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RESEARCH ARTICLE

A NAVEL METHOD FOR SOLVING FUZZY TRANSPORTATION PROBLEM INVOLVING INTUITIONISTIC TRAPEZOIDAL FUZZY NUMBERS

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ABSTRACT

In this paper 'A new method is proposed for finding a fuzzy optimal solution of fuzzy transportation problem. In this work intuitionistic trapezoidal fuzzy numbers (ITFN's) are employed to represent the transportation cost. Numerical examples are also provided to illustrate the proposed algorithm. It can be seen that the proposed algorithm gives a better fuzzy optimal solution to the given transportation Problem.

INTRODUCTION

In real world, we frequently deal with vague or imprecise information. Information available is sometimes vague, sometimes inexact or sometimes insufficient out of several higher order fuzzy sets, intuitionistic fuzzy set (IFS) first introduced by Atanassov (1986,1989) have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of membership values is not possible up to our satisfaction. Due to the some reason evaluation of non-membership values is not also always possible and consequently there remains a part in deterministic on which hesitation survives. Certainly fuzzy sets theory is not appropriate to deal with such problem, rather intuitionistic fuzzy set (IFS) theory is more suitable. Buhaesku (1989) studied some observations on intuitionistic fuzzy relations. Stoyanova (1993) presented Cartesian product over intuitionistic fuzzy sets. Mitchell (2004) discussed new ranking for intuitionistic fuzzy numbers. Recently Stephen Dinagar and Thiripurasundari (2012) studied solving linear programming problem with generalized intuitionistic trapezoidal fuzzy numbers with new arithmetic operations and Ranking functions.

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The basic transportation problem was originally developed by Hitchcock (1941). The transportation problems can be modeled as a standard linear programming problem, which can then be solved by the simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to the transportation problem can be made quite efficient in terms of how to evaluate the necessary simplex method information (variable to enter the basis, variable to leave the basis and optimality conditions). Charnes and Cooper (1954) developed the stepping stone method which provides an alternative way of determining the simplex method information. Dantzig and Thapa (1963) used the simplex method to the transportation problem as the primal simplex transportation method. An initial basic feasible solution (IBFS) for the transportation problem can be obtained by using the north-west corner rule, row minima, column minima, matrix minima, or the Vogel approximation method. The modified distribution method is useful for finding the optimal solution for the transportation problem. Deshmukh (2012) proposed an innovative method for solving transportation problem. In general, transportation problems are solved with the assumptions that the coefficients or cost parameters are specified in a precise way i.e., in crisp environment. In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. This type of imprecise data is not always well represented by random variable selected from a probability distribution.

In this paper, a new algorithm is proposed for solving fuzzy transportation problem. In the proposed algorithm transportation costs are represented by intuitionistic trapezoidal fuzzy numbers. To illustrate the proposed algorithm a numerical example is included. The paper is organized as follows. Firstly in section-2 of this paper, we recall the definition of intuitionistic trapezoidal fuzzy number and some operations. In section-3, we presented algorithm and Basic definitions. In section 4 - numerical example is given. Finally in section -5, conclusion is also included.

2. Preliminaries

In this section, some basic definitions and arithmetic operations are reviewed.

2.1. Intuitionistic fuzzy number

Let a set X be fixed an Ifs $A^{\sim i}$ in X is an object having the form $A^{\sim i} = \{(X, \mu_{A^{\sim i}}(x), \vartheta_{A^{\sim i}}(x))/x \in X\}$ where $\mu_{A^{\sim i}}(x): X \rightarrow [0,1]$ and $\vartheta_{A^{\sim i}}(x): X \rightarrow [0,1]$,define the degree of membership and degree of non-membership respectively of the element x∈X to the set $A^{\sim i}$,which is a subset of X, for every element of x∈X , $0 \leq \mu_{A^{\sim i}}(x) + \vartheta_{A^{\sim i}}(x) \leq 1$.

2.2. Definition

A IFS $A^{\sim i}$, defined on the universal set of real numbers R, is said to be generalized IFN if its membership and non-membership function has the following characteristics:

(i)$\mu_{A^{\sim i}}(x)$: R→ [0, 1]is continuous.

(ii)$\mu_{A^{\sim i}}(x) = 0$ for all x∈ $(-\infty, a_1] \cup [a_4,\infty)$.

(iii)$\mu_{A^{\sim i}}(x)$ is strictly increasing on $[a_1,a_2]$ and strictly decreasing on (a_3,a_4).

(iv)$\mu_{A^{\sim i}}(x) = w_1$ for all x ∈ $[a_2,a_3]$.

(v)$\nu_{A^{\sim i}}(x)$: R→ [0, 1] is continuous.

(vi)$\nu_{A^{\sim i}}(x) = w_2$ for all x ∈ $[b_2,b_3]$.

