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RESEARCH ARTICLE

LEAST COST DESIGN OF A PRISMATIC SIMPLY SUPPORTED SINGLY REINFORCED RECTANGULAR CONCRETE BEAM

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ABSTRACT

In this paper least cost design of a prismatic simple supported singly reinforced rectangular beam with varied spans of 5m, 7m, 9m, 11m and 13m and uniform concentrated load of 100KN based on flexural constraint of the American Concrete Institute (ACI 318/95) is reported. The cost function of the beam was developed based on the unit costs of 1:2:4 concrete mix, formwork, steel and link reinforcements. The unit cost of 1:2:4 concrete mix used was 30,000 per m^3 of concrete, 1,000 per m^2 of formwork, 300 per kg of steel reinforcement. The unit cost per kg of link reinforcement was assumed to be half the unit cost of steel reinforcement. The design variables were the depth and steel reinforcement ratio. The width of the beam was kept constant as 0.225m. The deign variables were appended to the Lagrangian multiplier which was used an the optimization process and a FORTRAN program was used to facilitate the needed optimal solutions. It was shown among other findings that the depth of the beam decreased as the steel ratio increased but the total cost per unit length reduced showing that it is cheaper to increase the area of steel reinforcement and reduce the depth of beam irrespective of its length.

INTRODUCTION

Optimization techniques play an important role in structural design. One of the design philosophies is that the structure must be economical both in terms of construction and maintenance cost[1]. This can be achieved through the process of optimization. The optimal design of beams was first proposed by Galileo[2] although his calculations were wrong. Apparently, the doctoral dissertation by E.J. Haug Jr. [3] (see also [4]) in 1966 was one of the first modern attempts to use a digital computer as a tool for the optimal design of this structural element. Haug reduced the non-linear optimal design problem to Lagrange problem in the Calculus of Variations with inequality constraints. His model considered a beam made of a linearly elastic material of known density with two supports and a certain given load. The control variables were the values of cross sections at different points along the beam, and constraints on the stress, shear and deflection were imposed. Haug used an iterative method based on the generalized Newton's algorithm to solve statically determinate beams. Venkayya[5] developed a method based on an energy criterion and a search procedure based on constraint gradient values for the design of structures subjected to static loading. His method can handle very efficiently: (a) design for multiple loading conditions, (b) stress constraints, (c) displacement constraints and (d) limits on sizes of the elements. This method also has been successfully applied to the design of trusses, frames and beams. In these cases, the weight of the structural element is the parameter to be minimized. Karihaloo [6] presented a model to minimize the maximum deflection of a simply supported beam under a transverse concentrated load. Haug and Arora [7] used the gradient projection method to optimize the design of simply supported and clamped beams with constraints on stress, deflection, natural frequency and bounds on the design variables. Again, the weight (volume) of the beam is the parameter to be minimized. Saouma et al.[8] developed a method for minimum cost design of simply supported, uniformly loaded, partially prestressed concrete beams. This model uses nine design variables: six geometrical dimensions, area of prestressing steel and area of tensile and compressive mild reinforcement. The imposed constraints are on the four flexural stresses, initial camber, dead and live load deflections, ultimate shear and ultimate moment capacity with respect to both the cracking moment and the applied load. This model was solved using the Penalty- Functions method coupled with Quasi-Newton unconstrained optimization techniques. Das Gupta et al.[9] applied generalized geometric programming to the optimal design of a modular floor system, which consisted of reinforced solid concrete and voided slab units supported on steel beams. One of the most remarkable characteristics of this model is that it defines a function representing the cost of

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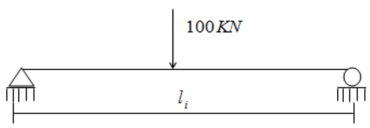
the floor system in terms of design variables, length, width and thickness of components, and other engineering cost parameters. This function is minimized subject to various constraints depending on stresses and deflections, and a dual-based algorithm was used to solve it.

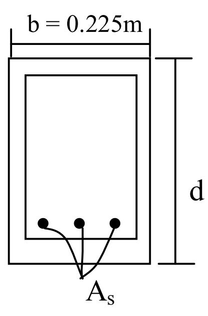
Some other authors have used multiobjective optimization techniques to deal with this problem. For example, Rao[10] studied a cantilever beam with a hollow rectangular cross section and tip mass, for which he minimized its structural mass and its fatigue damage, while he maximized its natural frequency. After using many different approaches (i.e., global criterion, game theory, goal attainment, utility function, etc.), he concluded that game theory gave the best results. Also, Osyczka [11] used the min-max method for the optimal design of an I-beam. More recently, Lounis and Cohn [12] considered the design of a posttensioned floor slab and a pretensioned highway bridge system for two conflicting objectives: minimum cost and minimum initial camber. They used the $\varepsilon -$constraint method to transform this multiobjective optimization problem into a single nonlinear optimization problem that they solved using the projected Lagrangian method. The objective of this paper is to formulate a model for least cost design of a singly reinforced concrete rectangular beam. The design variables were the depth and steel ratio. The model was solved using the method of Lagrangian multiplier.

