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RESEARCH ARTICLE

COMPROMISE ALLOCATIONS OF BIOBJECTIVE NONLINEAR PROGRAMMING PROBLEM UNDER PARTIAL RESPONSES

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ABSTRACT

In this article, a problem of bivariate stratified sample surveys in presence of partial responses has been considered. The population in each stratum is divided into three group's i.e. complete non-respondents, respondents of questions of category I or partial respondents and complete respondents. It is assumed that the respondents of the questions of category II always reply the questions of category I but not necessarily the vice versa. The problem of finding optimum allocations is formulated as Bi-objective Nonlinear Programming Problem (BONLPP). Since the problem is bi-objective the optimum solution cannot be obtained because the optimum solution of one objective may or may not be optimum for the second objective, so we obtain compromise optimum solution using four different methods i.e. Value function, Fuzzy programming, Goal programming and Chebychev approximation. For the demonstration purpose an illustrative example has been solved and also an R simulation study has been carried out to show the efficiency of the methods. Comparison of the solutions obtained by four different methods is shown graphically in the figure.

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INTRODUCTION

Non response is becoming a grooming concern in survey research. Non response is the phenomenon where persons in the sample from the population do not provide the requested information or provide information that is not usable. Two types of non responses can be distinguished. First is unit non response. This occurs when a selected person does not answer any question. The questionnaire form remains completely empty. Second is item non response. This occurs when some questions have been answered, but no answer is given to other, possibly sensitive questions. So, the questionnaire form has been partially completed. Item non response or partial non response may occur when persons refuse to answer a question (e.g., because they do not want to answer a sensitive question) or when they do not know the answer. If a paper questionnaire is used for data collection, some persons could also have accidentally skipped a question. Item non response can also occur as a consequence of a data editing procedure, when an incorrect value is detected and no correct value is available. Non response problem has been firstly discussed by Hansen and Hurwitz (1946) and in 1956 El-Badry extends his technique. After that several authors discuss the problem of complete non response in univariate as well as in multivariate case such as Khare (1987), Fabian and Hyunshik (2000), Najmussehar and Bari (2002) etc. Recently, problem of complete non response formulated as mathematical programming by some authors such as Khan *et al*. (2008), Varshney *et al*. (2011), Raghav *et al*. (2012), Gupta *et al*. (2012) etc. The second type of non response i.e. partial non response was first discussed by Tripathi and Khare (1997). They estimate the population mean in presence of partial response and after that Maqbool and Pirzada (2005) discuss it in two variate stratified sample surveys and find out the optimum sample size and sub-sampling fraction for a fixed budget. In this article, problem of stratified sample surveys in presence of partial response is considered which is formulated as Bi-objective nonlinear programming problem in section 2. Section 3 describes four different optimization techniques to obtain the compromise allocations of the formulated BONLPP. In section 4 an

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illustrative numerical example has been solved whereas in section 5, a simulation study has been carried out. Finally section 6 concludes the work with some suggestions for future work.

2 Problem Formulation of sample surveys in presence of partial response

The sampling scheme used in formulation is as in Maqbool and Pirzada (2005). However, for the sake of continuity they are reproduced here.

Let $Y_{hj1}, Y_{hj2}, \ldots, Y_{hjN_h}$; $j = 1,2\ldots, p$; $h = 1,2,\ldots, L$ be the measurement of N_h units who respond to j^{th} character in h^{th} stratum. Questionnaire is assumed to have the questions of two categories. Character I are measured by questions of category I and character II by those of category II.

First of all in phase one select a random sample from each stratum and send a mail questionnaire to all of the selected units in each stratum. After that identify the partial respondents (those who reply the questions of category I only) and the complete respondents (those who reply the questions of both the categories) in each stratum. Now by personnel interview or through some additional efforts collect data from the selected non-respondents and the partial respondents from each stratum in the sub sample. To make sure that a respondent to questions of category II always responds to questions of category I, it is assumed that the questions of category I are simple. Therefore the whole population is divided into three groups viz. non response, partial response and complete response. In second attempt it is assumed that through extra efforts information from non respondents and partial respondents in each stratum are collected and each unit of the sub sample yields information on both the categories.

Here subscripts designate the attempts 1 and 2 while superscripts designate characters. The superscripts with bar will stand for the character under study corresponding to non respondents.

