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RESEARCH ARTICLE

THEORETICAL STUDY FOR THE GLOBAL PARAMETERS COVERING THE D-D THERMONUCLEAR FUSION REACTION

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ABSTRACT

Adjusting the calculation related with the parameters that control the physical behaviors for the fusion phenomena occurs in the controlled thermonuclear fusion reactions needs to fix a very suitable and compatible data or formula for the common factor such as cross sections and/or astrophysical S-factor that play important rules for arriving in acceptable theoretical results for other parameters that give a good agreement with corresponding experimental data.

Key words:

Astrophysical S-factor, D-D thermonuclear fusion reaction, Cross-section, Reactivity, Reaction rate, Cold fusion.

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INTRODUCTION

The nuclear fusion cross sections of proton and deuteron induced reactions at low energies are of particular interest from the points of view of the stellar nucleosynthesis and the nuclear energy production. These cross sections are measured at laboratory energies and extrapolated to thermal energies because of their small values at such low energies. This extrapolation is done by introducing the astrophysical S-factor:

$$S(E) = \sigma(E)Ee^{2\pi\eta(E)} \quad \text{--- (1)}$$

Where $\sigma(E)$ is the reaction cross section at the incident center of mass energy E and $\eta(E) = Z_T Z_P \alpha \sqrt{\frac{\mu c}{2E}}$ with Z_T , Z_P , and μ denoting the atomic numbers and the reduced mass of the target and the projectile; α and c are the fine structure constant and the speed of light, respectively (Angulo *et al.*, 1999; Yan *et al.*, 1997). The exponential term in the equation represents the Coulomb barrier penetrability. Because one has a factor of the strong energy dependence of $\sigma(E)$ due to the barrier penetrability. The S-factor could be approximated by a smooth energy dependence in the absence of low-energy resonance. A lot of efforts have been made to extract the S-factor in the low-energy region in particular for transferring reactions experimentally.

Although, the low-energy is known for S-factor of radiative capture reactions, still one needs the extrapolation with theoretical formulas. Owing to this experimental effort one can, in principle, determine the S-factor in the energy region of astrophysical interest directly. However, at such a low energy, it is known that electrons around the target nucleus have an effect on the fusion cross section.

In constant, in the stellar nucleosynthesis, nuclei are almost fully ionized and are surrounded by the plasma electrons. The nuclear reactions in such a circumstance are affected by a different mechanism of the plasma electron screening. Hence, the screening effects of the bound electrons should be removed from the S-factor data to assess the reaction rate in the stellar site correctly. The enhancement by the bound electrons is discussed in terms of a constant potential shift (screening potential U_e) (Sachie Kimura and Aldo Bonasera 2007).

In this rapid communication, we investigate in detail the physical origins of the bare S-factor. We propose visualizing the fusion mechanism under the assumption of the compound nucleus (CN) formation as a two-step process:

Step 1: The nuclear fusion under the Coulomb barrier occurs with a given probability that depends crucially on nuclear effects (e.g., the nuclear potential, nuclear sizes and an excitation of collective modes). A CN is formed at a given excitation energy. Because we are interested in determining the S-factor in the limit of zero bombarding energies, the excitation energy is essentially given by the Q-value of the reaction that leads to the CN.

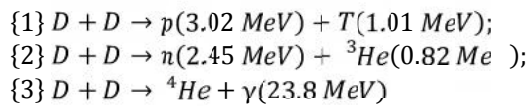
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Step 2: The CN decays into a given channel. We assume this decay process to be statistical and we make use of the Weisskopf model (Weisskopf 1937; Bonasera *et al.*, 1988) to evaluate its probability.

Low-energy behaviors of the S-factor have been studied by means of the R-matrix theory (Angulo and Descouvemont 1998; Barker 2002), which is also based on a description of the nuclear reaction through the intermediate CN. In the nuclear reactions in general, it is known that this CN process coexists with the direct process mechanism (Thomas 1955).