(vii)$\nu_{A^{\sim i}}(x)$ is strictly decreasing on $[b_1,b_2]$ and strictly increasing on (b_3,b_4).

(viii)$\nu_{A^{\sim i}}(x) = w_1$, for all x∈ $(-\infty, b_1] \cup [b_4,\infty)$ and $w = w_1+w_2$, $0 < w \leq 1$.

2.3. Definition

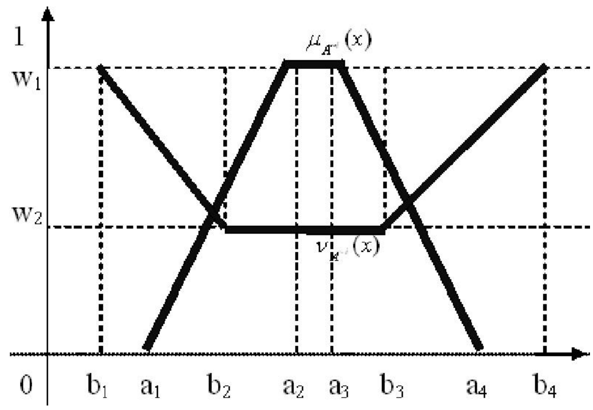
A generalized intuitionistic fuzzy number $A^{\sim i}$ is said to be a generalized trapezoidal intuitionistic fuzzy number with parameters, $b_1\leq a_1\leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ and denoted by $A^{\sim i} = (b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4;w_1,w_2)$ if its membership and non-membership function is given by

$$\mu_{A^{\sim i}}(x) = \begin{cases} \frac{w_1(x-a_1)}{(a_2-a_1)}, a_1 \leq x \leq a_2 \\ w_1, \quad a_2 \leq x \leq a_3 \\ \frac{w_1(x-a_4)}{(a_3-a_4)}, a_3 \leq x \leq a_4 \\ 0, \quad otherwise \end{cases}$$

and

$$\nu_{A^{\sim i}}(x) = \begin{cases} \frac{w_2(b_2-x)}{(b_2-b_1)}, b_1 \leq x \leq b_2 \\ w_2, \quad b_2 \leq x \leq b_3 \\ \frac{w_2(x-b_3)}{(b_4-b_3)}, b_3 \leq x \leq b_4 \\ w_1, \quad otherwise \end{cases}$$

Generalized trapezoidal intuitionistic fuzzy number is denoted by $A^{\sim i}_{GITrFN}$ = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4; w_1, w_2). Fig.1 membership and non-membership function of GITrFN.



2.4 Definition

We define a ranking function$\Re$: F(R) → R which maps each fuzzy number in to the real line; F(R) represents the set of all intuitionistic trapezoidal fuzzy numbers. If $\Re$ be any linear ranking function, then $\Re(A^{\sim i}) = \frac{(b_1+a_1+b_2+a_2+a_3+b_3+a_4+b_4)}{8}$.

2.5 Arithmetic operations

In this section, arithmetic operations between two intuitionistic trapezoidal fuzzy numbers, defined on universal set of real numbers R.

Let $A^{\sim i} = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$ and $B^{\sim i} = (d_1, c_1, d_2, c_2, c_3, d_3, c_4, d_4)$ intuitionistic trapezoidal fuzzy numbers, are as follows:

- Image$A^{\sim i} = (-b_4, -a_4, -b_3, -a_3, -a_2, -b_2, -a_1, -b_1)$.
- $A^{\sim i} + B^{\sim i} = (b_1 + d_1, a_1 + c_1, b_2 + d_2, a_2 + c_2, a_3 + c_3, b_3 + d_3, a_4 + c_4, b_4 + d_4$.
- $A^{\sim i} - B^{\sim i} = (b_1 - d_4, a_1 - c_4, b_2 - d_3, a_2 - c_3, a_3 - c_2, b_3 - d_2, a_4 - c_1, b_4 - d_1)$.
- if λ is any scalar , then $\lambda A^{\sim i} = (\lambda b_1, \lambda a_1, \lambda b_2, \lambda a_2, \lambda a_3, \lambda b_3, \lambda a_4, \lambda b_4). if\ \lambda > 0$.
 $= (\lambda b_4, \lambda a_4, \lambda b_3, \lambda a_3, \lambda a_2, \lambda b_2, \lambda a_1, \lambda b_1). if\ \lambda < 0$.
- $A^{\sim i} \times B^{\sim i} =$
 $(b_1\sigma, a_1\sigma, b_2\sigma, a_2\sigma, a_3\sigma, b_3\sigma, a_4\sigma, b_4\sigma)\ if\ \Re(B^{\sim i}) > 0$
 = $(b_4\sigma, a_4\sigma, b_3\sigma, a_3\sigma, a_2\sigma, b_2\sigma, a_1\sigma, b_1\sigma), if\ \Re(B^{\sim i}) < 0$
- $A^{\sim i} \div B^{\sim i} = \left(\frac{b_1}{\sigma}\frac{a_1}{\sigma}, \frac{b_2}{\sigma}, \frac{a_2}{\sigma}, \frac{a_3}{\sigma}, \frac{b_3}{\sigma}, \frac{a_4}{\sigma}, \frac{b_4}{\sigma}\right) if \Re(B^{\sim i}) \neq 0\ \Re(B^{\sim i}) > 0$,
 $= (\frac{b_4}{\sigma}, \frac{a_4}{\sigma}, \frac{b_3}{\sigma}, \frac{a_3}{\sigma}, \frac{a_2}{\sigma}, \frac{b_2}{\sigma}, \frac{a_1}{\sigma}, \frac{b_1}{\sigma})\ if \Re(B^{\sim i}) \neq 0, \Re(B^{\sim i}) < 0$.

