $= 0.9$. For the both the profiles, upto the value of depth ratio $\hat{d} = 0.4$ there is no effect of porosity on the thermal Marangoni number. For the values of the depth ratio $\hat{d} \geq 0.4$ the curves are diverging and for a fixed value of depth ratio, increase in the value of porosity decreases the thermal Marangoni number, that is to destabilize the system. In other words the increase in the void volume of the porous layer decreases the thermal Marangoni number and hence destabilizes the system.

Conclusion

The behavior of the system is unaltered for the both the Parabolic and Inverted Parabolic profiles. For smaller values of depth ratio of the composite layer, the increase in the values of the Porous parameter , the Chandrasekhar number Q, the viscosity ratio $\hat{«}$ and the decrease in the value of horizontal wave number a, increases the thermal Marangoni number, that is to stabilize the system for both the parabolic and inverted parabolic temperature profiles and hence to delay the Surface tension driven convection. For larger of depth ratio of the composite layer, the increase in the values of the Porous parameter , the Chandrasekhar number Q, and the decrease in the values of Horizontal wave number 'a' , the viscosity ratio $\hat{«}$ and the porosity ¤ increases the thermal Marangoni number for both the parabolic and inverted parabolic temperature profiles, hence their effect is to delay the surface tension driven convection i.e., to stabilize the system.

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