Theory

The conventional deuterium fusion in free space proceeds via the following nuclear reactions:



The cross-sections for reactions {1}-{3} are expected to be extremely small at low energies (≤ 10 eV) due to the Gamow factor arising from Coulomb barrier between two deuterons. The measured cross-sections have branching ratios:

$$(\sigma\{1\}, \sigma\{2\}, \sigma\{3\}) \approx (0.5, 0.5, \sim 10^{-6}).$$

Experimental values of the conventional hot-fusion cross section $\sigma(E)$ for reaction {1} or {2} have been conventionally parameterized as:

$$S(E) = \frac{S}{E} e^{-2\pi\eta} = \frac{S}{E} e^{-\sqrt{E_G/E}} \quad \text{--- (2)}$$

With

$$\eta = \frac{1}{2kr_B}, \quad r_B = \frac{\hbar}{2\mu e^2}, \quad \mu = m/2$$

$e^{-2\pi\eta}$ is known as the "Gamow factor", and E_G is the "Gamow energy" given by:

$$E_G = \frac{(2\pi\alpha Z_d Z_a)\mu c^2}{2} \quad \text{or} \quad \sqrt{E_G} \approx 31.39\sqrt{\text{keV}}$$

For the reduced mass $\mu = \frac{m}{2}$ for reactions {1} or {2}. The value E is measured in keV in the center-of-mass (CM) reference frame. The S-factor, $S(E)$, is extracted from experimentally measured values of the cross section $\sigma(E)$ for $E \geq 4 \text{ keV}$ and is nearly constant; $S(E) \approx 52.9 \text{ keV} \cdot \text{barn}$, for reactions {1} or {2}, in the energy range of interest here. $E \leq 100 \text{ keV}$. The S-factor is known as "astrophysical S-factor" (Yeong E. Kim 2012). For nuclear reactions in which the reactants must penetrate a barrier, which is the case with the thermonuclear reactions, the cross-section can be expressed in the form:

$$\sigma = K v^{-k} \exp(-L/v) \quad \text{--- (3)}$$

Where k , K , and L are constants. In particular for the D-D reaction

$$\sigma_{DD} = \frac{288}{E_d} \exp\left(-45.8/\sqrt{E_d}\right) \quad \text{--- (4)}$$

Where E_d is the deuteron bombarding energy in keV, and σ_{DD} is the total cross section in barns (David J. Rose and Melville Clark 1961). Also there is another relation to compute total cross sections for D-D reaction as follows:

$$\sigma_{DD} = \exp(4.727 - 0.03154 E_d) \quad \text{--- (5)}$$

Where E_d is the deuteron bombarding energy in MeV, and σ_{DD} is the total cross section in mbarns (Drosg 1978).

In particular, for the D-D reactions and in our works we focus on a more generalized empirical formula used in calculating the total cross sections for the D-D fusion reaction given above. The Astrophysical factor $S(E)$ is calculated according to the following formula:

$$S(E) = \sigma(E) E e^{2\pi\eta(E)}$$

After testing many other empirical formulas, we finally found that the above formula get more accurate results. Finally, the reaction rate is an integral of the total cross-section weighted by a Maxwellian distribution and taken over all energies. For energies given in mega-electron volts and cross-sections in barns, the reaction rate $N_a \langle \sigma v \rangle$ ($\text{cm}^3/\text{mol}^1/\text{s}^{-1}$) is given by:

$$N_a \langle \sigma v \rangle = 3.731 \times 10^{10} \mu^{-1/2} T_9^{-3/2} \int_0^\infty \sigma E e^{-11.604 \frac{E}{T_9}} dE \quad \text{--- (6)}$$

Where μ is the reduced mass in atomic mass unit and T_9 is the temperature in units of 10^9 K . For the integral to converge at temperatures relevant to BBN (Big Bang Nucleosynthesis), it is found that $T_9 < 2$. (Leonard *et al.*, 2006) The mean free path of D-D reaction $\lambda_{DD} = 1/n_i$ (E_D)

Where n_i is the ion density of the target and λ_{DD} is the cross-section of the DD reaction. (Kubes *et al.*, 2007)

