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RESEARCH ARTICLE

APPLICATION OF MARKOV CHAIN TO MODEL AND FORECAST STOCK MARKET TREND:
A STUDY OF SAFARICOM SHARES IN NAIROBI SECURITIES EXCHANGE, KENYA

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ABSTRACT

Wealth creation is the goal for every investor. The stock market is one attractive area for investment. Nairobi Securities Exchange being an emerging market in the region, it is considered that both foreign and local investors will seize the opportunity and invest in the stock market. However, this has not been the case for many potential investors due to inability to make informed investment decisions based on future expectations of the stock market. An understanding of the stock market trend in terms of predicting price movements is important for investment decisions. Markov Chain model has been widely applied in predicting stock market trend. In many applications, it has been applied in predicting stock index for a group of stock but little has been done for a single stock. Moreover, the model has had limited application in emerging stock markets. The overall objective of this study therefore, was to apply Markov Chain to model and forecast trend of Safaricom shares trading in Nairobi Securities Exchange, Kenya. The study was conducted through a longitudinal case study design. Secondary quantitative data on the daily closing share prices of Safaricom was obtained from NSE website over a period covering 1st April 2008 to 30th April 2012 forming a 784 days trading data panel. A markov chain model was determined based on probability transition matrix and initial state vector. In the long run, irrespective of the current state of share price, the model predicted that the Safaricom share prices would depreciate, maintain value or appreciate with a probability of 0.3, 0.1 and 0.5 respectively.

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INTRODUCTION

A stock market is an established legal framework surrounding the trading of shares of many companies or organizations. When a stock market is on the rise, it is considered to be an up and coming economy. It is often considered as the primary indicator of a country's economic strength and development. A rise in share index and share prices are usually associated with increased business investment and vice versa, and this also affects the wealth of households and their consumption (Obodos, 2007). Typically, stock market indices and prices are the performance indicators for the entire market. They act as barometer, which enable us to get an idea about the performance of the entire market in general. Interested investors all around the globe keep track of the movement of various stock market indices and prices to get an idea about how the global markets are moving.

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With globalization of the capital markets, we see that investors are not restricting their investments to a single market. Investments are made in different markets, enabling investors to enjoy the benefit of diversification of markets (Vasanthi et al., 2011). Investors base their investments and make a choice of their investment destinations based on the movements of the stock markets. Investors worldwide try to watch the movements of market indices and have always shown keen interest in trying to predict the share market trend. Stock market forecasts focus on developing a successful approach for predicting index values or stock prices. The ultimate aim is to earn high profit using well defined trading strategies. However, forecasting stock indices and stock prices is very difficult because of the market volatility that needs accurate forecast model. The stock market indices and prices are highly fluctuating, that is rising and falling randomly. Such fluctuations affect the investor's belief. Therefore determining more effective ways of stock market index and stock price prediction is important for stock market investor in order to make more informed and accurate investment decisions

(Preethi and Santhi, 2005). Several different approaches have been used to predict the trend of stock markets. These approaches range from neural network, Data mining, moving averages, Regression analysis, Autoregressive Integrated Moving Average (ARIMA) models to Markov Chain analysis. Studies indicate that the operational status of the stock market is subject to the influence of various factors from the market such as multiple market forces from both sides, the fundamentals state of the stock itself, macroeconomic policy, trade and economic degrees and psychological factors of investors. Therefore no single method can accurately predict changes in the stock market every day. Markov Chain analysis which this study aims at exploring, is a prediction method based on probability forecasting approach that has no after-effect and may be effectively used to analyze and predict the stock market index and closing stock price under the above market mechanisms (Deju Zhang and Xiaomin Zhang, 2009).

A number of studies have been conducted to predict the stock market trend of financial markets. Majority of these studies have tried to forecast financial time series of developed markets like the US, China and UK, not many empirical studies have been done to forecast stock market trend of emerging markets such as Nairobi Securities Exchange market in Kenya. There seems to be a lot of potential in the emerging markets and both local and overseas investors are looking forward to investing heavily in the stock markets. This is evidenced in the oversubscribed safaricom shares that were floated to investors in an Initial Public Offer (IPO) at a share price of Ksh.5 in March 2008 and started trading in Nairobi Securities Exchange market subsequent month. However, a few months later the Safaricom share price came down drastically to as low as Ksh.2 that saw the many investors pull out, Safaricom Kenya Ltd. being the most profitable company in East Africa notwithstanding. Given this unique trend for Safaricom, unlike the other 65 listed companies trading in NSE, this study therefore sought to predict the stock market trend of Safaricom shares trading in NSE in Kenya as an emerging market, using Markov chain analysis. The specific objectives were: (1) to study the trend of safaricom share prices in Nairobi Securities exchange; (2) to determine the Markov model for forecasting safaricom share price in Nairobi Securities exchange and (3) to forecast safaricom share price trend in Nairobi Securities Exchange.

Basic Concepts

This section presents the concepts and theories underpinning the study. This study was anchored on the theory of Markov Chains. A Markov chain is a special type of stochastic process. The concepts of stochastic process, Markov Chain, Transition matrix, Properties of Markov Chains and Classification of States are presented in sub sections as outlined below.

Stochastic Process

A stochastic process is a random process, that is, a change in the state of some system over time whose course depends on chance and for which the probability of a particular course is defined. It is basically a family of random variables, $X(t): t \in T$ defined on a given probability space, indexed by the time variable t , where t varies over an index set T . (Ryan, T,1973).

A stochastic process may be continuous or discrete. A stochastic process is said to be a discrete time process if set T is finite or countable. That is, if $T = \{0,1,2,3,4,\dots, n\}$ resulting in the time process $X(0), X(1), X(2), X(3), X(4), \dots, X(n)$, recorded at time $0,1,2,3,4,\dots, n$ respectively. On the other hand stochastic processes $X(t): t \in T$ is considered a continuous time process if T is not finite or countable. That is, if $T = [0, \infty)$ or $T = [0, k]$ for some value k .

