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RESEARCH ARTICLE

AN APPLICATION OF MULTIVARIATE STATISTICAL PROCESS CONTROL IN DANA STEEL COMPANY

***Ibrahim Mu'awiyya Idris**

Umaru Musa Yar'adua University Katsina, Nigeria

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ABSTRACT

Multivariate statistical process control is a branch of industrial statistics that involve monitoring quality specification of related variables simultaneously. Quality is the most essential target for manufacturing engineers and which mostly involves more than one variable in industry,i.e., a vector of variables (that conform to specification for measurement) which may be correlated. When these quality variables are correlated then the most well-known approach for multivariate process monitoring is the Hotelling's T-square control chart. In this research, a multivariate data in sub-groups consisting of five quality characteristics obtained from Dana Steel Company Limitedkatsina is analyzed for quality. Retrospective analysis shows that, the production process from which the data were obtained is in statistical control.

INTRODUCTION

Multivariate statistical process control is a branch of industrial statistics that deals with two or more related quality variables (quality characteristics), the first pioneering work in the study of multivariate quality control was done in Hotelling (1931). Hotelling wrote a paper on T-square test procedures for multivariate population and subsequently in Hotelling (1947), hotelling developed an extension of the T-square to control charts. Often there exist variability in the raw materials and variability in the production process due to natural and assignable causes, in addition the effect of these variabilities directly affects the products conforming to quality specification. The growing interest in multivariate quality control started in the last three decades when computer technology record its advancement that makes it possible to monitor multiples of quality variable all together in production process. An extensive review of literature for multivariate quality control (MQC) which includes multivariate cumulative sum (MCUSUM) and Multivariate exponentially weighted moving average (MEWMA) are discussed in Lowry and Montgomery (1995) and also in Alt (1985). In Timothy and Paul (1999), the multivariate control chart of Hotelling T-square was applied to a wood industry to monitor the vertical density profile of the wood which is influenced by many quality variables in the manufacturing process, there are indeed many application of Hotelling T-square multivariate control chart in literature.

***Corresponding author: Ibrahim Mu'awiyyaIdris**
Umaru Musa Yar'adua University Katsina, Nigeria.

This research work focuses on the application of Hotelling T-square multivariate control chart in Dana steel rolling industry katsina state, Nigeria. Recently there were sudden collapses of buildings across the nation and which claimed lives and properties worth millions of naira, research and investigations revealed that many factors contributed to the disaster and one of these factors is whether or not steel rods used during construction processes met the quality requirements. The data used in this research is a secondary data in ten subgroups of ten observations each obtained from Dana steel company limited Katsina, it comprises of five quality variables (Weight KG/M, Area MMSQ, Breaking force KGF/MMSQ, Tensile strength N/MMSQ and Yield strength N/MMSQ).

METHODS

Hotelling T-square control chart for sub-grouped data is used, the test statistics is given as

$$T_i^2 = n\left(\bar{X}_i - \bar{\bar{X}}\right)' S_p^{-1}\left(\bar{X}_i - \bar{\bar{X}}\right) \qquad (1)$$

Where $\bar{X}_i$is the sample mean of size n for the m subgroupsfor which $(i = 1,2,\dots,p)$, $\bar{\bar{X}}$is an unbiased estimator for the unknown population mean$\mu$and$S_p^{-1}$is the inverse of the pooled covariance matrix which is obtained by averaging the subgroup covariance matrices over the k subgroups. There two phases for control chart usage, the control limit for phase (I) known as the retrospective analysis is used for monitoring the in-control condition for the process under investigation

$$UCL = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{a(p,mn-m-p+1)} \quad (2)$$

$$LCL = 0 \quad (3)$$

Phase (II) which is known as the prospective analysis is used to set a control limit for future production having identified the quality attribute responsible for the out of control signal, in addition, this stage is known as the stage of improving quality, Ryan (1989). The control limit for this phase is given as

$$UCL = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{a(p,mn-m-p+1)} \quad (4)$$

$$LCL = 0 \quad (5)$$

Analysis of the sub grouped data (retrospective analysis)

The five p quality variables are jointly monitored from the sample data consisting of m subgroups with n observations each, the analysis is presented in the following steps

