REVIEW ARTICLE

A META-GRAPHIC FRAMEWORK FOR MAJOR THEORIES OF POVERTY WITH DYNAMIC AND INVARIANT MAPPING ANALYSIS

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ARTICLE INFO	ABSTRACT
	This paper presents a functional framework to study diverse theories of poverty in the current literature. Because of the complex nature of different schools of thought on poverty, one needs a meta-graph that can relate element in the poverty domain of a function to elements in the egalitarian range of the function. This paper presents such a framework that can compare and contrast different theories of poverty. Two novelties of the paper are dynamic analyses for a group of countries, and a correspondence relationship that allows us to look for invariant relationships in models of poverty. The former applies a new dynamic principle in economics, and the latter fills a gap in the literature identified by the Nobel Laureate Amartya Sen's theory of poverty.

INTRODUCTION

What is poverty? A definition presumes that one should have available a necessary amount of wealth or income. To nail down the necessary amount is a constant struggle among economists because of the broadness of the concept. Coming from another angle, we find answers that try to specify a sufficient amount of wealth and income to meet a basic set of needs, such as the poverty line measure of the US Census, or the socio-economic measure of the UNDP's HDI index. A search for an ideal definition seems out of reach at this time, but appears to be impaled in the interval between the necessary and sufficient level of wealth or income that would keep a person above poverty.
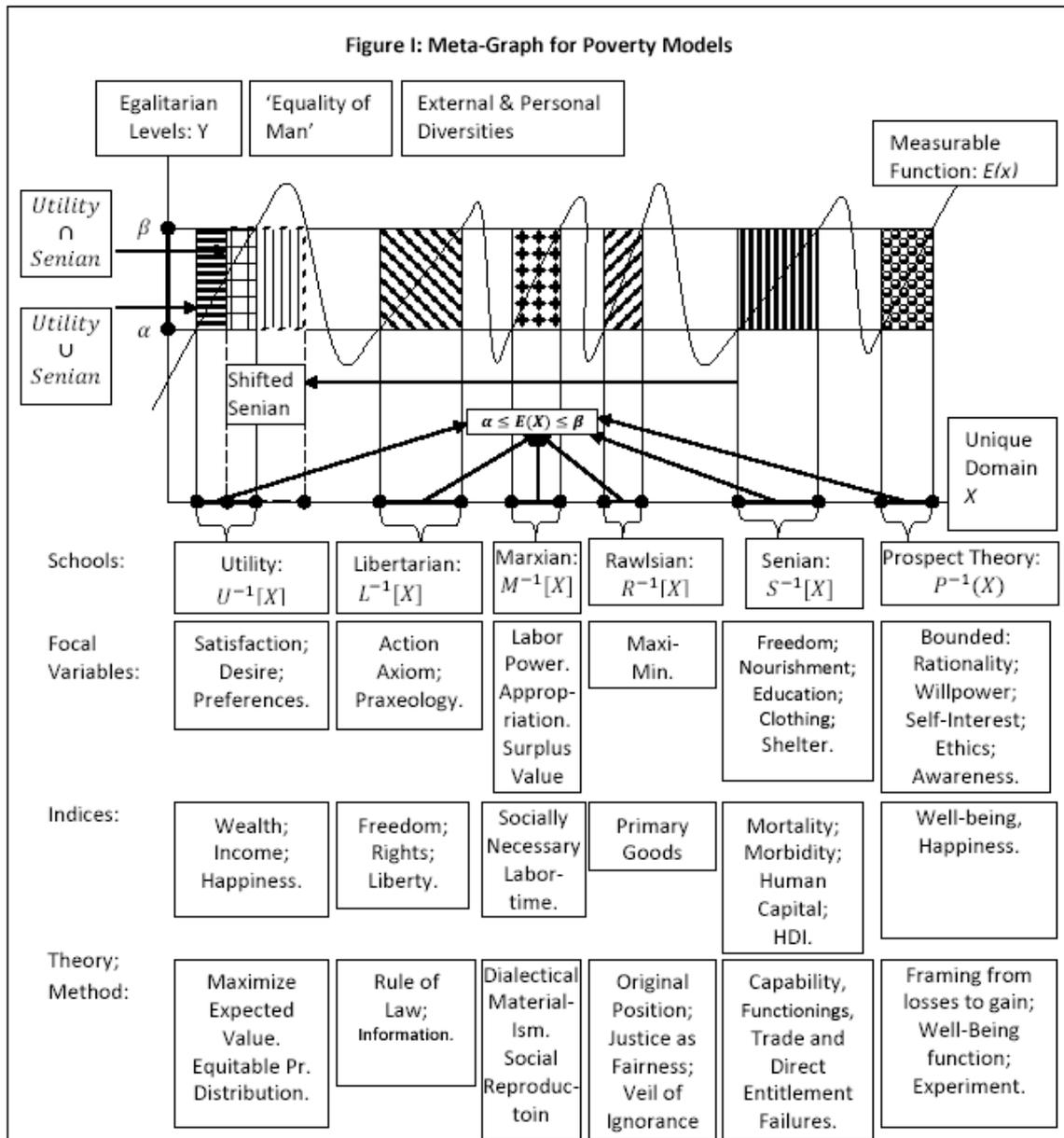
This paper proposes a meta-graph framework for that interval of study, and illustrates how a major theory of povertycan be viewed for that functional perspective as well as its extension to a correspondence relation. The first part of the paper presents the meta-function framework. It then takes up the illustration of Sen's theory of poverty in that framework, illustrating dynamic relations for developing countries. Finally, the paper ends with a brief history of the theories of poverty in the meta-graph.

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A Unified Model of Poverty

Figure 1 below is the meta-graph that brings the main theories of poverties into focus. The height of the bars are shown to be equal, but ideally, they reflect some objective (income) and subjective (utility) measure of poverty. The need of a measure comes up when we analyze standard of living as well. According to Milton Friedman, "The levels of living regarded as poverty are always judged by any society relative to the general level of living." (Friedman, 2008, pp. 255-256) Victor Fuchs puts the matter this way: …"Low" income or "poverty" in the United States in the 1960's is largely a matter of economic distance. When most Americans have a great deal, those who have much less are poor regardless of their absolute level of income". (Fuchs 1969, pp. 198; 199)

The vertical axis of Figure 1 represents an egalitarian space. Such a space is intended to capture the widely held views of poverty, including the Marxian view of social needs, Amartya Sen's equity concept, John Rawls's contractarian-cum-original position view, John Harsanyi's equiprobable distribution view for the utilitarian perspective, Libertarian equal Action Axiom-cum-rights concepts, and the well-being concept of prospect theory. The horizontal axis of Figure 1 represents the unique characteristics in the domain of the different schools of thought. For instance Marxists and Capitalists have different views of reality that do not meet at the hard-core level. Marx adopted the view of Dialectical Materialism where disorder is the bases of reality. On the other hand, capitalists tend to follow order in a step-by-step, nomological, way.