CALCULATIONS AND RESULTS

It is necessary to note that there exists many experimental or empirical formulas for measuring the fusion cross section for D-D reaction; and we found that each formula gives a different data or results corresponding to other. For this case, we test these formulas and indeed we choose a formula which gives suitable compatible agreement with the published common cross section described in Fig.(1). (Decreton *et al.*, 2007)

The calculated results about the total D-D fusion reaction are presented in Table (1) and described in Fig.(2). The astrophysical factor $S(E)$ shows a strongly dependence on the total cross sections and this fact needs to select a suitable data about the cross sections in order to arrive to good physical behaviors for the $S(E)$ factor. The calculated astrophysical factor are presented in Table (1) and described in Fig.(3).

Table 1. Energy dependence cross-section and astrophysical facts for D-D nuclear fusion reaction

Deuteron Energy E_d (keV)	Total Cross Sect $\sigma(E)$ (Barns)	Astrophysical Factor $S(E)$ (keV Barns)
112.2	0.034008	73.958316
172.2	0.051002	96.121073
232.8	0.061484	112.076309
314.6	0.069216	127.8805676
379.2	0.072289	137.479835
470.2	0.074099	148.246924
557.3	0.074256	156.487658
646.1	0.073546	163.439665

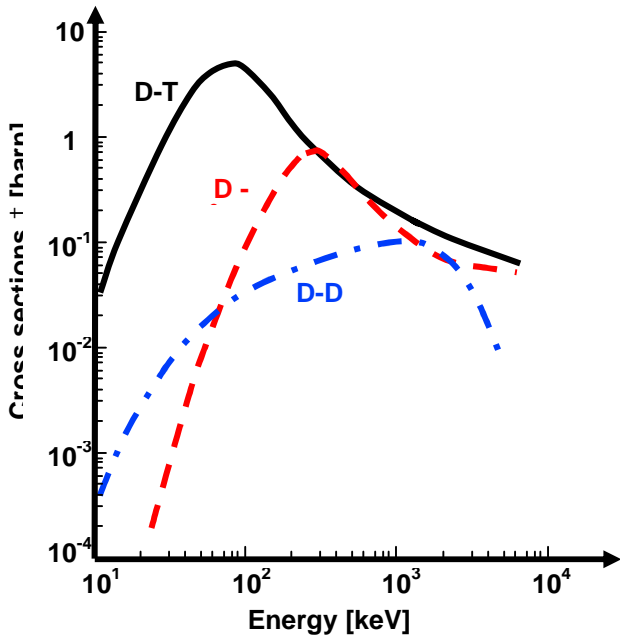


Figure 1. Measured cross sections for different fusion reactions as a function of the averaged centre of mass energy. Reaction cross sections are measured in barn. (1 barn= 10⁻²⁸ m²)

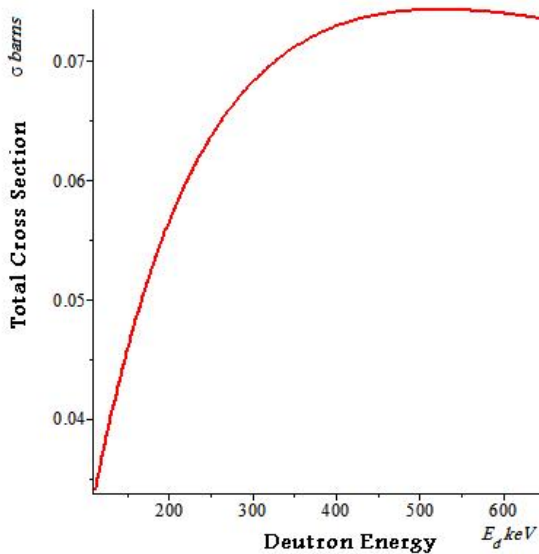


Figure 2. Variation of the total cross section for D-D fusion reaction vs. the incident deuteron energy