A state space S is the set of states that a stochastic process can be in. The states can be finite or countable hence the state space S is discrete, that is $S = \{1, 2, 3, \dots, N\}$. Otherwise the space S is continuous (Doubleday, Kevin and Julius Esunge, 2013). A stochastic process is said to be a stationary process (or strictly or strongly stationary process) if its joint probability distribution does not change when shifted in time. Consequently, parameters such as the mean and variance, if they are present, also do not change over time and do not follow any trends. If $\{X_t\}$ is a stochastic process and let $F_X(x_{t_1+\tau}, \dots, x_{t_k+\tau})$ represent the cumulative distribution function of the joint distribution of $\{X_t\}$ at times $t_1 + \tau, \dots, t_k + \tau$. Then, $\{X_t\}$ is said to be stationary if, for all t , for all T , and for all t_1, \dots, t_k ,

$$F_X(x_{t_1+\tau}, \dots, x_{t_k+\tau}) = F_X(x_{t_1}, \dots, x_{t_k}).$$

Since T does not affect $F_X(\cdot)$, F_X is not a function of time. (Honarkhah, M.; Caers, J. 2010)

The opening and closing prices and indices of Stock trading in Nairobi Securities exchange varies or fluctuates in a random manner due to influence of various factors from the market such as multiple market forces from both sides, the fundamentals state of the stock itself, macroeconomic policy, trade and economic degrees to psychological factors of investors.

Markov Chains

Markov chains are a type of Stochastic Process with special property that probabilities involving how the process will evolve in the future depend only on the present state of the process, and so are independent of events in the past. A stochastic process $\{X_t\}$ is said to have the Markovian property if $P\{X_{t+1} = j | X_0 = k_0, X_1 = k_1, \dots, X_{t-1} = k_{t-1}, X_t = i\} = P\{X_{t+1} = j | X_t = i\}$, for $t = 0,1,\dots$ and every sequence $i, j, k_0, k_1, \dots, k_{t-1}$.

In other words this Markovian property says that the conditional probability of any future “event”, given any past “event” and the present state $X_t = i$, is independent of the past “event” and depends only upon the present state. The conditional probabilities $P(X_{t+1} = j | X_t = i) = p_{ij}$ are called transition probabilities. (Doubleday et al., 2013).

Transition Probability Matrix

The conditional probabilities $P(X_{t+1} = j | X_t = i) = p_{ij}$ are called transition probabilities and can be arranged in the form of a $n \times n$ matrix known as the Transition Probability Matrix. It is given

by: $P = \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \dots & p_{nn} \end{bmatrix}$ which can be denoted as $P = P_{ij}$.

The matrix has the following properties,

- $P_{ij} > 0$ for all i and j .
- For all i and j , (i.e) sum of the element in each row is equal to 1. This is true because the sum represents total probability of transition from state i to itself or any other state.
- The diagonal element represents transition from one state to same state.

Markov Chain models are useful in studying the evolution of systems over repeated trials. The repeated trials are often successive time periods where the state of the system in any particular period cannot be determined with certainty. Rather, transition probabilities are used to describe the manner in which the system makes transitions from one period to the next. It helps us to determine the probability of the system being in a particular state at a given period of time. (Vasanthi Subha and Nambi, 2011)

Properties of Markov Chains and Classification of States

Reducibility

A state j is said to be accessible from a state i (written $i \rightarrow j$) if a system started in state i has a non-zero probability of transitioning into state j at some point. Formally, state j is accessible from state i if there exists an integer $n_{ij} \geq 0$ such that

$$\Pr(X_n = j | X_0 = i) = p_{ij}^{(n_{ij})} > 0.$$

This integer is allowed to be different for each pair of states, hence the subscripts in n_{ij} . Allowing n to be zero means that every state is defined to be accessible from itself. A state i is said to communicate with state j (written $i \leftrightarrow j$) if both $i \rightarrow j$ and $j \rightarrow i$. A set of states C is a communicating class if every pair of states in C communicates with each other, and no state in C communicates with any state not in C . It can be shown that communication in this sense is an equivalence relation and thus that communicating classes are the equivalence classes of this relation. A communicating class is closed if the probability of leaving the class is zero, namely that if i is in C but j is not, then j is not accessible from i .

A state i is said to be essential or final if for all j such that $i \rightarrow j$ it is also true that $j \rightarrow i$. A state i is inessential if it is not essential (Asher Levin, David, 2009).

A Markov chain is said to be irreducible if its state space is a single communicating class; in other words, if it is possible to get to any state from any state.

Periodicity

A state i has period k if any return to state i must occur in multiples of k time steps. Formally, the period of a state is defined as

$$k = \gcd\{n : \Pr(X_n = i | X_0 = i) > 0\}$$

(where "gcd" is the greatest common divisor). Note that even though a state has period k , it may not be possible to reach the state in k steps. For example, suppose it is possible to return to the state in $\{6, 8, 10, 12, \dots\}$ time steps; k would be 2, even though 2 does not appear in this list.

If $k = 1$, then the state is said to be aperiodic: returns to state i can occur at irregular times. In other words, a state i is aperiodic if there exists n such that for all $n' \geq n$,

$$\Pr(X_{n'} = i | X_0 = i) > 0.$$

Otherwise ($k > 1$), the state is said to be periodic with period k . A Markov chain is aperiodic if every state is aperiodic. An irreducible Markov chain only needs one aperiodic state to imply all states are aperiodic.

Recurrence

A state i is said to be transient if, given that we start in state i , there is a non-zero probability that we will never return to i . Formally, let the random variable T_i be the first return time to state i (the "hitting time"):

$$T_i = \inf\{n \geq 1 : X_n = i | X_0 = i\}.$$

The number

$$f_{ii}^{(n)} = \Pr(T_i = n)$$

is the probability that we return to state i for the first time after n steps. Therefore, state i is transient if:

$$\Pr(T_i < \infty) = \sum_{n=1}^{\infty} f_{ii}^{(n)} < 1.$$

State i is recurrent (or persistent) if it is not transient. Recurrent states are guaranteed to have a finite hitting time.