Figure I: Meta-Graph for Poverty Models

Schools:	Utility: $U^{-1}[X]$	Libertarian: $L^{-1}[X]$	Marxian: $M^{-1}[X]$	Rawlsian: $R^{-1}[X]$	Senian: $S^{-1}[X]$	Prospect Theory: $P^{-1}(X)$
Focal Variables:	Satisfaction; Desire; Preferences.	Action Axiom; Praxeology.	Labor Power. Appropriation. Surplus Value	Maxi-Min.	Freedom; Nourishment; Education; Clothing; Shelter.	Bounded: Rationality; Willpower; Self-Interest; Ethics; Awareness.
Indices:	Wealth; Income; Happiness.	Freedom; Rights; Liberty.	Socially Necessary Labor-time.	Primary Goods	Mortality; Morbidity; Human Capital; HDI.	Well-being, Happiness.
Theory; Method:	Maximize Expected Value. Equitable Pr. Distribution.	Rule of Law; Information.	Dialectical Material-Ism. Social Reproductoin	Original Position; Justice as Fairness; Veil of Ignorance	Capability, Functionings, Trade and Direct Entitlement Failures.	Framing from losses to gain; Well-Being function; Experiment.

At the fundamental level, we are secured in the prediction that the base of the rectangles in Figure 1 should not intersect for Marxists and Capitalists views.

For schools within the capitalist paradigm, the framework requires further study of the bases on which individual schools can intersect in their domain elements. For instance, one distinction of Sen's thought from say a Utilitarian perspective is that a "person choice" and "personal welfare" are separated in the former, but is an identity in the latter. As Sen puts it "The characteristic of commitment with which I am most concerned here is the fact that it drives a wedge between personal choice and personal welfare, and much of traditional economic theory relies on the identity of the two." (Sen 1977, RF p. 329). The crossed area in Figure 1 represents the Cartesian product of the unique aspect of each school measured on the X-axis, with their welfare contribution measured on the Y-axis. As shown, the Cartesian product does not overlap for schools. But Sen himself gives a nested utility function that

suggests overlap with the utility model. To show overlap in the Sen's case, we moved Sen's representation to overlap with the utility representation. Now, the double crossed area indicates the case where the utilitarian and Sen's views overlap or intersect (∩), while the perimeter of the two boxes show their addition, or union (∪).

The schools are represented as separate inverse functions from the graph. If the inverse exists, then it can be used to measure the contribution of each school egalitarian contribution, which is especially useful where schools overlap as in the case of Sen and Utilitarian schools. One technical aspect of Figure 1 is regarding the space we are dealing with. We deal with Euclidian space in common parlance where a function maps a point on the domain of X-axes to one point on the range of the Y-axes. In a correspondence, X and Y can be sets of points, which will also be used in this paper.

A Specialization of the Meta-Graph Model

Within the framework of Figure 1, one can analyze various aspects of a particular theory. We illustrate how this can be done for Sen's theory of poverty. In Figure 2, the domain of the function X, is partitioned into human capital and mortality aspects of poverty. Each domain element is an injection into the functioning set. Human Capital is mapped to a variable capability set with elements of college, on-the-job training, and high school level education. Mortality is mapped to a variable capability set with elements of caste, race, and gender. Because each capability set has multiple elements, the function is not constant.

We first notice that the domain set X is partitioned into mortality and human capital. This partition can be represented as a union of elements of the set, namely, $X = x \cup x'$. In economics the list of elements may be extended to include natural resources, income, and assistance in daily living and so on. For growth theory analysis, the domain elements are treated as independent variables, which purges any idea such as multi-colinearity between them.

implying that for $x, x' \in X, \varphi(x) \neq \varphi(x')$, yielding a one-to-one (1:1) mapping.

One finds a variety of image sets in the literature. While a neoclassical economists will write a utility function for a number of commodities, x_i, Sen writes a nested function $u_i = h_i(f_i(c(x_i)))$, where c is a characteristic function, f is a utilization function, h is a happiness function for the commodity x for the individual i. (Sen 1999, p. 7) Basically, Sen segments the function as $u_i = h_i(b_i)$ where b is a person being or well-being. It put a value to whether the person is "well-nourished, well-clothed, mobile, taking part in the life of the community" (Ibid., p. 8). Emphasis on the image set tends to be associated with ranking of a person's well-being. In traditional preference analysis, one can value b by constructing a scalar value or an index. Sen has proposed an ordinal measure of poverty, which focused on "the available information on the poor". (Sen 1976, p. 219) For example, we may note that the United Nations Development Program (UNDP) created the Human Development Index (HDI) to include social and economic objectives.
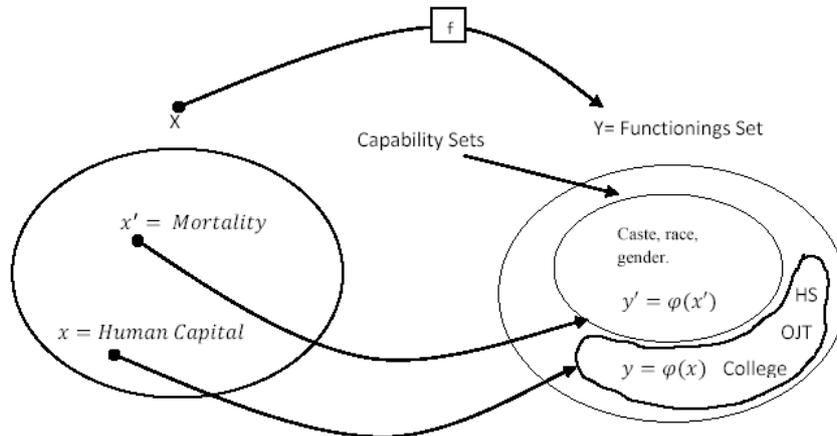

Figure 2: Set-value Mapping

Let A_a be a collection of the capability sets in the image of the mapping, and let B be the functioning set. Then the capability sets are subsets of the functioning set, $A_a \subset B$. One can then state the obvious relationship: $A_a \cap B = A_a$; $A_a \cup B = B$. This set argument, however, masks a subtle relationship between the capability sets and the functioning set. While the capability sets depict attributes or things one can own, acquire or produce, the functioning set is the resulting features of the states one has attained from acquiring capabilities. Capability sets appear as inputs to functioning sets. (Sen 1987, STD, p. xii). It seems natural to describe the elements of the capability set in terms of things one own or should own. Then we can use a dummy variable where 1 can mean that a person owns that attribute, and 0 can mean that a person does not own that attribute but needs to acquire it. Then a capability set can consist of all the real numbers between 0 and 1, which is denoted by $A = \{y | 0 \leq y \leq 1\}$, if the set is closed, or by $A = \{y | 0 < y < 1\}$ if the set is open. We show the capability sets as distinct,

HDI gives equal weights to life expectancy, educational index, and the per capita GDP of a country.

Mapping between Capability and Function sets

One could easily have split the functioning set into a third set, say Z, and have a composite mapping from X to Y to Z. In this composite case, the mapping between X and Y can be treated as a sequence map. A sequence map numbers the elements in X, and make a list of all $y \in Y$. Then, we can assign for k^{th} term in X, the function $f(k) = y_k$. (Binmore 1981, p. 169) The mapping between X and Z, however, shows a relationship between capability and functioning. One can therefore, compare capability sets and the functioning they generate across regions and countries. Doing binary comparisons of a similar capability set for two countries or regions, one may check for a 1:1 correspondence between the two set. If they are 1:1, they have the same cardinality or can be put in pairs, then

they have the same number of elements. On may take the same number of elements present in a region's or country's capability set as one sense of equality, but some better measures would include a measure of sameness, intensity, or level of the comparison. Immediately, one can see that, using the human capital capability set, two countries sets may not have the same number of elements. Some poor countries may have up to high school level education only, while other countries may have levels of HS levels and beyond. In other words, the paring for those two sets will break down, revealing a sense of inequality. The country with excess unpaired elements in its capability set can be said to enjoy a ranking of being better off.