Where $\sigma = (d_1 + c_1 + d_2 + c_2 + c_3 + d_3 + c_4 + d_4)/8$.

3. Mathematical formulation of a intuitionistic fuzzy transportation problem

Let us assume that there are m sources and n destinations. Let $a_i^{\sim i}$ be the fuzzy supply at i, $b_j^{\sim i}$ be the fuzzy demand at destination j , $c_{ij}^{\sim i}$ be the unit fuzzy transportation cost from source i to destination j and $x_{ij}^{\sim i}$ be the number of units shifted from source i to destination j.

Then the fuzzy transportation problem can be expressed mathematically as

$$\min z^{\sim i} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{\sim i}\ x_{ij}^{\sim i}$$

Subject to the constraints

$\sum_{j=1}^{n} x_{ij}^{\sim i} = a_i^{\sim i}$, i = 1,2,3,….,m
$\sum_{i=1}^{m} x_{ij}^{\sim i} = b_j^{\sim i}$, j = 1,2,3,….,n

And $x_{ij}^{\sim i} \geq 0$ for all i and j.

A fuzzy transportation problem is said to be balanced if the total fuzzy supply from all sources equal to the total fuzzy demand in all destination $\sum_{i=1}^{m} a_i^{\sim i} = \sum_{j=1}^{n} b_j^{\sim i}$,otherwise is called unbalanced.

3.1 Fuzzy feasible solution

A set of non negative allocation $x_{ij}^{\sim i} \geq 0$ which satisfies the row and column restriction is known as fuzzy feasible solution.

3.2 Fuzzy basic feasible solution

A fuzzy solution to a m × n fuzzy transportation problem that contains no more than m+n-1 non negative allocations is called fuzzy basic feasible solution to the fuzzy transportation problem.

3.3 Fuzzy optimal solution

A fuzzy basic feasible solution is said to be fuzzy optimal solution if it minimizes the total fuzzy transportation costs.

3.4 Algorithm for fuzzy transportation problem by using intuitionistic trapezoidal fuzzy number

Step 1
Construct the fuzzy transportation matrix from the given fuzzy transportation problem.
Step 2
Find the rank value of each cell $c_{ij}^{\sim i}$ by using the ranking procedure and select minimum fuzzy odd cost from all fuzzy cost in the matrix.
Step 3
Subtract the selected minimum fuzzy odd cost only from fuzzy odd cost in the matrix.
Step 4
Now there will be rank of one fuzzy cost is zero and remaining all rank value of fuzzy costs become even.
Step 5
Start the allocation from minimum of supply and demand. Allocated this minimum of supply/demand at the place of rank value of fuzzy cost is zero.
Step 6
Multiply by ½ each column (i.e 1/2 $c_{ij}^{\sim i}$) and again find the rank value of each cell and select the minimum fuzzy odd cost in the remaining matrix except rank value of fuzzy cost is zero in the matrix.
Step 7
Go to step 3 and 4.
Step 8
Repeat the step 5 and 6 for remaining sources and destinations t_{ij} (m+n-1) cells are allocated.
Step 9
Finally total minimum fuzzy cost is calculated as sum of the product of fuzzy cost corresponding allocated fuzzy value of supply/demand
i.e. Total fuzzy cost = $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{\sim i}\ x_{ij}^{\sim i}$

4. Examples

Fuzzy transportation problem having three origins and three destinations

	D_1	D_2	D_3	supply
O_1	(2,3,4,5,7,8,9,10)	(0,1,2,3, 5,6,7,8)	(-4,-2,-1,1,2,3,4,5)	(-3,0,2,3, 4,5,6,7)
O_2	(-3,0,2,3,4,5,6,7)	(-6,4,6,8,10, 12,14,16)	(-10,-6,5,6,10,15, 17,19)	(-4,-2,-1,1,2,3, 4,5)
O_3	(0,1,2,3,5,6,7,8)	(0,1,2,3,5,6,7,8)	(-3,-1,0,2,3,4,5,6)	(0,1,2,3, 5,6,7,8)
Demand	(-4,-2,-1,1,2,3,4,5)	(0,1,2,3,5,6,7,8)	(-3,0,2,3,4,5,6,7)	