Ergodicity

A state i is said to be ergodic if it is aperiodic and positive recurrent. In other words, a state i is ergodic if it is recurrent, has a period of 1 and it has finite mean recurrence time. If all states in an irreducible Markov chain are ergodic, then the chain is said to be ergodic. It can be shown that a finite state irreducible Markov chain is ergodic if it has an aperiodic state. A model has the ergodic property if there's a finite number N such that any state can be reached from any other state in exactly N steps. In case of a fully connected transition matrix where all transitions have a non-zero probability, this condition is fulfilled with $N=1$. That is i.e. a Markov chain is ergodic if there exists some finite k such that;

$$\Pr\{X(t+k) = j | X(t) = i\} > 0 \text{ for all } i \text{ and } j \text{ (Tommi Jaakkola, 2006).}$$

A model with more than one state and just one out-going transition per state cannot be ergodic (Meyn and Tweedie, 2005).

Absorbing states

A state i is called absorbing if it is impossible to leave this state. Therefore, the state i is absorbing if and only if

$$P_{ii} = 1 \text{ and } P_{ij} = 0 \text{ for } i \neq j.$$

If every state can reach an absorbing state, then the Markov chain is an absorbing Markov chain. In an absorbing Markov chain, a state that is not absorbing is called transient. There can be continuous-time absorbing Markov chains with an infinite state space, or discrete-time absorbing Markov chains with a finite discrete-state-space.

Canonical form of the transition matrix

Based on the classifications of the states, the transition matrix P can be partitioned into its canonical form. Let an absorbing Markov chain with transition matrix P have t transient states and r absorbing states. Then

$$P = \begin{pmatrix} Q & R \\ \mathbf{0} & I_r \end{pmatrix},$$

Where, Q is a t -by- t matrix,

R is a nonzero t -by- r matrix,

$\mathbf{0}$ is an r -by- t zero matrix, and

I_r is the r -by- r identity matrix.

Thus, Q describes the probability of transitioning from some transient state to another while R describes the probability of transitioning from some transient state to some absorbing state.

Fundamental matrix

A basic property about an absorbing Markov chain is the expected number of visits to a transient state j starting from a transient state i (before being absorbed). The probability of transitioning from i to j in exactly k steps is the (i,j) -entry of Q^k . Summing this for all k (from 0 to ∞) yields the desired matrix, called the fundamental matrix and denoted by N . It is easy to prove that

$$N = \sum_{k=0}^{\infty} Q^k = (I_t - Q)^{-1},$$

Where, I_t is the t -by- t identity matrix. The (i,j) entry of matrix N is the expected number of times the chain is in state j , given that the chain started in state i .

Literature Review

Many approaches have been employed over the years in forecasting stock market trends. These approaches range from artificial neural network (Halbert white; Jing Tao Yao and chew Lim tan; 2009), Data mining technology(Senthamarai Kannan, P. Sailapathi Sekar, M. Mohamed Sathik and P. Arumugam,2010), Moving Averages (Abdulsalam et al., 2010), Regression analysis, Autoregressive Integrated Moving Average Models, (Hsieh Don-Lin Yang and Jungpin Wu, 2003), moving average autoregressive exogenous (Kuang Yu et al., 2008) to Markov Chain analysis.

Deju Zhang et al.(2009), in their study on forecasting the stock market trend based on stochastic analysis method in Chinese Stock market found that the Markov chain has no after effect and that it could predict the stock market index and closing price more effectively under the market mechanisms. However, the study found that the Markov chain prediction method is only a probability forecasting method giving results expressed as probability of certain state of stock prices in future rather than being in absolute state. They concluded that investors can combine the results of forecasts from using Markov chain to predict with other factors and see it as a basis for decision making. The study suggested that future research could be done on forecasting individual stock closing prices especially for shares of robust companies.

Vasanthi S, Subha V. and Nambi T(2011) did an empirical study on stock index trend prediction using Markov chain analysis. The study used first order Markov chain model to predict the daily trend of various global stock indices and compared the results with that of traditional forecasting methods. The results showed that the Markov model outperformed the traditional models used in the study. The markov model exhibited high levels of accuracy with the one year data used to predict the trend. The study concluded that the model could help researchers in identifying the future trends in the stock markets, and serve as a useful indicator for investors to make better investment decisions. In terms of further studies, the study suggested that future research could be conducted using higher order Markov chains to gain better insight into the behavior of the market.

Rafiu Hassan and Baikunthu Nath (2005) used Hidden Markov Models (HMM) approach to forecasting stock price for interrelated markets. HMM was used for pattern recognition and classification problems because of its proven suitability for modeling dynamic system. The author summarized the advantage of the HMM was strong statistical foundation. It's able to handle new data robustly and computationally efficient to develop and evaluate similar patterns. The author decides to develop hybrid system using AI paradigms with HMM improve the accuracy and efficiency of forecast the stock market.

Yi-Fan Wang et al. (2010) used Markov chain concepts into fuzzy stochastic prediction of stock indexes to achieve better accuracy and confidence. They examined the comparison of ANN and Markov model and concluded that the later has major advantages. It generates high accurate result and requires only one input of data. The first hour's stock index data was used as the input and it lead the prediction of the probable index at any given hour. This approach did not require the standard deviation of the prediction. The approach provided not only improved profit performance but also used to determine stock-losses with greater confidence. Niederhoffer and Osborne (1966) used Markov Chains to show some non-random behaviour in transaction to transaction stock prices resulting from investors tendency to place buy and sell orders at integers (23), halves (23 1/2) and quarters in descending preferences.