Equality by cardinality has two implications for economists. One needs to distinguish between the formal primitive notions of sets, and sets as containers. This is a necessary distinction to be made for answering the question "equality of what". According to Sen, many different variables are involved in judging equality, which creates a "plurality of spaces" in which to judge equality. Plurality of spaces includes variables such as income, wealth, utility, resource, liberty, rights, and quality of life. Equality is not congruent in these spaces. (Sen 1991) Now, if we were to analyze cardinal equality using the primitive notion of sets, where the term sets and members of a set are undefined primitives, then we would miss a comparison of attributes that the people in the comparison have or contains.

From a cognitive point of view, Lakoff and Nunex have argued that the concept of set as in the Zermelo-Fraenkel axioms are of the formal type, that is, they are not containers, because of the paradox that an absolute set can contain itself. (Lakoff and Nunez 2000, pp. 145-146) An absolute set is paradoxical because it is a set of all sets, and cannot contain itself. But we need only deal with consistent sets. If one is concerned with capability and functioning, one has to study persons as containers. People have different characteristics such as age, sex, physical, and mental abilities. We must treat them as containers of these abilities. Speaking in a sense of abused terminology, people contain education differently. One attempt by the philosopher Mortimer Adler to address varying level of intelligence in education is to offer all students the best education. In his words "Human differences in capacity for education can be thought of in terms of containers of different sizes… Treating them equally does not mean trying to does the impossible, i.e., trying to put a quart or gallon of education into a half-pint container. It means, first of all, trying to fill each container up to the brim. To fill each to its capacity is to treat them equally, yet with full acknowledgement of their unequal capacities...But more than that is involved in treating them equally. "We will not be on the path of democratic education until we discover a way of pouring the richest cream we know how to concoct into all the containers, large and small, each right up to its capacity. Nothing less than that is equal educational opportunity." (Adler 1957, pp. 30-31).

There is something in Adler model that might not be acceptable. Adler is looking at the commodity space, the best quality of education, and not the functioning space, does the person have the ability to understand. Maximization of utility assigns the same weights to all individual gains, and therefore

do not accentuate differences, embracing the formal set model. John Rawls idea of primary goods and a new idea of Ronald Dworkin of equality in resources are seen as "instruments for giving people equal opportunity to pursue their end. (Dworking 1981; Sen 1992, IR, p. 19) Similarly, the libertarian would have us assign equal rights to everyone, which would not mean equality in the outcome spaces such as in wealth and income. (Sen 1991, p. 9) Prospect theory which underscores cognitive biases such as framing, and bounds on rationality, awareness, will-power, and ethics is focusing on the individual biases, testing them under laboratory conditions. Sen argued that "diagnosis precedes policy choice", which will place him squarely in the container side of things. (Sen 1995, IR, p. 107) Economists are interested in something else besides equality in the primitive and container aspect of cardinal equality. First, economists are concerned how the capability sets, say Human Capital is accumulated in a country and across countries. For purpose of economic growth, one also is concerned with whether or not the A_a sets can overlap, or are equivalent for say different countries. One reason one would ask these questions is that and appropriate index for capability, such as the HDI index is out to imitate is applied all over the world.

Second, for discrimination based on caste, race, and gender, we may have relationships in the sense that some people having HS but cannot afford college and find it hard to get a job, so OJT is low. Some people may be going to school on a part-time basis while they work, so that OJT and education proceed in a parallel fashion. Empirically, we know that some people may become successful in these parallel ventures and become more capable, and other people may not be successful in one or both. One can even think of people diversifying their efforts over capability sets in order to increase the probability of one's success, just as pharmaceutical companies may involve in a dozen or so parallel R&D projects to ensure one successful new chemical entity is attained. Third, a set of values in a country, such as the USA, may be injected into the states of the union, or imitated in other countries. Gone are the days when mother countries have transplanted cultures to their colonies. To the extent that cultures are shared, one can map capability sets of one country into another. Health, education, income, freedom and liberties are good candidate to look for if one is interested in invariance across time and space, the future and globally. (Dasgupta 1993, p. 70) It seems that one has to proceed in this way, in order to spotlight the kind of research Willard Quine had suggested to Sen, which we can put as a hypothesis:

Hypothesis [Quine]

In classifying ethical theories of social arrangements, one should compare invariants of equalities under factual transformation on the one hand, with properties of space that are invariant under group transformation as in the Erlanger Programm. (Sen 1995, IR, p. x)

We will try to address some of these concerns below. Our main concern will be to spotlight the invariance aspect of the discussion, and be less concern with whether capability sets can overlap.

The mappings between capability sets can be two-ways. Traditionally, invariance envisions a map, say f of a capability set for human capital that is spatially internal: $r: USA \rightarrow States$ or spatially external: $G: USA \rightarrow UK$. In the spatially internal case, if *States* send money (tax revenues) to the *USA* (Fed Tax), $q: States \rightarrow USA$, which are then returned to the *States* as entitlement for education in human capital, then composition map $q \circ r$, can operate back and forth between the *USA* and its *States* without reaching a stable invariant level. In mathematical language, an invariant requires that no retraction, r, should take place. It seems that the hypothesis is predicting or explaining that an invariant would most likely occur under a G map scenario that assumed only market activities. In other words, this would seem to imply that an invariant scheme under the hypothesis would require ultra-self-interest, with no aids aboard, and would most likely flourish under a pure market system as advocated under the utilitarian or libertarian regime. It cannot occur under prospect theory because people have cognitive biases. Nor do we expect it to occur under Sen's system because of its leaning toward institutional policies.

A group version of the mapping, again two-ways, may be considered under the Cantor-Schroeder-Bernstein theorem for sets. We start out with two capability sets, A and B avoiding the internal and external distinction. Let one map be: $f: A \rightarrow B$, and let the other map be $g^{-1}: B \rightarrow A$., that goes from a capability to a partitioning of the domain set. If both maps are injective then we can find an equivalent relation between A and B. Figure 3 below suggests such a mapping. The figure shows two capability sets, one each for two regions or countries. Citizens from the two regions or countries partition the two sets similarly, that is, they know how members of each country partition their capability sets. A directed arc connects $x \in A$, to $f(x) \in B$; and $y \in B$, to $g(y) \in A$.

The members of A are carried by an injective maps into B, and vice versa, members of B are carried by an injective maps into A. Because f and g are injective, one can find a bijective map $h: A \leftrightarrow B$, that is an onto map between A and B, establishing the same cardinality between the two sets. To visual the whole story, we can think of the zeroes as women, and the dots as men. A match maker wants to pair off couples for marriage. If all women are acquainted with a man, and vice versa, and the match maker can make those matches, then the two sets would be mapped onto each other. (Rosenholtz 2000, p. 22)

While the Cantor-Schroeder-Bernstein theorem illustrated in Figure 3 establishes the same cardinality between capability sets, it must be postulated that the partition of the sets between the two regions or countries are the same, converging, or diverging. Taking the Human Capital set as an illustration, each resident of one area knows how each residents in another area is valuing education at HS, College, and OJT levels. This knowledge helps one to value the elements in the partiitons. We recall that Sen created a vector value of the components and looked for their bounds, maximum or supremium. But knowledge of the partitions by residents in different areas allows another measure, namely, how to analize their dynamics of change.