Step1: Compute the sample mean $\bar{X}$ for each of the 10 subgroups

Subg1 $\bar{X} = [0.8542 \quad 108.7000 \quad 7808.0000 \quad 719.2000 \quad 451.8000]$
Subg2 $\bar{X} = [0.8661 \quad 110.4000 \quad 7190.0000 \quad 650.9000 \quad 430.6000]$
Subg3 $\bar{X} = [0.8503 \quad 108.3000 \quad 7172.5000 \quad 662.6000 \quad 430.9000]$
Subg4 $\bar{X} = [0.8528 \quad 108.5000 \quad 7360.0000 \quad 678.2000 \quad 431.0000]$
Subg5 $\bar{X} = [0.8561 \quad 109.0000 \quad 7283.0000 \quad 669.7000 \quad 435.7000]$
Subg6 $\bar{X} = [0.8689 \quad 110.6000 \quad 7572.5000 \quad 680.7000 \quad 442.1000]$
Subg7 $\bar{X} = [0.8585 \quad 109.3000 \quad 7872.5000 \quad 719.6000 \quad 458.9000]$
Subg8 $\bar{X} = [0.8631 \quad 110.0000 \quad 7561.0000 \quad 687.8000 \quad 439.1000]$
Subg9 $\bar{X} = [0.8792 \quad 112.0000 \quad 7265.0000 \quad 648.6000 \quad 422.3000]$
Subg10 $\bar{X} = [0.8666 \quad 110.3000 \quad 7464.0000 \quad 679.0000 \quad 428.2000]$

Step2: Compute the sample mean average $\bar{\bar{X}}$ (Grand mean) of the subgroups by averaging over the sub grouped means.

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \cdots + \bar{X}_n}{n}$$

$\bar{\bar{X}} = (0.86158 \quad 109.71000 \quad 7453.35000 \quad 679.56000 \quad 437.06000)$

Step3: Set the matrices $(\bar{X} - \bar{\bar{X}})$ and transpose the matrices $(\bar{X} - \bar{\bar{X}})'$

Subg1

$(\bar{X} - \bar{\bar{X}}) = [-0.01158 \quad -0.71000 \quad 2196.65000 \quad 214.44000 \quad -6.06000]$

$$(\bar{X} - \bar{\bar{X}})' = \begin{bmatrix} -0.01158 \\ -0.71000 \\ 2196.65000 \\ 214.44000 \\ -6.06000 \end{bmatrix}$$

This is computed for the remaining subgroups. Summing up the matrices $(\bar{X} - \bar{\bar{X}})$ and $(\bar{X} - \bar{\bar{X}})'$ we obtain the following matrices

$(\bar{X} - \bar{\bar{X}}) = [1.435967 \quad 182.850000 \quad 12422.250000 \quad 1132.600000 \quad 728.433333]$

$$(\bar{X} - \bar{\bar{X}})' = \begin{bmatrix} 1.435967 \\ 182.850000 \\ 12422.250000 \\ 1132.600000 \\ 728.433333 \end{bmatrix}$$

Step4: Compute the sample variance-covariance matrix S for each of the subgroups. For subgroup 1, the covariance is computed as

$$\begin{bmatrix} 0.0001081778 & 0.01195556 & -8.468444e+00 & -8.618222e-01 & 9.155556e+03 \\ 0.0119555556 & 1.34444444 & -9.934444e+02 & -1.007111e+02 & 2.488889e+00 \\ -8.4684444444 & -993.44444444 & 1.140762e+06 & 1.117910e+05 & -1.492044e+04 \\ -0.8618222222 & -100.71111111 & 1.117910e+05 & 1.097929e+04 & -1.375178e+03 \\ 0.0091555556 & 2.48888889 & -1.492044e+04 & -1.375178e+03 & 8.559556e+02 \end{bmatrix}$$

The covariance matrices for the remaining $n-1$ subgroups are computed respectively.

Step5: Set the pooled covariance matrix S_{pooled} by averaging the covariance matrices of the n subgroups and compute the inverse pooled covariance matrix S_{pooled}^{-1} which is given as

$$\begin{bmatrix} 488.788637 & 3.865702627 & 4.202171e+00 & -6.363880e-01 & -2.280665e+00 \\ 3.865703 & 0.029972525 & 2.706089e-02 & -4.895473e-03 & -1.577886e-02 \\ 4.202171 & 0.027060887 & 3.088102e-07 & 3.363124e-06 & 7.814394e-05 \\ -0.636388 & -0.004895473 & 3.363124e-06 & 3.465877e-05 & 6.269687e-04 \\ -2.280665 & -0.015778862 & 7.814394e-05 & 6.269687e-04 & 3.984987e-04 \end{bmatrix}$$