Least cost design model derivation

Consider an ith simply supported reinforced concrete rectangular beam shown below in figure 1.



Where l_i = 5m, 7m, 9m, 11m and 13m. The cross- section of all the beams including reinforcements is uniform since the beam is prismatic.



The objective function is the algebraic sum of all the costs of the elements of the beam given as

$$L = C_c\ A_c + C_f\,(b+2d) + C_c\ A_s + \tfrac{1}{2} C_s\ A_s \qquad [1]$$

Where

A_c = Area of concrete

C_c = Cost coefficient of concrete

A_s = Area of steel reinforcement

C_s = Cost coefficient of steel

A_f = Area of formwork

C_f = Cost coefficient of formwork

b = breadth of beam

d = depth of beam

$$A_c = \text{bd} \qquad [2]$$

$$A_s = \rho bd \qquad [3]$$

Substitution of equations [2] and [3] in equation [1] transforms equation [1] to

$$L = C_c\ bd + C_f\,(b+2d) + C_c \rho bd + \tfrac{1}{2} C_s\ \rho bd \qquad [4]$$

Where

ρ = steel ratio

The moment of resistance of the AC1 – 318/95 is used as the design constraint. This represents the ultimate moment of resistance of all the beams cross-section given as

$$MR = \phi A_s\ f_y \left(d - \frac{a}{2} \right) \qquad [5]$$

where ϕ = strength reduction factor = 0.90 for flexure.

Substituting for A_s transforms equation [5] to

$$M_R = 0.90\, \rho bdf_y \left(d - \frac{a}{2} \right) \qquad [6]$$

The steel ratio, ρ is restricted to

$$\rho_{\min} \leq \rho_b < \rho_{\max} \qquad [7]$$

Where

$$\rho_{\min} = \frac{200}{f_y}, (f_y\ \ in\ \ psi) \qquad [8]$$

and

$$\rho_{\max} = 0.75\, \rho_b \qquad [9]$$

Steel ratio that produces the balance strain condition is given as

$$\rho_b = \frac{0.85 \beta f_c\ x}{f_y}\ \frac{87000}{87000 + f_y}, \quad (f_y\ \ in\ \ psi) \qquad [10]$$

Where

$\beta = 0.85$ for $f^1{}_c \leq 4000 psi$

a = depth of equivalent rectangular stress block.

From equation [8],

$$\rho_{\max} = \frac{200}{59450} = 0.00336$$

From equation [10],

$$\rho_b = \frac{0.85\,x\,0.85\,x\,3625}{59450} \left(\frac{87000}{87000 + 59450} \right)$$

$$= 0.0262$$

From equation [9],

$$\rho_{\max} = 0.75\,x\,0.0262 = 0.0196$$

From equation [7], steel ratio is bounded on the interval of

$$0.00336 \leq \rho_b \leq 0.0196 \qquad [11]$$

Taking ρ at an interval of 0.005, the values of ρ used in the least cost design are
0.0034, 0.0084, 0.0134, 0.0184, 0.0196.
From equation (6),

$$a=\frac{A_s f_y}{0.85f^1{}_c b}=\frac{\rho bdf_y}{0.85f^1{}_c b}=\frac{\rho df_y}{0.85f^1{}_c} \quad [12]$$

Substituting for a in equation [6] gives
Where

$f_y,\ f^1{}_c =$ Characteristic strength of steel reinforcement and concrete respectively.

$b, d =$ Width and depth of beam respectively.

$$M_R=0.90\rho bdf_y\left(d-\frac{\rho df_y}{0.85f_c}\right) \quad [13]$$

$$M_R=0.90\rho bd^2 f_y-\frac{1.06\rho^2 fy^2}{f_c} \quad [14]$$

The objective function is to optimize the cost of reinforced concrete beam subject to the constraint on the moment given by

$$0.90\rho bd^2 f_y-\frac{1.06\rho^2 bdfy^2}{f_c}-M_u=0 \quad [15]$$

Where

$M_u =$ Applied bending moment due to 100KN load at centre of span of the individual beams.