Dynamcs of Change and Invariance through Graph Theory

To illustrate dynamics of changes among same capability sets for different areas, while again looking for invariants as suggested by Quine, we again appeal to graph theory. The dynamic view looks for adjustment to a common value. Moving out of poverty is like looking for common language, or the reason why herds congregate to avoid predators, or the reason for why birds fly with the same velocity, or why animals flock together in general.
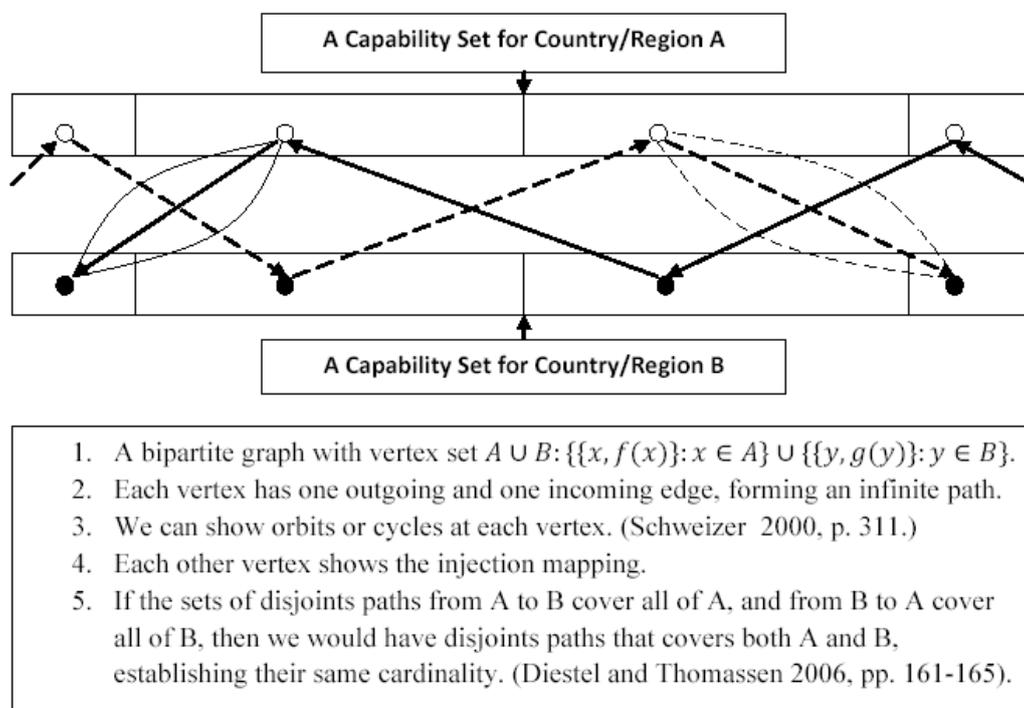
A Capability Set for Country/Region A

A Capability Set for Country/Region B

1. A bipartite graph with vertex set $A \cup B$: $\{\{x, f(x)\}: x \in A\} \cup \{\{y, g(y)\}: y \in B\}$.
2. Each vertex has one outgoing and one incoming edge, forming an infinite path.
3. We can show orbits or cycles at each vertex. (Schweizer 2000, p. 311.)
4. Each other vertex shows the injection mapping.
5. If the sets of disjoints paths from A to B cover all of A, and from B to A cover all of B, then we would have disjoints paths that covers both A and B, establishing their same cardinality. (Diestel and Thomassen 2006, pp. 161-165).

Figure 3: Comparison of two Capability Sets for two Countries or Regions

The main idea is to apply Laplace Matrix to specify possible graph invariant solution to the convergence to a common value problem.

Capability sets of similar varieties must be converging or diverging with a velocity, v_i. We want to deterimine how the capability sets denoted by, c_i, for $i = 1,2,...,k$, relates to the speed of adjustment (velocity), v_i, so that $\frac{d c_i}{dt} = c' = v_i$. The focus now is on how to derive changes in velocity as the differences in the capability elements, h. Cucker and Smale (2007) suggested the following equation expression: $v_j(t+h) - v_i(t) = h\sum_j a_{i,j}(v_j - v_i)$, The expression basically states that the ith person in an area wants to adjust its rate of changes in capability to the jth person rate in a different area, where the average rate is characterized by the coefficient $a_{i,j}$. This coefficient depends on level of capabilities. We can think of people communicating for instance through the ways technology diffuses across the globe and training to use it follows where the technology goes. When such diffusions are strong, the difference in capability associated with that type of human capital will be small. Convergence to a common level or capability can be specified as convergence to the diagonal of a matrix $D_{i,i} = \sum_j a_{i,j}$.The elements of an adjacency matrix, A_p, is specified as: $a_{i,j} = \frac{K}{(1+\| x_i - x_j \|^2)^\beta}$.

The Laplace matrix to study convergence is then defined as $L_p = D_p - A_p$. And the problem of convergence then becomes one of solving the system of differential equations:

$$c' = v$$
$$v' = -L_p v \quad \cdots\cdots\cdots\cdots\text{(System 1)}$$

Dynamic Application to Developing Countries

Dynamic solutions for this system (1), exist and are unique. To illustrate, we use three blocks of developing areas, Latin America, Middle East and North Africa, and Eastern Europe. We examine income in the capability set. A recent report looks at the number of people living on less than \$1.50 per day during 1960-2000. (Campano and Salvatore 2006, p. 124). We assume that because the citizens of these areas are informed about levels of income, the areas is under the force of competition to keep their poverty income in line of convergence. We are after the possible relationships between the variables v and c for the areas. The Laplacian matrix is most easily understood from the perspective of graph theory. We can then examine the Laplacian matrix to spotlight some strategic behavior of firms. One can draw oval around these three areas to represent vertices, and connect them by line or edges to represent the idea that the people in each area are looking at each other. Each area will have two edges, and the graph of the Laplacian Matrix of this relationship is given by the expression:

$$l_{i,j} = \begin{cases} \deg(v_i) & for \quad i = j \\ -1 & for \quad i \ne j \quad for \quad adjacent \quad values \\ 0 & otherwise \end{cases}$$

We can now form a Laplacian Matrix to study the behavior of the areas as:

$$A = \begin{bmatrix} & Lat.Am & ME\ \&\ N.Af & E.Europe \\ \hline Lat.Am & 2 & -1 & -1 \\ ME\ \&\ N.Af & -1 & 2 & -1 \\ E.Europe & -1 & -1 & 2 \end{bmatrix}$$

The characteristic polynomial for this matrix is: $\chi(x) = x^3 - 6x^2 + 9x$, whose roots are 0, and 3+0.00i, and 3-0.00i. Therefore, if we use $\beta = 0$, and $K = 3$, then the equation $a_{i,j} = \frac{K}{(1+\| x_i - x_j \|^2)^\beta}$ is satisfied. But we are still required to fill in values for the x_i, and x_j. Differences of the poverty values for every 10 years from 1960 to 2000 ranges from 6 to 27 million for Latin America vs. Eastern Europe, and from 3 to 10 million Latin America and Mid East and N. Africa. For the purpose of this study we assume the former as 30, and the latter as 10. (Campano and Salvatore, op. cit) The dynamics of the System (1) differential equations with values we discuss has the following field of solutions. Capability convergence measured by in come on the x-axis, and velocity on the y-axes. The isoclines show the path of velocity approaching a limit point of zero on the vertical axis. The steeper the curve, the faster will the equilibrium point be reached.