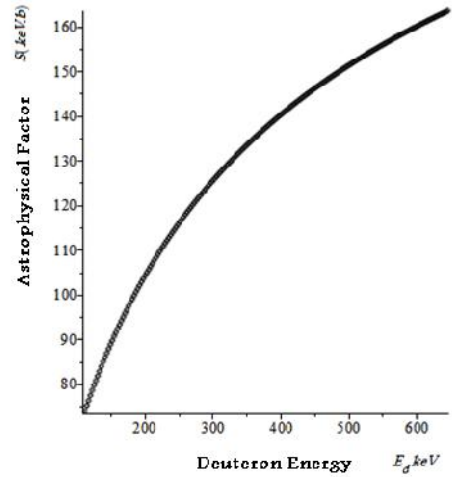


Figure 3. Variation of the astrophysical factor vs. the incident deuteron energy

The D-D fusion reaction rate as a function of the deuteron energy and deuteron temperature is calculated according to Eq.(6) by integrating the equation over energies range from (112.2-646.1 KeV) per each temperature and their calculated results are completely presented in Table (2) and described in Fig.(4). From (112.2-646.1 KeV) we calculated the mean free path and their results are completely presented in Table (3) and described in Fig.(5).

Table 2. Temperature dependence fusion reaction rate for D-D nuclear fusion reaction

Deuteron Temperature $T_d (\times 10^9) (K)$	Fusion Reaction Rate $\langle \sigma v \rangle < (\text{cm}^3/\text{mol}^1/\text{s}^1)$
0.2	0.09403
0.3	0.11517
0.4	0.13298
0.5	0.14868
0.6	0.16287
0.7	0.17592
0.8	0.18807
0.9	0.19948
1.0	0.21027
1.1	0.22053
1.2	0.23034
1.3	0.23975
1.4	0.24880
1.5	0.25753
1.6	0.26597
1.7	0.27416
1.8	0.28211
1.9	0.28984
2.0	0.29737

Table 3. Energy dependence mean free path for the D-D nuclear fusion reaction

Deuteron Energy E_d (keV)	Mean Free Path λ_{DD} (m)
112.2	7.1823×10^{-4}
172.2	4.3957×10^{-4}
232.8	2.9919×10^{-4}
314.6	1.8871×10^{-4}
379.2	1.3355×10^{-5}
470.2	8.2884×10^{-5}
557.3	5.2682×10^{-5}
646.1	3.3193×10^{-5}

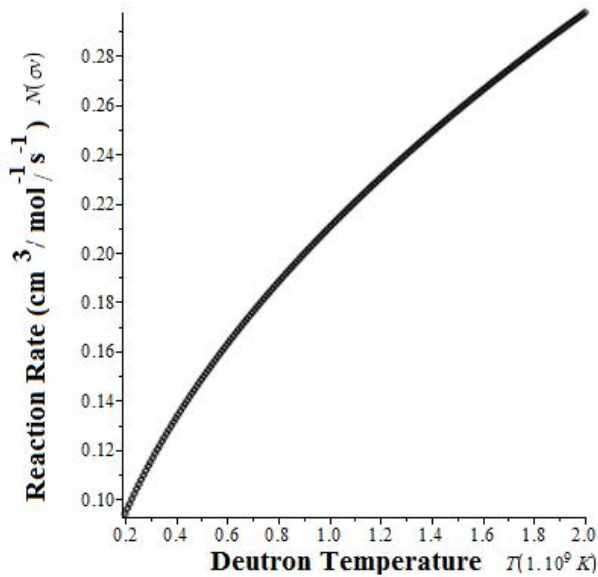


Figure 4. Variation of fusion reaction rate for D-D fusion reaction vs. the deuteron temperature