We take a random sample of size $n_h\ (h = 1, 2, \ldots, L)$ from h^{th} stratum using simple random sampling without replacement, which is partitioned as $n_h = n_{h1}^{-(1)} + n_{h1}^{(1,2)} + n_{h1}^{-(1,2)}$, say

where

$n_{h1}^{-(1)} \rightarrow$ number of respondents to questions of category I only in h^{th} stratum at first phase which is also non-respondents to category II questions in h^{th} stratum at first phase ,

$n_{h1}^{(1,2)} \rightarrow$ number of complete respondents to questions of categories I and II both in h^{th} stratum at first phase,

$n_{h1}^{-(1,2)} \rightarrow$ number of complete non-respondents in h^{th} stratum at first phase.

In second phase by personnel interview or through other extensive methods information is collected from the complete non-respondents and partial respondents to both the category questions.

$n_{h2}^{(1,2)} \rightarrow$ sub-sample in h^{th} stratum out of $n_{h1}^{-(1,2)}$ (complete non respondents), all of which respond to questions of both the categories at second attempt.

Let $k_h = \dfrac{n_{h1}^{-(1,2)}}{n_{h2}^{(1,2)}}$

$$\Rightarrow n_{h2}^{(1,2)} = \frac{n_{h1}^{-(1,2)}}{k_h}$$

Also

$n_{h2}^{(2)} = \dfrac{n_{h1}^{-(1)}}{k_h} \rightarrow$ sub-sample out of $n_{h1}^{-(1)}$, all of which respond to questions of category II at second attempt.

Numbers of units who respond to questions of category I in the h^{th} stratum are:

$\overline{n}_{h1}^{(1)} + n_{h1}^{(1,2)} = n_{h1}^{*}$ (say) at phase I

$n_{h2}^{(1,2)}$ at phase II

$\overline{n}_{h1}^{(1,2)}$ non respondents at phase I.

Numbers of respondents to questions of category II are:

$n_{h1}^{(1,2)}$ at phase I

$n_{h2}^{(2)} + n_{h2}^{(1,2)}$ at phase II

$\overline{n}_{h1}^{(1,2)} + \overline{n}_{h1}^{(1)} = n_{h2}^{*}$ (say) non respondents only at phase I.

Here proportion of units viz k_h selected for second attempt out of the partial respondents and the total non respondents are assumed to be same.

Let us denote the population mean of characters I and II by $\bar{Y}^{(1)}$ and $\bar{Y}^{(2)}$. We define the estimators of $\bar{Y}^{(1)}$ and $\bar{Y}^{(2)}$ respectively by

$$\overline{y}^{(1)} = \sum_{h=1}^{L} \frac{P_h}{n_h}\left[\overline{n}_{h1}^{(1)} + n_{h1}^{(1,2)}{}_{h2}^{(1,2)}\right]$$

$$\overline{y}^{(2)} = \sum_{h=1}^{L} \frac{P_h}{n}\left[\overline{n}_{h1}^{(1)} + n_{h1}^{(1,2)}{}_{h2}^{(1,2)}\right]$$

where

$\bar{y}_{h1}$ = mean of respondents to questions of category I for character I based on $\overline{n}_{h1}^{(1)} + n_{h1}^{(1,2)}$ units at I[st] attempt.

$\overline{y}_{h1}^{(1,2)*}$ = sub-sample mean of respondents to questions of category I at second attempt based on $n_{h2}^{(1,2)}$ units taken out of $\overline{n}_{h1}^{(1,2)}$ non-respondents.

$\overline{y}_{h2}$ = mean of respondents to questions of category II (character II) based on $n_{h1}^{(1,2)}$ at first attempt.

$\overline{y}_{h2}^{(2)*}$ = sub-sample mean of respondents to questions of category II at second attempt based on $n_{h2}^{(1,2)}$ units.

Then the variances of the two estimators $\bar{y}^{(1)}$ and $\bar{y}^{(2)}$ corresponding to the character I and II are given by

$$V(\bar{y}^{(1)}) = \sum_{h=1}^{L}\left[\left(\frac{N_h - n_h}{N_h n_h}\right) + \left(\frac{k_h - 1}{n_h}\right) w_{h3}\right] P_h^2 S_{h1}^2 \qquad (1)$$

$$V(\bar{y}^{(2)}) = \sum_{h=1}^{L}\left[\left(\frac{N_h - n_h}{N_h n_h}\right) + \left(\frac{k_h - 1}{n_h}\right) w_{h4}\right] P_h^2 S_{h2}^2 \qquad (2)$$

where $P_h = N_h/N$ and S_{h1}^2, S_{h2}^2 are the variances of the non-response classes for the characters I and II respectively.