The dynamics for income capability set is only a first step. One needs to consider a plurality of them. For instance, we were able to gather some data for nutritional intake from the years 1969-71 and 1979-81. (Dasgupta 1993, p. 440) The data is only partial, but we can take and approximate difference for the regions to be 30 and 45. One must also make an assumption about the relation of income to nutritional intake. Assuming they are independent and have same weights in System (1), therefore, we enter them as product in the system. The results show the joint results for $(ci - cj)$ representing differences in per capita incomes, and $(zi - zj)$ representing differences in nutritional deficiencies of millions of person in the areas. The results indicate an unstable situation, where as zero is approach from below, escape from zero happen above zero. This solution may be found acceptable for those who think that economic behavior are more in the form of hyperbolic solution, particularly in the specification of utility curves.

We take note that using the same method, one can make another kind of comparisons among 1. The U.S. Relative to the poorest Country, 2. The U. S. Relative to the 10th Poorest Countries, and The 20 Richest Countries relative to the 20 poorest countries. Such a comparison is worthwhile because instead of convergence, one finds divergences
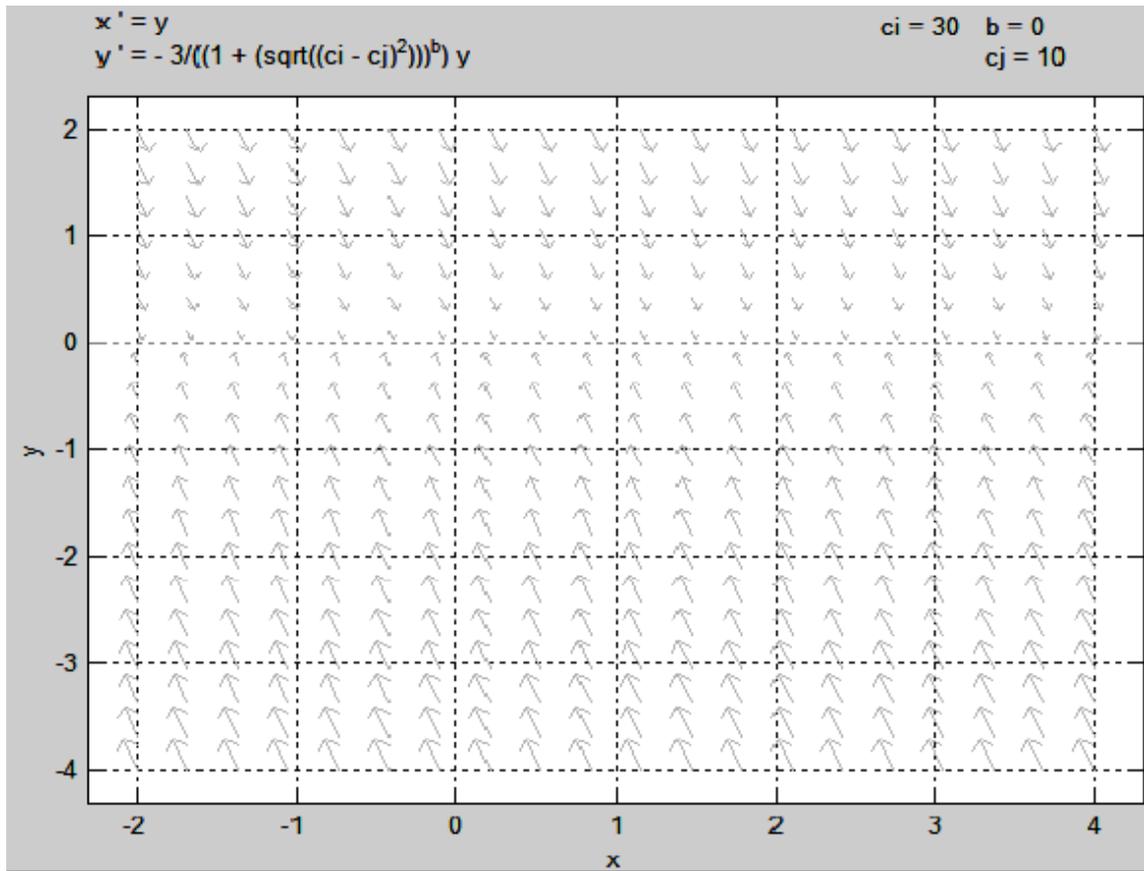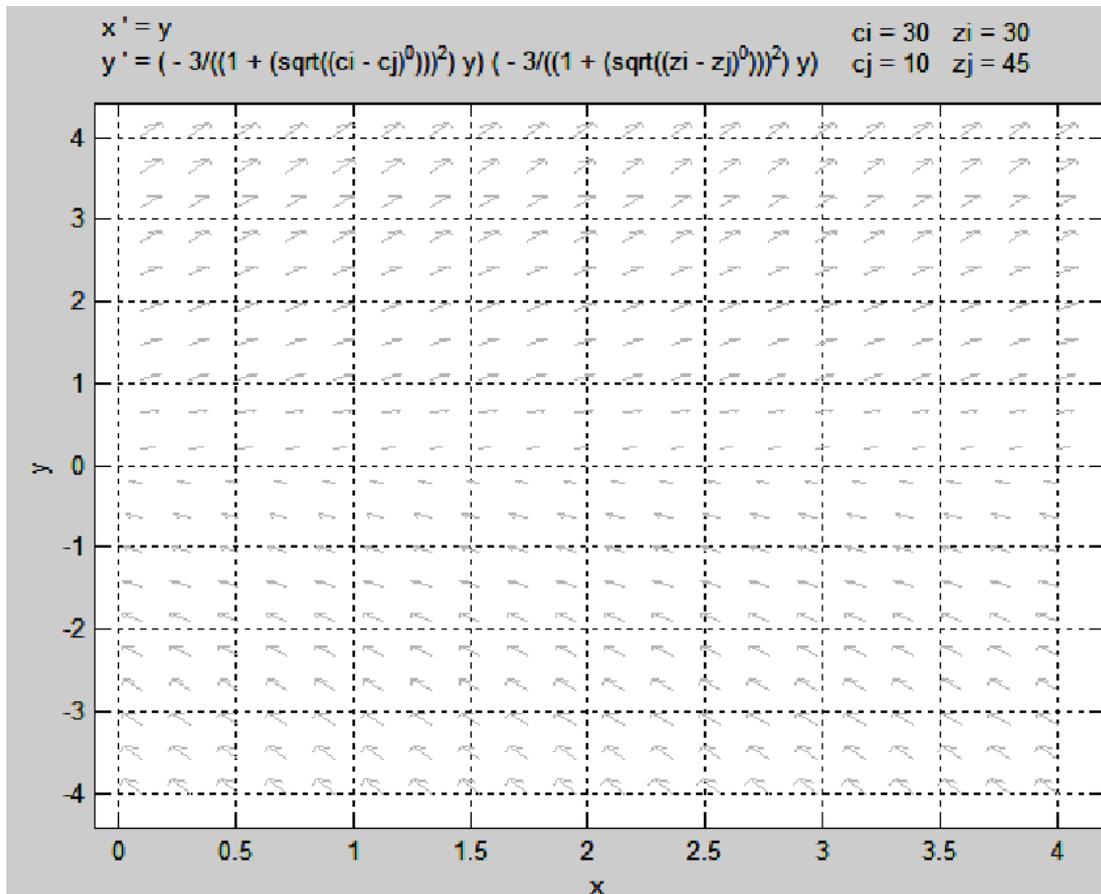
Fig. 4.