The unconstrained cost optimization problem is given as a lagrange function as

$$F(\rho,d,\lambda)=C_c bd+c_f(b+2d)+C_s\rho bd+\frac{1}{2}C_s\rho bd$$
$$+\lambda\left(0.9\rho bd^2 f_y-\frac{1.06\rho^2 bd^2 f_y{}^2}{f_c}-M_u\right) \quad [16]$$

The necessary conditions for minimum of $F(\rho,d,\lambda)$ are

$$\frac{\partial F}{\partial\lambda}=0.9f_y\rho bd^2-\frac{1.06\rho^2 bd^2 f_y{}^2}{f_c}-M_u=0 \quad [17]$$

$$\frac{\partial F}{\partial\rho}=C_s bd+\frac{1}{2}C_s bd+0.9f_y\rho bd^2\lambda-\frac{2.12\rho bd^2 f_y{}^2\lambda}{f_y}=0 \quad [18]$$

$$\frac{\partial F}{\partial d}=C_c b+2C_f b+C_s\rho b+\frac{1}{2}C_s\rho b+1.8f_y\rho bd-2.12\rho bd^2 f_y{}^2=0 \quad [19]$$

Equations [17] [18] and [19] are called the stationary points.

From structural analysis, the maximum moment M_u for each loaded span is

$$\frac{PL_i}{4} \quad [20]$$

Where

$L_i = 5m, 7m, 9m, 11m \text{ and } 13m$

A fortran program was used to obtain the optimal results. From the results of the executed program, it can be seen that the process of arriving at the least cost design of a simply supported rectangular reinforced concrete beam is an iterative process which terminates once the least cost is arrived at, but this was done within the limit of steel ratio. It is observed that at different lengths, the depth of the beam decreases as the steel ratio increases, but the total cost per unit length reduces. Conclusively, for any length of beam, the least cost can be gotten by increasing the area of steel reinforcement and reducing the depth of the beam as the depth of the beam decreases with increase in steel ratio.

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FRED.FOR

```
C
C NAME: OSEMOBOR WILFRED
C DEPT: CIVIL AND ENVIRONMENTAL ENGINEERING
C MAT. NO: U2005/3010267
C COURSE COMPUTER PROGRAMMING FOR COST OPTIMIZATION OF
C                    A SIMPLY SUPPORTED BEAM
C
C DATE: 30/10/2010
C
      REAL L
      N=0
   66 WRITE(6,*)'PLEASE INPUT A VALUE FOR L'
      READ(5,*)L
      LEAP=0
   31 LEAP=LEAP+1
      B=0.225
      CC=30000
      CF=1000
      CS=300
      FC=25000
      FY=410000
      P=100
      MU=P*L**2/4
      GOTO(10,20,30,40,50)LEAP
C
C FIRST ITERATION
C
   10 D=SQRT=MU/(0.9*FY*0.0034*B-(1.06*0.0034**2*B*FY**2/FC))
C
      Z1=CC*B*D
      Z2=CF*(B+2*D)
      Z3=CS*0.0034**2*B*D
      Z4=1/2*CS*0.0034*B*D
      Z5=N*(0.9*FY*0.0034*B*D**2)
      Z6=1.06*0.0034**2*FY**2/FC
      F1=Z1+Z2+Z3+Z4+Z5-Z6-MU
C
      WRITE(6,101)L, F1
  101 FORMAT(//2X,'WHEN L=',F5.1,' AND RHO= 0.0034, F=',F12.4/)
      IF(LEAP .LT. 5)GOTO 31
C
C SECOND ITERATION
C
   20 D=SQRT=MU/(0.9*FY*0.0084*B-(1.06*0.0084**2*B*FY**2/FC))
C
      Z1=CC*B*D
      Z2=CF*(B+2*D)
      Z3=CS*0.0084**2*B*D
      Z4=1/2*CS*00.0084*B*D
      Z5=N*(0.9*FY*0.0084*B*D**2)
      Z6=1.06*0.0084**2*FY**2/FC
      F2=Z1+Z2+Z3+Z4+Z5-Z6-MU
C
      WRITE(6,102)L, F2
  102 FORMAT(//2X,'WHEN L=',F5.1,' AND RHO= 0.0084, F=',F12.4/)
      IF(LEAP .LT. 5)GOTO 31
C
C THIRD ITERATION
C
   30 D=SQRT=MU/(0.9*FY*0.0134*B-(1.06*0.0134**2*B*FY**2/FC))
C
      Z1=CC*B*D
      Z2=CF*(B+2*D)
      Z3=CS*0.0134**2*B*D
```