After ignoring the terms independent of n_h in variances of two estimators (1) and (2) can be written as:

$$V(\overline{y}^{(1)}) = \sum_{h=1}^{L}\left[\left(\frac{1}{n_h}\right) + \left(\frac{k_h - 1}{n_h}\right) w_{h3}\right] P_h^2 S_{h1}^2 \qquad (3)$$

$$V(\overline{y}^{(2)}) = \sum_{h=1}^{L} \left[\left(\frac{1}{n_h} \right) + \left(\frac{k_h - 1}{n_h} \right) w_{h4} \right] P_h^2 S_{h2}^2 \quad (4)$$

The cost function is defined as:

$$C = co + \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} c_{h1}^{(1)} + \sum_{h=1}^{L} c_{h1}^{(1)} (n_{h1}^{(1)} + n_{h1}^{(1.2)}) + \sum_{h=1}^{L} c_{h1}^{(2)} n_{h1}^{(1.2)} + \sum_{h=1}^{L} c_{h2} (n_{h2}^{(2)} + n_{h2}^{(1.2)}) \quad (5)$$

or $C_0 = C - c_0 = \left[\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} c_{h1}^{(1)} + \sum_{h=1}^{L} c_{h1}^{(1)} (n_{h1}^{(1)} + n_{h1}^{(1.2)}) + \sum_{h=1}^{L} c_{h1}^{(2)} n_{h1}^{(1.2)} + \sum_{h=1}^{L} c_{h2} (n_{h2}^{(2)} + n_{h2}^{(1.2)}) \right]$

where c_0= overhead cost

c_h= cost of including a unit in the sample in h^{th} stratum.
$c_{h1}^{(1)}$= cost incurred/unit in enumerating questions of category I in h^{th} stratum in first attempt.
$c_{h1}^{(2)}$= cost incurred/unit in enumerating questions of category II in h^{th} stratum in first attempt.
c_{h2}= cost incurred/unit in h^{th} stratum in enumerating both the characters in second attempt.

It is understood that the values of $\overline{n}_{h1}^{(1)}$ and $\overline{n}_{h1}^{(1,2)}$ are not known until the first attempt is made, the expected cost is used in planning the sample. Therefore the expected values of

$n_{h1}^{*} = n_h w_{h1}$, $n_{h1}^{(1,2)} = n_h w_{h2}$, $n_{h2}^{(1,2)} = \frac{n_h w_{h3}}{k_h}$, and $n_{h2}^{(2)} = \frac{n_h w_{h4}}{k_h}$ and hence the total expected cost is given by

$$C_0 = \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} c_{h1}^{(1)} n_h w_{h1} + \sum_{h=1}^{L} c_{h1}^{(2)} n_h w_{h2} + \sum_{h=1}^{L} c_{h2} n_h \left(\frac{w_{h3}}{k_h} + \frac{w_{h4}}{k_h} \right) \quad (6)$$

where w_{hj} are the proportion of respondents and non respondents in h^{th} stratum to questions of both the categories such that

$$\omega_{\eta 1} + \omega_{\eta 3} = 1$$
$$\omega_{\eta 2} + \omega_{\eta 4} = 1$$

In order to obtain the optimum allocation (n_h^*) and sub sampling fraction (k_h^*) for the characteristics under study, we formulate a Bi-Objective Nonlinear programming Problem (BONLPP) as follows:

$$\left.\begin{aligned}
&\text{Minimize } V(\overline{y}^{(1)}) = \sum_{h=1}^{L} \left[\left(\frac{1}{n_h} \right) + \left(\frac{k_h - 1}{n_h} \right) w_{h3} \right] P_h^2 S_{h1}^2 \\
&\text{Minimize } V(\overline{y}^{(2)}) = \sum_{h=1}^{L} \left[\left(\frac{1}{n_h} \right) + \left(\frac{k_h - 1}{n_h} \right) w_{h4} \right] P_h^2 S_{h2}^2 \\
&\text{Subject to } \sum_{h=1}^{L} c_h n_h + \sum c_{h1}^{(1)} n_h w_{h1} + \sum c_{h1}^{(2)} n_h w_{h2} + \sum c_{h2} n_h \left(\frac{w_{h3}}{k_h} + \frac{w_{h4}}{k_h} \right) \le C_0 \\
&\qquad 2 \le n_h \le N_h;\ k_h \ge 0 \\
&\text{and} \qquad n_h \text{ are integers; } h = 1,2,\ldots,L.
\end{aligned}\right\} \quad (7)$$