Fig. 5.

(Campano and Salvatore 2006, pp. 122-123). More complex convergence situations may look at all the developing areas. One will need a full array of capability sets over longer time periods. As the cluster of regions increase, the directed graph can bring out more realistic relationships such as leader vs. follower strategies, where the head of the arrow points to the leader. In this case, we will have to distinguish between in-degree, $Deg^+(v)$, and out-degree $Deg^-(v)$.

A Brief Background on the Schools Represented in Figure 1.

The Utilitarian Perspective

Early utilitarianisms was concerned with happiness while the modern concerned is with actual preferences, and ethical concerns are with informed preference, while people are really concerned with their well-being. (Harsanyi 1997, p. 129) Harsanyi followed the "informed-preference theory, interpreting a person's utility level as the degree to which both his subjective experiences and his objective conditions satisfy his informed preferences, i.e. his preferences as freed from the distorting effects of factual errors." (ibid., p. 140) Generally a person's utility depends on a commodity, $U_i(x)$, where the Us are person's i utilities derived from chosen alternatives, x. One can denote each individual utilities as $U_1(x) = u_1$. (Harsanyi 1955, p. 313). Then a social welfare function for n individuals can be written as $W(u_1,\cdots,u_n)$. A social welfare function becomes $SWF_1 = W(U_1(x),\cdots,U_n(x))$. For another alternative, y, we get $SWF_2 = W(U_1(y),\cdots,U_n(y))$. On ethical ground, social choice may render SWF_1 preferred to SWF_2.

Harsanyi made the utility function of the individual obey postulates such as those developed by John von Neumann and Oskar Morgenstern (NM). The result was that the SWF was a linear combinations of individual utilities, $W(u_1,\cdots,u_n) = \sum a_i U_i$, where the $a > 0$ for all individuals. (Harsanyi 1955, p. 314). These are straight line utility function.

Tversky and Kahneman Utility Perspective

Prospect theory is posited in the domain of Hedonic psychology, studying the factors that may explain why we are pleasant or unpleasant. (Kahneman *et al.,* 2003, p. ix) Prospect Theory attacks the utilitarian perspective, founded on NM utility axiom. Kahneman and Tversky have shown that the framing of choices can bias the outcome: To quote them, "…people underweigh outcomes that are merely probable in comparison with outcomes that are obtained with certainty. This tendency, called the certainty effect, contributes to risk aversion in choices involving sure gains and to risk seeking in choices involving sure losses. In addition, people generally discard components that are shared by all prospects under consideration. This tendency, called the isolation effect, leads to inconsistent preferences when the same choice is presented in different forms "(Kahneman and Tversky, 1979, p. 263).

Libertarian Perspective

Ludwig Von Mises wrote that: "Action is will put into operation and transformed into an agency, is aiming at ends and goals, is ego's meaningful response to stimuli and to the conditions of its environment; it is a person's conscious adjustment to the state of the universe that determines his life." (Mises 1963, p. 11) One of his faithful students wrote: "The Fundamental praxeological axiom is that individual human being *act*." He goes on to say that "This axiom of action is indisputable and important truth, and must form the basis for social theory." (Rothbard, 1951, p. 943) Hayek claims that the axiom's core feature is "…logically the statements of theories are independent of any particular experience." (Hayek, Vol. IV, 1992, p. 148). While the implication of the action axiom is that everyone is responsible for the consequences of their action, the libertarians are wedded to an ends and means paradigm, and also they speak of a utility scale. On the 'means' side, "…the fact that people act to achieve goals implies that there is a scarcity of means to attain them…Scarcity implies cost, which in a monetary system…are reflected in prices, and so forth." (Rothbard, 1973, p. 315) On the 'ends' side, the actor passes a value judgment each time it ranks and chooses between various ends. "These scales of preference may be called *happiness* or *welfare* or *utility* or *satisfaction* or *contentment*." (Rothbard, 1970, p. 15)

John Rawls Perspective

Rawls 1951 dissertation at Princeton University was after "a reasonable decision procedure" to adjudicate competition. (Rawls 1999, p. 1) He looked for a "just and right" analogous to inductive logics, which would use reasonable principles of conduct. (ibid., p. 2) He did not find the formula, but leave his method up for the judgment of the public. Its assumes "moral insight" which is scaled from normal to brilliant intelligence; a judge with sympathetic knowledge, who "does not provide any reason for his judgment" but judges according to the facts; facts are ranked in the order of M, N, O…etc., so that M would be preferred over N,O, etc., and explications that seek "invariant" judgments of competent judges. In his 1971book, *A Theory of Justice*, Rawls expounded "The Difference Principle" regarding Justice as Fairness and Distributive Justice. The principle uses production possibility and indifference curves, which are standard tools of neoclassical economics. A Leontief (L-shaped) like indifference curve is postulated with vertex on a 45 degrees line between "the most favored representative man" on the X-axes, and "the least advantage man" on the Y-axes. (Rawls 1971, p. 76) The Difference Principle holds that the improvement of one person's situation does not represent an improvement to society unless the other person situation is improve as well. Gains occurs when the L-shaped curves move in the NE direction. Rawls also used a production possibility curve (PPC) to show that the principle of efficiency must be subordinate to a principle of justice to find an efficient distribution of commodities (Ibid, pp. 68-60). A point on the PPC shows that once the distribution to say the "the most favored" person is made, the distribution of the "the least advantaged" person is simultaneously determined.

Rawls tried to answer the difficulty of interpersonal comparisons of the expectation of primary goods. (Rawls 1971, p. 92) Primary goods are defined to be "things that every rational man is presumed to want". It includes social primary

goods such as "rights and liberties, powers and opportunities, income and wealth" and natural primary goods such as "health and vigor, intelligence and imagination". One can, for instance, give up some liberty for social or economic gains, providing only that everyone's position is improved. (Ibid., p. 62) In the preface to the French Edition, 1987, Rawls redefined the concept of primary good to leave out "natural facts of human psychology", resting the definition only on "moral conception of the person". "Primary good are now characterized as what persons need in their status as free and equal citizens, and as normal and fully cooperating members of society over a complete life "(Rawls 1999, p. 415).