Step 1

Where $\sum a_i^{\sim i} \ \overset{=}{_R}\ b_j^{\sim i}$

Now we choose the minimum fuzzy odd cost value for that find the rank value of each cell

$R(c_{11}^{\sim i}) = 6$ $R(c_{12}^{\sim i}) = 4$ $R(c_{13}^{\sim i}) = 1$ $R(c_{21}^{\sim i}) = 3$ $R(c_{22}^{\sim i}) = 8$ $R(c_{23}^{\sim i}) = 7$ $R(c_{31}^{\sim i}) = 4$ $R(c_{32}^{\sim i}) = 4$ $R(c_{33}^{\sim i}) = 2$.
Here the minimum fuzzy odd cost (-4,-2,-1,1,2,3,4,5) subtract this minimum fuzzy odd cost from all fuzzy odd cost which shown in the following table

	D_1	D_2	D_3	supply
O_1	(2,3,4,5,7,8,9,10)	(0,1,2,3,5,6,7,8)	(-9,-6,-4,-1,1,4,6,9)	(-3,0,2, 3,4,5,6,7)
O_2	(-8,-4,-1,1,3,6,8,11)	(-6,4,6,8,10, 12,14,16)	(-15,-10,2,4, 9,16,19,23)	(-4,-2,-1,1, 2,3,4,5)
O_3	(0,1,2,3,5,6,7,8)	(0,1,2,3,5,6,7,8)	(-3,-1,0,2, 3,4,5,6)	(0,1,2,3, 5,6,7,8)
Demand	(-4,-2,-1,1,2,3,4,5)	(0,1,2,3,5,6,7,8)	(-3,0,2,3, 4,5,6,7)	

$c_{13}^{\sim i}$ = (-9,-6,-4,-1,1,4,6,9) $c_{21}^{\sim i}$ = (-8,-4,-1,1,3,6,8,11) $c_{23}^{\sim i}$ = (-15,-10,2,4,9,16,19,23)

Then R($c_{13}^{\sim i}$) = 0 R($c_{21}^{\sim i}$) = 2 R($c_{23}^{\sim i}$) = 6.
Here R($c_{13}^{\sim i}$) is the only zero cost so allocate the (O_1,D_3) and for choose min((-3.0,2,3,4,5,6,7),(-3.0,2,3,4,5,6,7) and R(O_1) = 3 and R(D_3) = 3.

We choose the minimum value of O_1 and D_3 .delete the column D_3 and row O_1 for demand exhausted supply as ((-3.0,2,3,4,5,6,7)-(-3.0,2,3,4,5,6,7)) = R(-10,-6,-3,-1,3,6,10) = 0

	D_1	D_2	D_3	supply
O_1	(2,3,4,5,7,8,9, 10)	(0,1,2,3,5,6, 7,8)	(-3,0,2,3,4,5,6,7) (-9,-6,-4,-1, 1,4,6,9)	(-3,0,2, 3,4,5, 6,7)
O_2	(-8,-4,- 1,1,3,6,8,11)	(-6,4,6,8,10, 12,14,16)	(-15,-10,2,4,9, 16,19,23)	(-4,-2,-1, 1,2,3,4,5)
O_3	(0,1,2,3,5,6,7, 8)	(0,1,2,3,5,6, 7,8)	(-3,-1,0,2,3,4,5,6)	(0,1,2,3,5,6, 7,8)
Demand	(-4,-2,- 1,1,2,3,4,5)	(0,1,2,3,5,6, 7,8)	(-3,0,2,3,4,5,6,7)	

Step 2

Multiply by ½ in D_1 and D_2 which shown in next table

	D1	D2	supply
O2	(-8,-4,- 1,1,3,6,8,11)	(-6,4,6,8,10,12,14,16)	(-4,-2,-1,1,2,3,4,5)
O3	(0,1,2,3,5,6,7,8)	(0,1,2,3,5,6,7,8)	(0,1,2,3,5,6,7,8)
Demand	(-4,-2,-1,1,2,3,4,5)	(0,1,2,3,5,6,7,8)	

Now again choose the minimum fuzzy odd cost continue this process finally we get
Total fuzzy cost = (-15,-2, 7, 18, 30, 38, 46, 54) = 22.

5. Conclusion

In this paper, a new distinct simple algorithm for solving fuzzy transportation problem has been developed. The proposed algorithm is easy to understand and apply. The fuzzy optimal solution obtained in this investigation is same as that of fuzzy MODI method or fuzzy Vogel's Approximation method.

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