3 Several optimization techniques to solve BONLPP

3.1 Value function

Many authors studied multi objective optimization in detail such as Kish, Miettinen etc. The problem (7) expressed under value function technique as (see Diaz-Garcia and Ulloa, 2008):

$$\left.\begin{aligned} &\text{Minimize } \varphi\left(\sum_{j=1}^{p} V_j\right) \\ &\text{Subject to } \sum_{h=1}^{L} c_h n_h + \sum c_{h1}^{(1)} n_h w_{h1} + \sum c_{h1}^{(2)} n_h w_{h2} + \sum c_{h2} n_h \left(\frac{w_{h3}}{k_h} + \frac{w_{h4}}{k_h}\right) \le C_0 \\ &\qquad\qquad 2 \le n_h \le N_h;\ k_h \ge 0 \\ &\text{and} \qquad n_h \text{ are integers; } h = 1,2,\ldots,L. \end{aligned}\right\} \tag{8}$$

where ϕ(.) is a scalar function that summarizes the importance of each of the variance of the p characteristics. ϕ(.) may take different forms for different problems amongst them one particular forms used here is weighted sum. Now under this approach the problem (8) can be expressed as:

$$\left.\begin{aligned} &\text{Minimize } \left(\sum_{j=1}^{p} \lambda_j V_j\right) \\ &\text{Subject to } \sum_{h=1}^{L} c_h n_h + \sum c_{h1}^{(1)} n_h w_{h1} + \sum c_{h1}^{(2)} n_h w_{h2} + \sum c_{h2} n_h \left(\frac{w_{h3}}{k_h} + \frac{w_{h4}}{k_h}\right) \le C_0 \\ &\qquad\qquad 2 \le n_h \le N_h;\ k_h \ge 0 \\ &\text{and} \qquad n_h \text{ are integers; } h = 1,2,\ldots,L. \end{aligned}\right\} \tag{9}$$

where $\lambda_j \ge 0;\ j = 1,2,\ldots,p$ are the weights according to the relative importance of the characteristics. Without loss of generality we can take $\sum_{j=1}^{p} \lambda_j = 1$.

3.2 Fuzzy programming

Let $V_j^{\star}$ be the optimal value of V_j obtained by solving the BONLPP (7). Further let $\tilde{V}_j$ denote the variance under compromise allocation, where $n_h;\ h = 1,2,\ldots,L$ are to be worked out.

Obviously, $\tilde{V}_j \ge V_j^{*}$ or $\tilde{V}_j - V_j^{*} \ge 0;\ j = 1,2,\ldots,p$ will give the increase in the variance due to not using the individual optimum allocation for j^{th} characteristic.

To obtain a fuzzy solution, we first compute the upper and lower tolerance limits for each objective i.e., $L_k = \min_j V_k(n_{hj}^{*})$ and $U_k = \max_j V_k(n_{hj}^{*})$, where $n_{hj}^{\star}$ denote the optimum allocation in the h^{th} stratum for the j^{th} characteristic.

The differences of the maximum and minimum values of V_k are denoted by $d_k = U_k - L_k;\ k = 1,2,\ldots p.$

Now the fuzzy programming formulation of the BONLPP is given as:

$$\left.\begin{array}{ll}\text{Minimize} & \delta \\ \text{Subject to} & \sum_{h=1}^{L}\left[\left(\frac{1}{n_h}\right)+\left(\frac{k_h-1}{n_h}\right)w_{hi}\right]P_h^2\, S_{hj}^2 - \delta d_k \le V_j^*,\ i=3,4 \\ & \sum_{h=1}^{L}c_h n_h + \sum_{h=1}^{L}c_{h1}^{(1)}n_h w_{h1} + \sum_{h=1}^{L}c_{h1}^{(2)}n_h w_{h2} + \sum_{h=1}^{L}c_{h2}n_h\left(\frac{w_{h3}}{k_h}+\frac{w_{h4}}{k_h}\right)\le C_0 \\ & 2\le n_h \le N_h;\ k_h \ge 0 \\ \text{and} & n_h \text{ are integers; } h=1,2,\ldots,L. \end{array}\right\};\ j=1,2,\ldots,p$$