Rawls emphasis on social primary goods is in the construction of an index for the least advantage group (Rawls 1971, p. 93). We use an intuitive approach by asking which primary good the representative of the least advantaged would prefer. We do not compare the choice of everyone since each justice is presupposed for everyone. The representative parties have an original position and an expectation about the distribution of primary goods. People are call on to say choose what kind of social institution they most desire. They are to deliberate behind a "veil of ignorance" namely that they do not know. They are ignorant about their socio-economic position, their personal talents, ability, and special interest. Because the representatives do not know the talents and abilities, ethnicity and gender, religion or belief system of the citizens they represent, they are unable to threaten them, and to render invalid the social contract they have with them to coordinate their activities. The original position leads to an egalitarian distribution, liken to the origin of the indifference curve diagram mentioned above. It is a position of uncertainty "by assumption, the participants would be uncertain about what their personal circumstances would be under any particular institutional framework to be agreed upon." (Harsanyi 1975, p. 494)

Contrary to Rawls, John Harsanyi who was simultaneously writing at that time said that it would lead to a utilitarian distribution. (Binmore 2004, p. 17)We may appeal to the game of victim and prey to analyze the original position. One party is preying on the benefit of the other. An agency is required to policing the original position, which for Rawls can be called "natural duty", and Harsanyi, "moral commitment". As Binmore points out, if no police is available, a self-policing mechanism such as convention would favor Rawls outcome of an egalitarian distribution, and not the utilitarian outcome. (Binmore 2004, p. 34)

Marxian Perspectives

Marx analysis is based on dialectical materialism. In terms of poverty, dialectics show up in the use of technology in society. As Ronald Meeks puts it, production relations leads to unearned income, allowing capital accumulation, which brings about technological progress. (Meek 1976, p. 116) In the form of capital deepening, technology displays both constructive and destructive tendencies. On the creative side, it sustains economic growth, but on the destructive side it displaces workers, worsening "… the historically inherited misery of the

masses and blocks the extension of the market, thus destroying stability and even jeopardizing profits." (Lowe 1965, p. 183) Marx based his economic theory on the value of a commodity, which can be transferred into prices. Some economists have built a theory of immiseration of the workers on that model. The theory of immiseration was inferred from statements in the *Communist Manifesto* and in *Capital*, but not explicitly developed. (Rosdolsky 1977, pp. 300-313) One inference is that growth or wealth increases poverty. We find in Joan Robinson work that "…growth in wealth is not at all the same thing as reducing poverty…Growth requires technical progress and technical progress alters the composition of the labor force, making more places for educated workers and fewer for uneducated". (Robinsion 1980, V. 4, p. 101)

One might find it strange that Marx use word such as "mental and physical capabilities" for producing use-values in his definition of labor-power, concepts that are still relevant over 100 years later as described in Figure 2 above. A Worker's "natural wants, such as food, clothing, fuel, and housing, vary according to the climatic and other physical conditions of his country." (Marx 1867, p. 171) The quote continues "…in a given country, at a given period, the average quantity of the means of subsistence necessary for the laborer is practically known. " (Ibid)

In his *Critique of the Gotha Programme*, Marx wrote that after making many deductions from the "proceeds of labor", one must also make "funds for those unable to work, etc., in short, what is included under so-called official poor relief today." (Marx 1875, p. 7) In another place he wrote that to expect equality of wages is an empty wish because "as the cost of producing laboring power of different quality differ, so must differ the value of the labor power in different trades. (Marx1897, WPP, p. 45). Further, because capitalism goes into cycles, "laid-off worker have no reserves, since they can subsist only when they are selling their labor- power, unemployment obviously condemns them to the starkest of poverty". (Mandel 2002, p. 43) In capitalism we find that "society splits up into two classes, capitalists here, wage-laborers there; heredity wealth on one side, heredity poverty on the other". (Engels 1902, p. 6)

Amartya Sen's Perspective

Sen's view argues that commodities and primary commodities requires capabilities and functions to benefit the poor. Functions are concerned with "… ranking different states of affairs from a 'social point of view', in the light of the assessment of the people involved." (Sen 2009, IJ, p. 95) Sen introduced the concept of a "universal domain", which requires that people should respect each other rights (Sen 2009, IJ, p. 111). The capability set bounds (contains) the function set: $Capability(x) \subseteq Function(x)$ as is illustrated in Figure 2. We illustrate this with Sen's famous example of one person fasting as a religious observance, and another fasting because of poverty. (Sen 1999, STD 1987, p. 37) Fasting is the only choice for the poor, but fasting is an alternative choice for the religious observer, while both are within the domain of capability. Again, Sen spoke of a "set of functioning vectors— 'the person's capability set'—is given by the value of the best

element in that set". (Sen 1999, CC, p. 39) Again, he wrote explicitly of "the value of the best element (functioning vector) in the capability set." (Ibid., p. 43) He looked for the maximum value of the vector, and lacking one, the supremum of that set.

Sen has expressed interest in fixed point theory of sets. He was advised by the logician Willard Quine to seek out invariant for equity principles that are preserved under transformation. (Sen 1995, IR, p. x) While Sen decided not to probe such invariants at that time, or subsequently as far as our knowledge of the record goes, it seems that such a task must take on the nature of a fixed point theory application. Sen delineated one-to-one mapping of capability sets to achieve complete ranking of alternatives choices, but he did not elevate the analysis to consider a fixed point in the sense of Brower Fixed Point theorem. Under Brower Theorem, the transformation has to be continuous. Sen has assumed continuous indifference curves, but followed only a limited analysis. He and other Senians allow indifference curves to cross, creating better and worse than sets in order to highlight partial ordering. (Sen 1999, CC, p. 24; Muellbauer 1987, STD, p. 45)). Usually, in one dimension, one has only to invoke the intermediate value theorem of mathematics to reveal a fixed point. In two dimensions, the mapping should not have a retraction if a fixed point is desired.

One senses of measurability is that for $\alpha < \beta$ on the egalitarian axis, we focus on the function on the domain of horizontal axis that falls between α and β. We say that the functions are measurable. Some have presented Sen's model as lying in the interval between utility and goods (commodity). (Muellbauer 1987, p. 40; Atkinson 1999, p. 185) Since the set of measurable function would include the set $[x|\alpha < E(x) < \beta]$, then such intervals excluding the end points are also measurable. At times, Sen seems to focus only on the maximum or supremum of the function. While other are concerned with a set of commodity vector, Sen is concerned with a functioning vector—a person's capability set. The elements of these sets determine freedom of agency and well-being, but the vector elements for Sen may not be a complete ordering. Selecting the best element of such a set, would be what Sen would call an "elementary evaluation". (Sen 1999, CC, p. 39) One may not be able to do that maximum evaluation because of partial ordering. (Ibid) Senian sought welfare criterion that would make ranking more complete. In the end, Sen admits that to make partial order more complete was "no more than a beginning". Obstacles stand in the way of achieving a maximum or supremum of the vector of capability elements. He found that extending the elements of the capability set from partial to complete ranking an "over demanding" task. (Sen 1999, CC, p. 41) Taking two set of functioning elements and using the technique of dominance to make one-to-one binary comparisons of elements was "limited" in the sense that the choice of elements are arbitrary, they also exclude the "quality" of elements. (Ibid., p. 44). What else can one do? Sen considered "incorporating aspects of freedom among the functionings". (ibid., p. 44) Senian have also argued for considering the addition of information on preference (Atkinson 1999, p. 179)

Conclusion

In this paper, the high theories of poverty are viewed from a meta-graphical point of view. The framework helps comparisons and contrasts of theories from a unified point of view. We have single out Sen's view of poverty to illustrate the operational nature of this framework. The analysis revealed that one needs to be concerned not only with inverse mappings that pull together the overlaps of the theories, but also with correspondence of their domain and image elements.

A novelty of the process is that it indicates the direction in which one can make more progress in poverty theory. For instance, correspondence relationship between capability and function of the poor is a clear direction one should take if one want to answer the Quine hypothesis of invariance highlighted in the paper. One can see the meta-graph as a research program to do comparative studies as we have done with Sen's theory. We also demonstrate how the framework can capture dynamic changes among countries.

REFERENCES

Adler, Mortimer, J. 2012. "Liberal Education in an Industrial Democracy, *Philosophy is Everybody's Business*, XIX, No 1, pp. 1-37.