Dryden (1969) applied Markov Chains to United Kingdom stocks which, at the time, were quoted as rising, falling, or remaining unchanged. Fielitz (1969), Fielitz and Bhargava (1973) and Fielitz (1975) showed that individual stocks tend to follow a first order, or higher order, Markov Chain for daily returns; however, the process is not stationary, neither are the chains homogenous. A two-state Markov Chain is used by Turner, Startz, and Nelson (1989) to model changes in the variance of stock returns and Cecchetti, Lam and Mark (1990) showed that if economic driving variables follow a Markov Chain process, then the negative serial correlation found in long horizons can be consistent with an equilibrium model of asset pricing.

Mcqueen and Thorley (1991) used a Markov Chain model to test the random walk hypothesis of stock prices. Given a time series of returns, they defined a Markov Chain by letting one state represent high returns and the other to represent low returns. The random walk hypothesis restricted the transition probabilities of the Markov Chain to be equal irrespective of the prior years. The results showed that annual real returns exhibited significant non-random walk behaviour in the sense that low (high) returns tended to follow runs of high returns for the period under consideration.

From the above literature review, it comes out that Markov Chain model has been widely applied in predicting stock market trend. In many applications, it has been applied in predicting stock index for a group of stock but little has been done for a single stock. Moreover, the model has had limited application in emerging stock markets. This study therefore, sought to apply Markov Chain model to study the trend of Safaricom shares trading in Nairobi Securities Exchange, Kenya as an emerging market.

MATERIALS AND METHODS

This study used a case study design. The Safaricom shares were floated for initial public offers in March 2008 and subsequently started trading in the NSE market in April 2008. The case study design was therefore longitudinal in nature covering four years period of share prices from 2008 to 2012. This study was carried out in Nairobi Securities Exchange Market. This market is located in Nairobi which is the capital city of Kenya. It is situated within the Central Business District at Nation Centre. Nairobi Securities Exchange is a capital market that engages in trading, clearing and settlement of equities, debt, derivatives and other associated instruments. The operations of this market is automated and comprises eight segments ranging from Agricultural sector, Commercial and Services, Telecommunications and Technology (Where Safaricom Kenya ltd belongs), Automobiles and Accessories, Banking, Insurance, Investment to Manufacturing and Allied. Secondary quantitative data on the daily closing stock prices of Safaricom Kenya limited was obtained over a period of four years covering 1st April 2008 to 30th April 2012 through document analysis to form a 768 trading days data panel. This data was gathered from the official website of the Nairobi Securities Exchange. Additionally, the Daily newspapers that capture daily trading of NSE market were used as data source.

Model development

To apply Markov process to share market behavior, share price can be viewed as a system toggling between bullish and bearing state. We construct a transition probability matrix from the past behavior of the system and this transition probability matrix in conjunction with the probability values of the present state of the system is used to determine the probabilities of the next state.

Determination of Initial State Vector

If each closing day is taken as a discrete time unit, then the closing share prices of Safaricom are divided into three states namely Decreases (D), Unchanged (U) and Increases (I). Let $x_1 = D$, $x_2 = U$ and $x_3 = I$, where x_i are the number of observations for the share prices in the named states gathered over the period of study, then the state space is $E(x_1, x_2, x_3)$. State probability is the possibility size of emergence of a variety of states. State vector is denoted by $n_{(i)} = (p_1, p_2, \dots, p_n)$ where $i = 1, 2, \dots, n$, p_j is the probability of x_j , $j = 1, 2, \dots, n$. Since there are 768 trading days in the four year period of study, the observations for D, U and I amounts to 768. So the probability of each state will be as follows:

$p_1 = x_1/768$, $p_2 = x_2/768$ and $p_3 = x_3/768$, giving:

$n_{(0)} = (x_1/768, x_2/768, x_3/768)$ as the initial state vector.

Establishment of the Three State Transition Matrix

The transition matrix for this study would involve three states only as the Safaricom stock assumes basically three states. The states are the chances that a stock decreases, that it remains the same (unchanged) and that it increases. The three states are stated as follows:

D = Safaricom share price decreases

U = Safaricom share price remains the same (Unchanged)

I = Safaricom share price increases.

Transition Probability Matrix provides a precise description of the behavior of a Markov chain. Each element in the matrix represents the probability of the transition from a particular state to the next state. The transition probabilities are usually determined empirically, that is based solely on experiment and observation rather than theory. In other words, relying or based on practical experience without reference to scientific principles.

This research was based on historical daily closing prices of Safaricom shares quoted on the Nairobi Securities Exchange. The data on share price of the Safaricom, was collected from the daily list published by the Nairobi Securities Exchange (also the daily Nation newspaper) from 2008 to 2012. The transitions from one state to another (that is the share price movement pattern, which could be that a decrease in price can be followed by another decrease or a decrease is followed by unchanged or a decrease followed by an increase etc) was observed from the data collected and the result for the period (4 years) under study was compiled in Table 3.1 as follows;

Each entry P_{ij} in the table refers to the number of times a transition has occurred from state i to state j . Consequently, from the share price movement compiled, the transition probabilities was computed to obtain transition matrix for Safaricom Shares as shown in Table 3.2. The states 1, 2 and 3 represent the Price Decreases, Unchanged and Increases respectively. The probability transition matrix P_{ij} is formed by dividing each element in every row by the sum of each row.