3.3 Goal programming

To solve the following BONLPP using goal programming, we first solve each objective subject to the system constraints separately

$$\left.\begin{array}{ll}\text{Minimize} & \sum_{h=1}^{L}\left[\left(\frac{1}{n_h}\right)+\left(\frac{k_h-1}{n_h}\right)w_{hi}\right]P_h^2\, S_{hj}^2;\ i=3,4 \\ \text{Subject to} & \sum_{h=1}^{L}c_h n_h + \sum_{h=1}^{L}c_{h1}^{(1)}n_h w_{h1} + \sum_{h=1}^{L}c_{h1}^{(2)}n_h w_{h2} \\ & +\sum_{h=1}^{L}c_{h2}n_h\left(\frac{w_{h3}}{k_h}+\frac{w_{h4}}{k_h}\right)\le C_0 \\ & 2\le n_h \le N_h;\ k_h \ge 0 \\ \text{and} & n_h \text{ are integers; } h=1,2,\ldots,L. \end{array}\right\};\ j=1,2,\ldots,p \quad (10)$$

Let $V_j^{\star}$ be the optimum value of V_j with the solution to the j^{th} NLPP as $\underline{n}_j^* = (n_{j1}^*, n_{j2}^*, \ldots, n_{jL}^*)$.

Further let $\tilde{V}_j$ the optimal value under compromise solution with $\underline{n}_c^* = (n_{1c}^*, n_{2c}^*, \ldots, n_{Lc}^*)$. the vector of optimum compromise allocations for the j^{th} characteristics.

Obviously,

$$\tilde{V}_j \ge V_j^* \ or\ \tilde{V}_j - V_j^* \ge 0;\ j=1,2,\ldots,p \quad (11)$$

A reasonable criterion to work out a compromise allocation may be to minimize the sum of increases in the variances, $V_j; j=1,2,\ldots,p$ due to the use of the compromise solution. The goal is to find the compromise allocation $\underline{n}_c^* = (n_{1c}^*, n_{2c}^*, \ldots, n_{Lc}^*)$ such that the increase in value of j^{th} variance due to the use of a compromise allocation should not exceed $x_j; j=1,2,\ldots,p$, where $x_j \ge 0$ are the unknown goal variables.

To achieve these goals x_{ij} must satisfy

$$\tilde{V}_j - V_j^* \le x_j;\ j=1,2,\ldots,p$$

or

$$\tilde{V}_j - x_j \leq V_j^*$$

$$\sum_{h=1}^{L}\left[\left(\frac{1}{n_h}\right)+\left(\frac{k_h-1}{n_h}\right)w_{hi}\right]P_h^2S_{hj}^2 - x_j \leq V_j^* \quad (12)$$

The value of $\sum_{j=1}^{p} x_j$ will give us total increase in the variances by not using the individual optimum allocations.

This suggests the following Goal Programming Problem (GPP) to solve:

$$\left.\begin{array}{ll}
\text{Minimize} & \sum_{j=1}^{p} x_j \\
\text{Subject to} & \sum_{h=1}^{L}\left[\left(\frac{1}{n_h}\right)+\left(\frac{k_h-1}{n_h}\right)w_{hi}\right]P_h^2 S_{hj}^2 - x_j \leq V_j^*,\ i=3,4 \\
& \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} c_{h1}^{(1)} n_h w_{h1} + \sum_{h=1}^{L} c_{h1}^{(2)} n_h w_{h2} \\
& + \sum_{h=1}^{L} c_{h2} n_h \left(\frac{w_{h3}}{k_h} + \frac{w_{h4}}{k_h}\right) \leq C_0 \\
& 2 \leq n_h \leq N_h;\ k_h \geq 0 \\
\text{and} & n_h \text{ are integers; } h = 1,2,\ldots,L.
\end{array}\right\};\ j = 1,2,\ldots,p \quad (13)$$

The GPP (13) may be solved by using the optimization software LINGO (LINGO- User's Guide). For more information one can visit the site: http://www.lindo.com