Page 1

FRED.FOR

```
      Z4=1/2*CS*0.0134*B*D
      Z5=N*(0.9*FY*0.0134*B*D**2)
      Z6=1.06*0.0134**2*FY**2/FC
      F3=Z1+Z2+Z3+Z4+Z5-Z6-MU
C
      WRITE(6,103)L, F3
  103 FORMAT(//2X,'WHEN L=',F5.1,' AND RHO= 0.0134, F=',F12.4/)
      IF(LEAP .LT. 5)GOTO 31
C
C FOURTH ITERATION
C
   40 D=SQRT=MU/(0.9*FY*0.0184*B-(1.06*0.0184**2*B*FY**2/FC))
C
      Z1=CC*B*D
      Z2=CF*(B+2*D)
      Z3=CS*0.0184**2*B*D
      Z4=1/2*CS*0.0184*B*D
      Z5=N*(0.9*FY*0.0184*B*D**2)
      Z6=1.06*0.0184**2*FY**2/FC
      F4=Z1+Z2+Z3+Z4+Z5-Z6-MU
C
      WRITE(6,104)L, F4
  104 FORMAT(//2X,'WHEN L=',F5.1,' AND RHO= 0.0184, F=',F12.4/)
      IF(LEAP .LT. 5)GOTO 31
C
C FIFTH ITERATION
C
   50 D=SQRT=MU/(0.9*FY*0.0196*B-(1.06*0.0196**2*B*FY**2/FC))
C
      Z1=CC*B*D
      Z2=CF*(B+2*D)
      Z3=CS*0.0196**2*B*D
      Z4=1/2*CS*0.0196*B*D
      Z5=N*(0.9*FY*0.0196*B*D**2)
      Z6=1.06*0.0196**2*FY**2/FC
      F5=Z1+Z2+Z3+Z4+Z5-Z6-MU
C
      WRITE(6,104)L, F5
  105 FORMAT(//2X,'WHEN L=',F5.1,' AND RHO= 0.0196, F=',F12.4/)
      IF(LEAP .LT. 5)GOTO 31
C
      WRITE(6,*)'IN SUMMARY'
      WRITE(6,111)F1,F2,F3,F4,F5
  111 FORMAT (/2X,5F12.2//)
      N=N+1
      IF(N .LT. 5)GOTO 66
C
      WRITE(6,*)'FIVE VALUES OF L PROCESSED... THANK YOU, RUN AGAIN!'
      STOP
      END
```

FRED PROGRAM.txt

```
RUN

FIRST ITERATION
WHEN L= 5.0 AND RHO= 0.0034, F= 20252.4800

WHEN L= 5.0 AND RHO= 0.0084, F= 8457.3080

WHEN L= 5.0 AND RHO= 0.0134, F= 4952.3760

WHEN L= 5.0 AND RHO= 0.0184, F= 2740.5570

WHWN L= 5.0 AND RHO= 0.0196, F= 2269.9960

IN SUMMARY

   20252.48    8457.31    4952.38    2740.56    2270.00


SECOND ITERATION
WHEN L= 7.0 AND RHO= 0.0034, F= 51737.0900

WHEN L= 7.0 AND RHO= 0.0084, F= 22974.8900

WHEN L= 7.0 AND RHO= 0.0134, F= 15630.0500

WHEN L= 7.0 AND RHO= 0.0184, F= 12200.3200

WHWN L= 7.0 AND RHO= 0.0196, F= 11637.8500

IN SUMMARY

   51737.09   22974.89   15630.05   12200.32   11637.85


THIRD ITERATION
WHEN L= 9.0 AND RHO= 0.0034, F= 115219.9000

WHEN L= 9.0 AND RHO= 0.0084, F= 53157.8200

WHEN L= 9.0 AND RHO= 0.0134, F= 38537.3600

WHEN L= 9.0 AND RHO= 0.0184, F= 33161.5000

WHWN L= 9.0 AND RHO= 0.0196, F= 32560.9300

IN SUMMARY

  115219.90   53157.82   38537.36   33161.50   32560.93


FOURTH ITERATION
WHEN L= 11.0 AND RHO= 0.0034, F= 246008.1000

WHEN L= 11.0 AND RHO= 0.0084, F= 116782.1000

WHEN L= 11.0 AND RHO= 0.0134, F= 87910.6600

WHEN L= 11.0 AND RHO= 0.0184, F= 79331.40000

WHWN L= 11.0 AND RHO= 0.0196, F= 78885.1500
IN SUMMARY

  246008.10  116782.10   87910.66   79331.40   78885.15
```

FRED PROGRAM.txt

```
FIFTH ITERATION
WHEN L= 13.0 AND RHO= 0.0034, F= 136085.4000

WHEN L= 13.0 AND RHO= 0.0084, F= 58772.1800

WHEN L= 13.0 AND RHO= 0.0134, F= 39553.7300

WHEN L= 13.0 AND RHO= 0.0184, F= 31129.4300

WHWN L= 13.0 AND RHO= 0.0196, F= 29820.5000
IN SUMMARY

  136085.40   58772.18   39553.73   31129.43   29820.50


FIVE VALUES OF L PROCESSED... THANK YOU, RUN AGAIN!
```

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