Table 3.1 The Share Price Movement of Safaricom Kenya Ltd

	Decrease in Share Price (D)	Unchanged in Share Price (U)	Increase in Share Price (I)
Decrease in Share Price (D)	P_{DD}	P_{DU}	P_{DI}
Unchanged in Share Price (U)	P_{UD}	P_{UU}	P_{UI}
Increase in Share Price (I)	P_{ID}	P_{IU}	P_{II}

Table 3.2 Transition Matrix P_{ij}

State	1	2	3	Sum of Row
1	P_{11}	P_{12}	P_{13}	T_1
2	P_{21}	P_{22}	P_{23}	T_2
3	P_{31}	P_{32}	P_{33}	T_3

The n-step Transition Matrix

The absolute probabilities at any stage where n is greater than unity is determined by the used of n -step transition probabilities. This is a higher order transition probability $P_{ij}^{(n)}$ of the transition matrix P_{ij} . The n -step matrix shows the behavior of share prices n -steps later. The elements of this matrix represent the probabilities that an object in a given state will be in the next state n -steps later. These repeated transitions are used to evaluate whether the transition probabilities converge over repeated iterations ie,

$$\lim_{t \rightarrow \infty} P_{ij}^{(n)}$$

This results in steady state probabilities which shows the probabilities of the shares prices increasing, remaining unchanged or decreasing regardless of the share’s most recent changes in daily closing price.

Calculating the State Probabilities of the subsequent closing days (forecasting)

Since the state probability in different periods are denoted by $n_{(i)}$, then $n_{(i+1)} = n_{(i)} * p_{(ij)}$ Thus $n_{(i+n)} = n_{(i)} * p_{(ij)}^{(n)}$.

So, depending on the state of the closing share price on the last day of trading period under study (ie the 768th day), with no follow –up information, it will be regarded as the initial vector. Suppose it is in U state, then, $n_{(0)} = (1,0,0)$. By virtue of the vector and transition matrix, we can predict or forecast state probabilities of various closing date in future. Hence, we can obtain the state probability of closing price on the 769th day as; $n_{(1)} = n_{(0)} * p_{(ij)}$. State probability vector of closing price on 770th day will be: $n_{(2)} = n_{(1)} * p_{(ij)}$, and so on.

RESULTS AND DISCUSSION

Trend of Safaricom Share Prices

The first research question sought to determine the trend of safaricom share prices in Nairobi Securities exchange. To address this, the 768 days trading data panel for daily closing prices of safaricom shares trading in Nairobi Securities exchange was gathered as illustrated in appendix I. A Four days weekly moving average of daily closing prices of safaricom shares was then computed and the results presented in Figure 4.1 below.

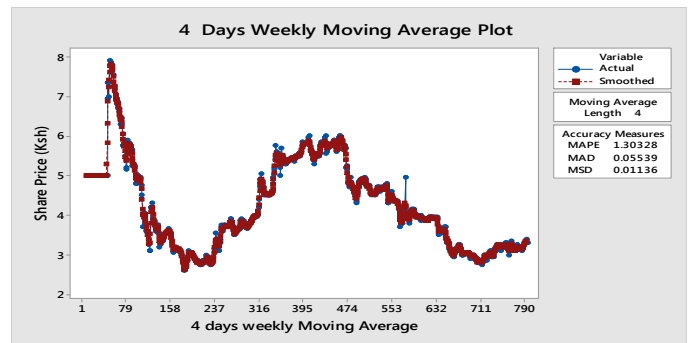


Figure 4.1: Four days weekly moving average of safaricom daily closing prices

Moving Average (MA) is a tool commonly used by market analysts, as popular as the use of trend lines and chart patterns to understand the price behaviour of stocks. The price of a stock can fluctuate wildly over time due to the frequent change in market sentiment, sector or industries in play and profit taking. This makes interpretation of the underlying price movement of the stock difficult. Therefore a Moving Average is usually taken by averaging the prices over a period of time producing a smoother line (Ng Ee Hwa, ChartNexus, 2007).

The results in Figure 1 above show the Safaricom stock prevailing trend. There are 3 types of trends namely “Uptrend”, “Downtrend” and “Sideway”. Each represents a different collective sentiment of the stock market participants, bullishness in an Uptrend, bearishness in a Downtrend and indecision in a Sideway market. Knowing the trend, a trader can more easily identify the tops and bottoms of the price movement for buying or shorting opportunities.

The 4 days weekly moving average starts with a sideways trend at a share price of Ksh. 5. This is followed by a bullish trend up to a peak of Share price of Ksh 8 and then engages a bearish trend to a low of less than Ksh. 3 share price. The trend then again takes a bullish pattern to a high of Ksh. 6 before experiencing a bearish trend and eventually a bullish trend from a low of below Ksh.3. The most common way to interpreting the price moving average is to compare its dynamics to the price action. When the instrument price rises above its moving average, a buy signal appears, if the price falls below its moving average, what we have is a sell signal. This trading system, which is based on the moving average, is not designed to provide entrance into the market right in its lowest point, and its exit right on the peak.

It allows acting according to the following trend: to buy soon after the prices reach the bottom, and to sell soon after the prices have reached their peak. (Neo Ease, 2009)

Model determination

The second research question sought to determine the Markov model for forecasting safaricom share price in Nairobi Securities exchange. To address this, a transition probability matrix from the past behavior of safaricom daily closing share prices was constructed and this transition probability matrix in conjunction with the probability values of the present state of the system was used to determine the probabilities of the next state.

Initial State Vector of Safaricom Share Price

Each closing day was taken as a discrete time unit and the closing share prices of Safaricom were divided into three states namely Decreases (D), Unchanged (U) and Increases (I). Let $x_1 = D$, $x_2 = U$ and $x_3 = I$, where x_i are the number of observations for the share prices in the named states gathered over the period of study, then the state space is $E(x_1, x_2, x_3)$. State probability is the possibility size of emergence of a variety of states. State vector is denoted by $n_{(i)} = (p_1, p_2, \dots, p_n)$ where $i = 1, 2, \dots, n$, p_j is the probability of x_j , $j = 1, 2, \dots, n$. The 784 trading day's data panel on daily closing prices of Safaricom shares obtained from NSE for the four year period were as shown in appendix I.