Atkinson, Anthony, B. 1999. "The Contributions of Amartya Sen to Welfare Economics," *Scand. J. of Economics,* Vol. 101, No. 2, pp. 173-190.

Binmore, K. G. 1981. *Topological Ideas*, (Cambridge, U. K.: Cambridge University Press).

Binmore, K. G. 2004. *Natural Justice*, (Oxford, U. K.: Oxford University Press).

Campano, Fred and Dominick Salvatore. 2006. *Income Distribution*, (Oxford, U. K.: Oxford University Press).

Cucker, Felipe and Steve Smale, 2007. "Emergent Behavior in Flocks," *IEEE Transactions on Automatic Control*, Vol. 52, No. 5, pp. 852-862.

Cucker, Felipe and Steve Smale, 2007. "On the Mathematics of Emergence," *Japanese Journal of Mathematics*, Vol. 2, pp. 197-227.

Dasgupta, Partha, 1993. *An Inquiry into Well-Being and Destitution*, (Oxford, U. K.: Clarendon Press).

Diestel, Reinhard and Carsten Thomassen, "A Cantor-Bernstein Theorem for Paths in Graphs," *The American Mathematical Monthly*, Vol. 113, No. 2 (Feb., 2006), pp. 161-165.

Dworkin, R.1981. "What is Equality? Part I: equality of Welfare" and "What is Equality? Part 2: Equality of Resources," *Philosophy and Public Affairs*, Volume 10, pp 185-246, and 283-345.

Engels, F. 1902. *Preface to Wage-Labor and Capital by Karl Marx*, trans., by Harriet M. D., (New York, N. Y.: New York Labor News Company).

Fuchs, Victor.1969. "Comments on Measuring the Low-Income Population," in Lee Soltow, edited, *Six Papers on the Size Distribution of Wealth and Income: Studies in Income and Wealth No. 33*, (New York, N.Y.: NBER).

Harsanyi, John, 1975. "A Theory of Justice: Review by: John C. Harsanyi," *The American Political Science Review,* Vol. 69, No. 2, pp. 594-606.

Harsanyi, John, C. 1955. "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility," *Journal of Political Economy*, Vol. 63, No. 4, pp. 309-321.

Harsanyi, John, C. 1997. "Utilities, Preferences, and Substantive Goods," *Social Choice and Welfare*, Vol. 14, No. 1, pp. 129-145.

Hayek, F. A.1992. *The Austrian School and the Fortunes of Liberalism. The Collected Works of F. A. Hayek, IV,* (Chicago, Ill.: The University of Chicago Press).

Kahneman, D. and A. Tversky, 1979. "Prospect Theory: An Analysis of Decisions under Risk," *Econometrica*, Vol. 47, pp. 313-317.

Kahneman, Daniel, Ed Diener, and Norbert Schwarz, edited. 2003. *Well-Being: The Foundations of Hedonic Psychology*, (New York, N. Y.: Russell Sage Foundation).

Lowe, Adolph.1965. *On Economic Knowledge*, (New York, N. Y.: Harper and Row, Publishers).

Mandel, Ernest, 2002. An *Introduction to Marxist Economic Theory*, (Australia: Resistance Book).

Marx, Karl, 1938 [1875].*Critique of the Gotha Programme*, (NYC, N. Y.: International Publisher).

Marx, Karl, 1967 [1867]. *Capital, Volume I: A Critical Analysis of Capitalist Production*, Edited by Frederick Engels, (NYC, N. Y., International Publishers).

Marx, Karl, 1973 [1897]. *Wages, Price and Profit*, (Peaking (Shangai), China: Foreign Language Press).

Meek, Ronald, 1976. "Karl Marx's Economic Method," in M. C. Howard and L. E. King, edited, *The Economics of Marx*, (New York, N. Y.: Penguin Books, Inc.), pp. 114-128.

Mises, Ludwig Von, 1963. *Human Action*, (Chicago: Contemporary Books, Inc., Third Edition).

Muellbauer, John, 1987. "Professor Sen on The Standard of Living," in Sen, Amartya, John Muellbauer, Ravi Kanbur, Keith Hart, and Bernard Williams, *The Standard of Living,* edited Goeffrey Harthorn, (Cambridge, U. K.: Cambridge University Press), pp. 39-58.

Rawls, John, 1971. *A Theory of Justice*, (Cambridge, Mass.: Cambridge University Press).

Rawls, John. 1999. *Collected Papers*, Edited by Samuel Freeman, (Cambridge, Mass.: Harvard University Press).

Robinson, Joan, 1980. *Collected Economic Papers*, Volume 1-5, (Cambridge, Mass.: The MIT Press).

Rosdolsky, Roman. 1977. *The Making of Marx's Capital*, translated By Pete Burgess, (London, U. K.: Pluto Press).

Rosenholtz, Ira. 2000. "One Point Determines a Line A Geometric Axiom of Choice," *Math Horizons*, Vol. 8, No. 2, pp. 20-24.

Rothbard, Murray N. 1970. *Man, Economy, and State: A Treatise on Economic Principles*, (LA: Nash Publishing) Vol. I.

Rothbard, Murray N. 1973. "Praxeology as the Method of Economics," in Maurice Natanson, Edited. "Phenomenology and the Social Sciences," Vol. 2, (Evanston: Northwestern University Press), pp. 311-339.

Rothbard, Murray, N. 1951. "Praxeology: Reply to Mr. Schuller," *American Economic Review*, vol. 41(5), pp. 943-946.

Schweizer, Berthold, "Cantor, Schroder, and Bernstein in Orbit,"*Mathematics Magazine*, Vol. 73, No. 4 (Oct., 2000), pp. 311-312.

Sen, Amartya K. 1976. "Poverty, an Ordinal Approach to Measurement, *Econometrica*, Volume 44, pp. 219-231.

Sen, Amartya K.1977. "Rational Fools: A Critique of the Behavioral Foundations of Economic Theory," *Philosophy and Public Affairs*, Vol. 6, No. 4, pp. 317-344.

Sen, Amartya, John Muellbauer, Ravi Kanbur, Keith Hart, and Bernard Williams, 1987. *The Standard of Living,* edited Goeffrey Harthorn, (Cambridge, U. K.: Cambridge University Press).

Sen, Amartya. 1981. *Poverty and Famines: An Essay on Entitlement and Deprivation*, (Oxford, U. K.: Clarendon Press).

Sen, Amartya. 1991. *Presidential Address: The Nature of Inequality*, in Kenneth J. Arrow, edited, *Issues in Contemporary Economics: Volume I. Markets and Welfare,* (New York, N. Y.: New York University Press), pp. 3-21.

Sen, Amartya. 1995. *Inequality Reexamined*, (Cambridge Mass.: Harvard University Press).

Sen, Amartya. 1999. *Commodities and Capabilities*, (Oxford, U. K.: Oxford University Press).

Sen, Amartya. 2002. "A Decade of Human Development," *Journal of Human Development*, Vol. 1, No.1,pp. 17-23.

Sen, Amartya. 2002. *Rationality and Freedom*, (Cambridge, Mass.: Harvard University Press).

Sen, Amartya. 2004. The *Argumentative Indian: writing on Indian History, Culture and Identity*, (New York, N. Y.: Picador).

Sen, Amartya. 2009. *The Idea of Justice*, (Cambridge, Mass.: Harvard University Press).

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