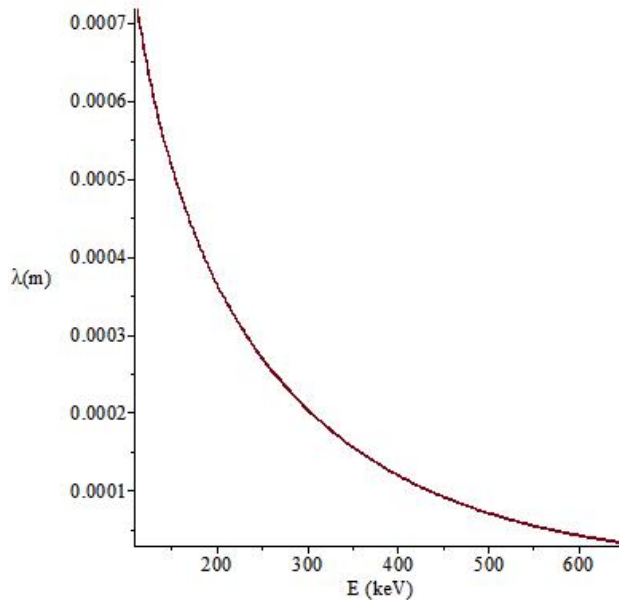


Figure 5. Variation of mean free for D-D fusion reaction vs. the deuteron energy

DISCUSSION AND CONCLUSION

It is clearly shown that the most important parameter in deducing the accuracy for our results is the fusion reaction rate since it is included the cross section in addition to the physical parameters that describe any fusion collision phenomena i.e, incident energy for the projectile, target energy, density and ignition temperature. By comparing the cross section behavior for the D-D fusion reaction presented in Fig.(4) with the corresponding experimental published.

We observed a right agreement between our results and experimental published and this behavior leads to a fact that there exists a real precession for the choices of cross section formula which in turn gives reflect on the results about both the two other parameters $S(E)$, $N_a < \sigma v >$ respectively. And the above conclusions are very clearly shown in the physical behaves for the D-D reaction rate in which we have very similar shapes with the other published results (Leonard *et al.*, 2006).

From the Table (3), we see that the mean free path decreases with the increasing of both the deuteron beam and considerably overcomes the dimensions of the pinched plasma. (Kubes *et al.*, 2007) Finally, we suggest to support our efforts by using the above physical description in the future work especially the works that deal with the general thermonuclear fusion reaction.

REFERENCES

- Angulo C. and P. Descouvemont, Nucl. Phys. A639, 733(1998)
 Angulo C., M. Amould, M. Rayet, P.Descouvemont, D.Baye, C.Leclercq-Willain, A. Coc, S. Barhoumi, P.Aguer, C.Rolfs *et al.* 1999. Nucl.Phys. A656,3.
 Barker F.C., Nucl.Phys. A707, 277(2002).
 Bonasera A., M.D. Toro, and C.Gregoire, Nucl.Phys. A483, 738(1988).
 David J.Rose, Melville Clark, JR" Plasmas and Controlled Fusion", John Wiley and Sons Inc., New York, London, (1961).
 Decreton M., V. Massaut, I.Uytendhouwen, J.Braet, F.Druyts, B. Brichard, and E.Laes " Controlled Nuclear Fusion: The energy of the stars on earth" , Open Report, SCK.CEN-BLG-1049, September, (2007).
 Drog M., "On the Energy Dependence of the Total Cross Section of the Reaction D(d,n) ^3He " Nucl. Sci. Eng.", Vol. 65, (1978).
 Kubes P., D.Klir, J.Kravarik, K.Rezac, B.Bienkowska, I. Ivanova – Stanik, L.Karpinski, M.Paduch, M.Scholz, H.Schmidt , M.J.Sadowski, K.Tomaszewski, D-Dreaction and fast deuterons in plasma focus facility, 28th ICPIG, July 15-20, (2007), Prague, Czech Republic. Topic number: 17.
 Leonard D.S., H.J. Karwowski, C. R. Brune, B.M. Fisher and E.J. Ludwig, Phys.Rev.C 73, 045801(2006).
 Sachie Kimura and Aldo Bonasera, Phys.Rev. C76, 031602(R)(2007).
 Thomas R.G., Phys. Rev. 100, 25(1955).
 Weisskopf V., Phys.Rev. 52,295(1937).
 Yan J., F. E. Cecil, J.A.McNeil, M.A.Hofstee, and P.D.Kunz , Phys.Rev. C55, 1890(1997).
 Yeong E. Kim, Purdue Nuclear and Many Body Theory Group (PNMBTG) Preprint PNMBTG-12—17 (July 2012).