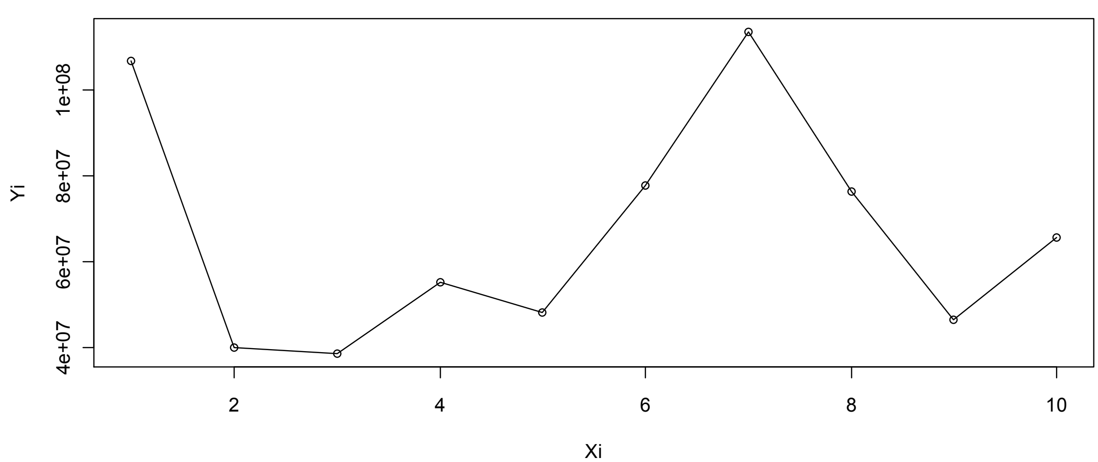

Fig. 1. Control Chart of Sub Grouped Data

Step 7: Compute the Hotelling T-square statistic using eqn (1)which is given by

$$\begin{bmatrix} 106843677 & 40133022 & 38688889 & 55294991 & 48106872 & 77674311 \\ & 113560407 & 76361994 & 46567964 & 65800985 & \end{bmatrix}$$

Step 8: Using eqn (2) and choosing$(a = 0.01)$, we compute the upper control limit where

$$UCL = 15.72$$

$$LCL = 0$$

The corresponding control chart for the retrospective analysis is given in Fig1, where $T^2 = Y_i$ values are plotted against the subgroupsX_i.

RESULTS AND DISCUSSION

In our analysis of the sub-grouped data, all the subgroups points fall within the control region which are below the upper control limit as indicated by the control chart (Fig. 1), therefore the production process that generated the data is said to be in state of statistical control. In the analysis of Hotelling T-square for sub-grouped data, since there are two distinct phases for control chart usage where phase I (retrospective analysis) is used in testing whether the process is in state of statistical control when m preliminary samples were drawn and the sample statistic $\bar{\bar{X}}$and S_{pooled}computed for the subgroups. In addition, the main objective is to obtain an in control set of observations so that control limit can be established for phase II,but if in examining the control chart of phase I (retrospective examination) and all points plot within the control limit as in Figure 1, i.e. no point exceed UCL, then the same control limit $UCL$for phase I is sufficient for monitoring the future process of production, hence, UCL for phase II will no longer be computed.

Conclusion

Quality is the most essential factor within market, and multivariate control chart procedure using Hotelling's T-square statistic provides a reasonable tool for monitoring production processes with multiple correlated variables. In this research we have analyzed a multivariate data obtained from Dana Steel Company limited katsina, Nigeria and the analysis carried out based on Hotelling's T-square revealed that the data was obtained from statistically controlled production process.

REFERENCES

Alt FB. 1985. Multivariate quality control. *The Encyclopedia of Statistical Sciences*, Kotz S, Johnson NL, Read CR (Eds.). Wiley: New York; 110–122.

Hotelling, H. 1931. The Economic of Exhaustible Resources. *The Journal of Political Economy*; 137-175.

Hotelling, H. 1947. Multivariate quality control-illustrated by the air testing of sample bombsights, in Techniques of Statistical Analysis, Eisenhart, C., Hastay, M.W. and Wallis, W.A. (eds.), McGraw-Hill, New York, NY, pp. 111–184.

Lowry CA, Montgomery D.C. 1995. A Review of Multivariate Control Charts. *IIE Transtions*. 27: 800-810.

Young, Timothy M and Winistorfer, Paul M. 1999. Multivariate Control Charts of MDF and OSB Vertical Density Profile Attributes. *Forest Product Journal*, Vol.49, Issue 5, page 79.

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