The proportions (probabilities) for share prices that Decreases (D), Unchanged (U) and Increased (I) were $(246/784) = 0.3137$; $(315/784) = 0.4017$ and $(223/784) = 0.2844$ respectively. This gives rise to initial state vector as: $n_{(i)} = (0.3137 \quad 0.4017 \quad 0.2844)$

Establishment of the Three State Transition Matrix

The transition matrix for this study involved three states only as the Safaricom stock assumes basically three states. The states are the chances that a stock decreases (D), that it remains the same /unchanged (U) and that it increases (I). The transitions from one state to another (that is the share price movement pattern, which could be that a decrease in price can be followed by another decrease or a decrease is followed by unchanged or a decrease followed by an increase etc) observed from the data panel in appendix I were compiled in Table 4.2 as follows;

Table 4.2 The Share Price Movement of Safaricom Kenya Ltd

	Decrease in Share Price (D)	Unchanged in Share Price (U)	Increase in Share Price (I)
Decrease in Share Price (D)	102	65	59
Unchanged in Share Price (U)	64	168	62
Increase in Share Price (I)	68	50	103

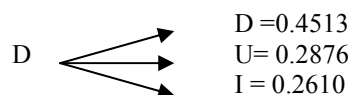
Each entry P_{ij} in the table refers to the number of times a transition has occurred from state i to state j .

Consequently, from the share price movement compiled, the transition probabilities were computed to obtain transition matrix for Safaricom Shares as shown below.

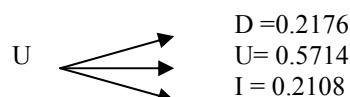
Transition Matrix P_{ij} For Safaricom Shares

$$P_{ij} = \begin{bmatrix} 0.4513 & 0.2876 & 0.2610 \\ 0.2176 & 0.5714 & 0.2108 \\ 0.3076 & 0.2262 & 0.4660 \end{bmatrix}$$

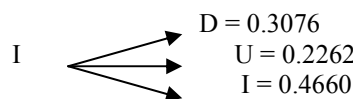
The values for each vector movement of Safaricom Shares are as follows:



This implies that 0.4513 Safaricom share price that decreases will still decrease; 0.2876 Safaricom share price that decrease will remain the same while 0.2610 Safaricom share price that decreases will increase.



This means that 0.2176 Safaricom share price that remains the same will decrease; 0.5714 Safaricom share price that remains the same will still remain the same whereas 0.2108 Safaricom share price that remain the same will increase.



This also means that 0.3076 Safaricom share price that increases will decrease; 0.2262 Safaricom share price that increases will remain unchanged while 0.4660 Safaricom share price that increase will still increase.

The above share price transition movements can be illustrated clearly by a transition diagram as shown in Figure 4.2.2 below.

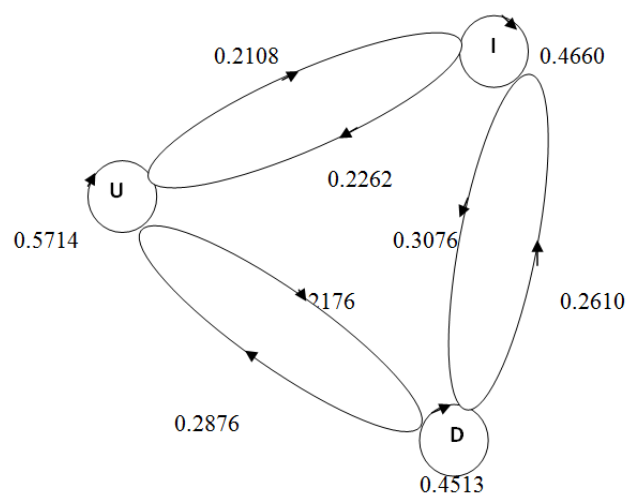


Figure 4.2.2 Transition Diagram for Safaricom Share Price

The n-step Transition Matrix

The absolute probabilities at any stage where n is greater than unity was determined by the use of n-step transition probabilities. This is a higher order transition probability $P_{ij}^{(n)}$ of the transition matrix P_{ij} . The n-step matrix shows the behavior of share prices n-steps later. The elements of this matrix represent the probabilities that an object in a given state will be in the next state n-steps later. These repeated transitions were used to evaluate whether the transition probabilities converge over repeated iterations ie,

$$\lim_{t \rightarrow \infty} P_{ij}^{(n)}$$

This results in steady state probabilities which shows the probabilities of the shares prices increasing, remaining unchanged or decreasing regardless of the share’s most recent changes in daily closing price. The higher order transition probability $P_{ij}^{(n)}$ of the transition probability matrix P_{ij} was calculated to observe the behavior of the share price and the results obtained using MATLAB Statistical Software are as shown below.

$$P_{ij} = \begin{bmatrix} 0.4513 & 0.2876 & 0.2610 \\ 0.2176 & 0.5714 & 0.2108 \\ 0.3076 & 0.2262 & 0.4660 \end{bmatrix}$$

$$P^{52} = \begin{bmatrix} 0.4497 & 0.2214 & 0.3289 \\ 0.3082 & 0.2055 & 0.4863 \\ 0.4481 & 0.1244 & 0.4275 \end{bmatrix}$$

$$P^{104} = \begin{bmatrix} 0.3440 & 0.1329 & 0.5231 \\ 0.3406 & 0.1398 & 0.5196 \\ 0.3439 & 0.1331 & 0.5230 \end{bmatrix}$$

$$P^{208} = \begin{bmatrix} 0.3435 & 0.1338 & 0.5227 \\ 0.3432 & 0.1344 & 0.5224 \\ 0.3435 & 0.1339 & 0.5226 \end{bmatrix}$$

$$P^{416} = \begin{bmatrix} 0.3435 & 0.1339 & 0.5226 \\ 0.3435 & 0.1339 & 0.5226 \\ 0.3435 & 0.1339 & 0.5226 \end{bmatrix}$$

$$P^{832} = \begin{bmatrix} 0.3435 & 0.1339 & 0.5226 \\ 0.3434 & 0.1339 & 0.5226 \\ 0.3435 & 0.1339 & 0.5226 \end{bmatrix}$$

$$P^{1664} = \begin{bmatrix} 0.3435 & 0.1339 & 0.5226 \\ 0.3434 & 0.1339 & 0.5226 \\ 0.3435 & 0.1339 & 0.5226 \end{bmatrix}$$

$$P^{2080} = \begin{bmatrix} 0.3435 & 0.1339 & 0.5226 \\ 0.3435 & 0.1339 & 0.5226 \\ 0.3435 & 0.1339 & 0.5226 \end{bmatrix}$$