3.4 Chebyshev approximation

Here we consider the problem given by equation (7). For using Chebyshev Approximation we have to convert the problem into convex programming problem so by making the transformation $n_h = \frac{1}{x_h};\ h = 1,2,\ldots,L$ and $a_{hj} = \left[1+(k_h-1)w_{hi}\right]P_h^2 S_{hj}^2; i = 3,4$ then the problem (8) is equivalent to minimizing the linear form (see Khan *et al*., 2011)

$$\left.\begin{array}{ll}
\text{Minimize} & V_j = \sum_{h=1}^{L} a_{hj} x_h \\
\text{Subject to} & \sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} c_{h1}^{(1)} n_h w_{h1} + \sum_{h=1}^{L} c_{h1}^{(2)} n_h w_{h2} \\
& + \sum_{h=1}^{L} c_{h2} n_h \left(\frac{w_{h3}}{k_h} + \frac{w_{h4}}{k_h}\right) \leq C_0 \\
& \frac{1}{N_h} \leq x_h \leq \frac{1}{2};\ k_h \geq 0
\end{array}\right\};\ j = 1,2,\ldots,p \quad (14)$$

Now the objective functions are linear and the single constraint is convex (see Kokan and Khan (1967)). So eq. (14) represents convex programming problems. The problem (14) is now equivalent to minimizing the linear form (see Ali *et al*., 2011)

$$\left.\begin{aligned}&\text{Minimize } Z = x_{L+1}\\&\text{Subject to } \; a_j V_j \le x_{L+1} \; or \; a_j \sum_{h=1}^{L} a_{hj} x_h - x_{L+1} \le 0;\; j = 1,2,\ldots,p\\&\sum_{h=1}^{L} c_h n_h + \sum_{h=1}^{L} c_{h1}^{(1)} n_h w_{h1} + \sum_{h=1}^{L} c_{h1}^{(2)} n_h w_{h2}\\&+\sum_{h=1}^{L} c_{h2} n_h \left(\frac{w_{h3}}{k_h} + \frac{w_{h4}}{k_h}\right) \le C_0\\&\frac{1}{N_h} \le x_h \le \frac{1}{2};\; k_h \ge 0\end{aligned}\right\} \tag{15}$$

where a_j are the weights assigned to the variances according to their importance.

4 Illustrative example

The following example illustrates the four methods discussed in above section. The data are taken from Maqbool and Pirzada (2005) in which a population is considered which is divided into four strata. The total amount available for conducting the survey is assumed to be C=3000 units with an expected overhead cost $c_0 = 1000$ units. This gives $C_0 = C - c_0 = 2000$ units. The proportion of respondents are w_{h1}=0.4 and w_{h2}=0.3 and the proportion of non-respondents are w_{h3}=0.6 and w_{h4}=0.7 for the character I and II respectively (Table 1).

Table 1. Input data for two characteristics and four strata

h	N_h	P_h	S_{h1}^2	S_{h2}^2	c_h	$c_{h1}^{(1)}$	$c_{h1}^{(2)}$	c_{h2}
1	20	0.2	3.5	3.2	0.5	8.5	8.7	25
2	30	0.3	5.5	4.8	0.7	7.4	7.6	20
3	40	0.4	6.5	6.2	0.4	7	7.2	18
4	10	0.1	5.5	5.3	0.6	9	9.2	25

For j=1 BONLPP takes the form:

$$\left.\begin{aligned}&\text{Minimize } \left(\frac{0.196}{n_1} + \frac{1.089}{n_2} + \frac{2.704}{n_3} + \frac{0.121}{n_4} + \frac{0.294k_1}{n_1} + \frac{1.63351k_2}{n_2} + \frac{4.056k_3}{n_3} + \frac{0.1815k_4}{n_4}\right)\\&\text{subject to } \; 6.51n_1 + 5.94n_2 + 5.36n_3 + \frac{32.5n_1}{k_1} + \frac{26n_2}{k_2} + \frac{23.4n_3}{k_3} + \frac{32.5n_4}{k_4} \le 2000\\&2 \le n_1 \le 20;\; 2 \le n_2 \le 30;\; 2 \le n_3 \le 40;\; 2 \le n_4 \le 10\\&k_h \ge 0\\&\text{and } \; n_h \text{ are integers; } h = 1,2,\ldots,4.\end{aligned}\right\}$$