From the above n-step transition matrix, it is noticed that after a period of one hundred and four (104) Trading days, the matrix begins to approach some constant probabilities. After extending matrix multiplication to the 416th power, representing the 416th closing trading day of the market, we see all values converge indicating that an equilibrium is attained.

$$P^{416} = \begin{bmatrix} 0.3435 & 0.1339 & 0.5226 \\ 0.3434 & 0.1340 & 0.5226 \\ 0.3435 & 0.1339 & 0.5226 \end{bmatrix}$$

The fact that the transition matrix converges to a steady state system means that the Markov chain is ergodic. This is clear as the three states in the matrix each with three nonzero

probabilities contained in their row vectors, reflecting the fact that each of the states can be reached from any of the states in a finite number of steps.

The following comments can be made from the above steady state transition matrix;

The probability that a share price will depreciate (D) given that it initially either depreciated (D), unchanged (U) or appreciated (I) is 0.3435

The probability that a share price will remain unchanged (U) given that it initially either depreciated (D), unchanged (U) or appreciated (I) is 0.1339

The probability that a share price will appreciate (I) given that it initially either depreciated (D), unchanged (U) or appreciated (I) is 0.5226

This shows that regardless of the stocks’ most recent changes in closing price, approximately 34.35% of the time they will drop in value tomorrow, 13.39 % will maintain the value and 52.26% will have increases in price. This means that for investors who held a portfolio made up of Safaricom shares during the period of study, averaged gains 433 out of 784 trading days.

Calculating the State Probabilities of the subsequent closing days (forecasting)

The third research question sought to determine the extent to which the Markov model can be used to forecast Safaricom share price trend in Nairobi Securities exchange.

To address this, we apply an initial state vector to the transition matrix and predict what state that initial vector will transition to after n iterations, or trading days.

Since the state probability in different periods are denoted by $n_{(i)}$, then $n_{(i+1)} = n_{(i)} * P_{(ij)}$ Thus $n_{(i+n)} = n_{(i)} * P_{(ij)}^{(n)}$.

So, By virtue of the initial satate vector and transition matrix, we can predict or forecast state probabilities of various closing date in future after the 784th trading day. Hence, we can obtain the state probability of closing price on the 785th day as; $n_{(1)} = n_{(0)} * P_{(ij)}$.

$$so, (0.3137 \ 0.4017 \ 0.2844) \begin{bmatrix} 0.4513 & 0.2876 & 0.2610 \\ 0.2176 & 0.5714 & 0.2108 \\ 0.3076 & 0.2262 & 0.4660 \end{bmatrix} = (0.3165 \ 0.3841 \ 0.2991)$$

State probability vector of closing price on 786th day is: $n_{(2)} = n_{(1)} * P_{(ij)}$

$$thus, (0.3165 \ 0.3841 \ 0.2991) \begin{bmatrix} 0.4513 & 0.2876 & 0.2610 \\ 0.2176 & 0.5714 & 0.2108 \\ 0.3076 & 0.2262 & 0.4660 \end{bmatrix} = (0.3184 \ 0.3782 \ 0.3030)$$

That is, state probability that the closing prices of the 785th and 786th trading day locate in states U and U, respectively, are maximum, and they are the same as the actual situation of 3.30 and 3.30. These results of daily closing price state interval after each day predicted is basically consistent with the actual situation.

By using the n-step probability transition matrix and state vectors, we forecast the probabilities of Safaricom share prices appreciating for some selected period of trading days in the future based on the formula: Thus $n_{(i+n)} = n_{(i)} * P_{(ij)}^{(n)}$. The results are posted in Table 4.3 as shown below.

Table 4.3. Forecasts of Safaricom daily closing share price

Period (some selected Trading days from May 2012	Probabilities of Safaricom Share price depreciating	Probabilities of Safaricom Share remaining unchanged	Probabilities of Safaricom Share Appreciating
104 (1/2 a year)	0.3425	0.1874	0.4606
208 (1 year)	0.3433	0.1340	0.5224
832 (4 year)	0.3434	0.1339	0.5225
1664 (8 years)	0.3434	0.1339	0.5225
2080(10 years)	0.3434	0.1339	0.5225

From the forecasts in Table 4.3 above, we notice that irrespective of current price today, the probability of the daily closing share price of Safaricom either depreciating, remaining unchanged or appreciating continue to increase until equilibrium was reached (after 832 trading days or 4 years) and then became constant.

Regardless of the Safaricom share price today, the probability of its share price appreciating in 4 years time (by 2016) is approximately 0.5. Compared to approximate probability of 0.3 and 0.1 of Shares depreciating or not gaining value respectively, investors or future investors would be informed that those who bought Safaricom shares or buy safaricom shares today have more chances of appreciating resulting into capital gains and hence better returns on investment.

Conclusions and Recommendations

Based on the first objective, it was concluded that based on the 4 days weekly moving average, the daily closing Safaricom share price had generally a bullish and bearish trend alternating, though with an initial sideways trend, indicating volatility of prices. The second objective was to determine the Markov model for forecasting safaricom share price in Nairobi Securities Exchange, it was concluded that the derived initial state vector and the transition matrix could be used to predict the states of Safaricom share price accurately as confirmed by the prediction of the 785th and 786th trading days. Additionally, the convergence of transition matrix to a steady state implying ergodicity that is a characteristic of stock market makes the model applicable. Finally, In the long run, irrespective of the current state of share price, the model predicted that the Safaricom share prices will depreciate maintain value or appreciate with a approximate probability of 0.3, 0.1 and 0.5 respectively.

The Markov Chain prediction method is purely a probability forecasting method as the predicted results is simply expressed probability of certain state of stock or shares prices in the future rather than be in absolute state. But because it has no after- effect, using this method to analyse and predict the stock market daily closing prices is relatively more effective under the market mechanism. This study shows how the Markov model fits the data and is able to predict trend due to its memoryless property and random walk capability, in that each state can be reached directly by every other state in the transition matrix, consequently giving good results.

Hence, this model will help both researchers and investors in identifying the future trends in Safaricom shares and stock markets in general thereby being able to make informed decisions regarding investment in the stock market. However, the operational status of the stock market is subject to the

influence of various factors from the market ranging from multiple market forces to psychological factors of investors. Therefore, no single method can accurately predict changes in the stock market. Markov Chain prediction method is no exception and therefore, a combination of results from using markov Chain to predict with other factors can be more useful as a basis for decision making. The current study was a case study on only one company trading in the Nairobi security Exchange. Furthermore, the study was conducted based on First order Markov Chains assuming only three possible states (Increased, unchanged and Decreased). This study therefore suggest that further studies could be conducted on several companies listed in the Nairobi Securities Exchange, using higher order Markov chains to gain better insight into the behavior of the stock market.

REFERENCES

- Abdulsalam sulaiman olaniyi, adewole, kayoed, Jimoh R.G. 2010. "Stock Trend Prediction using Regression Analysis – A Data Mining Approach", *AJSS Journal*, ISSN 2222-9833.
- Asher Levin, David 2009. *Markov chains and mixing times*. p. 16. ISBN 978-0-8218-4739-8.
- Cecchetti, S. C., Lam, P. and Mark, N. C. 1990. "Mean Reversion in Equilibrium Asset Prices". *The American Economic Review*, Vol. 80, 398-415.
- Deju Zhang, L., China and Xiaomin Z. 2009. Study on forecasting the stock market trend based on stochastic analysis method; *International journal of business mangement*; vol.4 no.6
- Doubleday, Kevin and Julius Esunge. 2013. 'Application of Markov Chains to stock trends. "*Journal of Mathematics and Statistics. Science Publications, n.d Web*.
- Fielitz, B. D. (1971), "Stationarity of Random Data: Some Implications for the Distribution of Stock Price Changes". *Journal of Financial and Quantitative Analysis*, Vol. 6, 1025-1034.
- Fielitz, B. D. 1969. "On the Behaviour of Stock Price Relatives as a Random Process with an Application to new York Stock Exchange Prices". Kent State University, USA.
- Fielitz, B. D. 1975. "On the Stationarity of Transition Probability Matrices of Common Stocks", *Journals of Financial and Quantitative Analysis*, Vol. 10, 327-339.
- Fielitz, B. D. and Bhargava, T. N. (1973), "The Behaviour of Stock-Price Relatives--A Markovian Analysis". *Operations Research*, Vol. 21, No. 6, 1183-1199.

- Halbert White, 2009. "Economic prediction using neural networks: the case of IBM daily stock returns" Department of Economics University of California, San Diego.
- Honarkhah, M. and Caers, J. 2010. "Stochastic Simulation of Patterns Using Distance-Based Pattern Modeling". *Mathematical Geosciences*, 42 (5): 487–517.
- Hsieh Y.L., Don-Lin Yang and Jungpin Wu, 2003. "Using Data Mining to study Upstream and Downstream causal relationship in stock Market"
- JingTao YAO and Chew Lim TAN. 2009. "Guidelines for Financial Prediction with Artificial neural networks".
- Kuang Yu Huang, Chuen-Juan Jane, 2009. "A hybrid model stock market forecasting and portfolio selection based on ARX, grey system and RS theories", *Expert systems with Applications*, pp.5387-5392 .
- Mcqueen, G. and Thorley, S. 1991. "Are Stocks Returns Predictable? A Test Using Markov Chains". *The Journal of Finance*, Vol. 56, No. 1, 49-60.
- Meyn S. P. and R.L. Tweedie, 2005. *Markov Chains and Stochastic Stability*.
- Neo Ease, 2009. *Indicator forex : how to interpret Moving Average indicator*; world press
- Ng Ee Hwa, Chart Nexus 2007. *Different uses of Moving Average* downloaded at http://www.chartnexus.com/learning/static/pulses_apr2007
- Niederhoffer, V. and Osborne, M. F. M. 1966. "Market Making and Reversal on the Stock Exchange" *Journal of American Statistical Association*, Vol. 61, 897-916.
- Obodos, E. 2007. "Predicting Stock Market Prices in Nigeria: A Preliminary Investigation". University of Benin, Benin City, Nigeria.
- Preethi G. and Santhi B. 2012. *Stock Market Forecasting Techniques: A Survey*. *Journal of Theoretical and Applied Information Technology*, December 2012. Vol. 46 No.1
- Rafiul Hassan and Baikunth Nath, 2005. "Stock market forecasting using hidden Markov model; A new approach". *Proceedings of 5th international conference on intelligence systems design and applications*. 7695-2286-06/05, IEEE 2005.
- Tommi Jaakkola, (2006). *course materials for 6.867 Machine Learning*, MIT OpenCourseWare (<http://ocw.mit.edu/>), Massachusetts Institute of Technology
- Turner, C. M., Startz, R. and Nelson, C. R. 1989. "A Markov Model of Heteroskadasticity, Risk & Learning in the Stock Market" *Journal of Financial Economics*, Vol. 25, 3-22.
- Vasanthi S, Subha V. and Nambi T. 2011. *An empirical analysis on stock index trend prediction using Markov chain analysis*; *Sri Krishna International research and Education consortium*, Vol. 1 issue 1. 2231-4288.