Above problem is solved by an optimizing software LINGO-13 and obtain the optimum solution as:

n_1=12,k_1=1.8660,n_2=29,k_2=1.7111,n_3=40,k_3=1.4210,n_4=9,k_4=1.7812

with the corresponding value of variance V_1=0.4570349

For j=2 BONLPP takes the form:

$$\left.\begin{array}{ll}
\text{Minimize} & \left(\begin{array}{l}\dfrac{0.12288}{n_1}+\dfrac{0.62208}{n_2}+\dfrac{1.84512}{n_3}+\dfrac{0.08427}{n_4}\\ +\dfrac{0.28672k_1}{n_1}+\dfrac{1.45152k_2}{n_2}+\dfrac{4.30528k_3}{n_3}+\dfrac{0.19663k_4}{n_4}\end{array}\right)\\
\text{subject to} & 6.51n_1+5.94n_2+5.36n_3+\dfrac{32.5n_1}{k_1}+\dfrac{26n_2}{k_2}+\dfrac{23.4n_3}{k_3}+\dfrac{32.5n_4}{k_4}\le 2000\\
 & 2\le n_1\le 20;\ 2\le n_2\le 30;\quad 2\le n_3\le 40;\quad 2\le n_4\le 10\\
 & k_h\ge 0\\
\text{and} & n_h \text{ are integers; } h=1,2,\ldots,4.
\end{array}\right\}$$

Above problem is solved by an optimizing software LINGO-13 and obtain the optimum solution as:

n_1=10,k_1=1.5160,n_2=23,k_2=1.3861,n_3=40,k_3=1.3279,n_4=8,k_4=1.4646

with the corresponding value of variance V_1=0.4058625

To obtained the compromise allocations four methods used and according to the procedure discussed in the section 3 problems are formulated and solved by an optimization software LINGO (2013). LINGO is a user's friendly package for constrained optimization developed by LINDO System Inc. A user's guide-LINGO User's Guide (2013) is also available. For more information one can visit the site http://www.lindo.com. And the results are summarized in Table 2 below:

Table 2. Compromise optimum solutions obtained by four different methods

	Allocations								Variances		
Approaches	n_1	k_1	n_2	k_2	n_3	k_3	n_4	k_4	V_1	V_2	Trace=V_1+V_2
Value function	11	1.6850	26	1.5455	40	1.3702	8	1.5187	0.4579560	0.4067659	0.8647220
Fuzzy programming	11	1.6853	26	1.5467	40	1.3697	8	1.5179	0.4579706	0.4067673	0.8647379
Goal programming	11	1.6850	26	1.5455	40	1.3702	8	1.5187	0.4579560	0.4067659	0.8647220
Chebychev approximation	12	1.8243	29	1.7082	40	1.4189	9	1.7644	0.4570259	0.4090924	0.8661183

5 R simulation study

For comparing the efficiency of the methods discussed in section 3, a simulation study has been carried out. The R language (2011) has been used to perform the simulations and data analysis. We have generated a population of size N=1050. From this population four strata are randomly generated. The characteristics for the two populations have generated in the following way:

$$2^{(1)}\sim N(500,150) \quad \alpha\nu\delta \quad 2^{(2)}\sim N(300,95)$$

Data obtained by simulation study is shown in table 1. In addition to the above, it is assumed that the relative value of the variances of the non-respondents and respondents, that is, $S^2_{jh2}/S^2_{jh}=0.25$ for $j=1,2$ and $h=1,2,3,4$. Further, let the total amount available for the survey be C_0=1700 units for the problem (7). The proportion of respondents is w_{h1}=0.4 and w_{h2}=0.3 and the proportion of non-respondents is w_{h3}=0.6 and w_{h4}=0.7 for the character I and II respectively.

Table 3. Input data for two characteristics and four strata

h	N_h	P_h	S^2_{h1}	S^2_{h2}	c_h	$c^{(1)}_{h1}$	$c^{(2)}_{h1}$	c_{h2}
1	306	0.291429	5800.17	2385.23	0.5	8.5	8.7	25
2	205	0.195238	555.248	160.637	0.7	7.4	7.6	20
3	353	0.33619	1592.11	684.948	0.4	7	7.2	18
4	186	0.177143	5464.03	1749.39	0.6	9	9.2	25

For j=1 BONLPP takes the form:

$$\left.\begin{array}{ll} \text{Minimize} & \left(\begin{array}{l} \dfrac{49.26123}{n_1}+\dfrac{2.116488}{n_2}+\dfrac{17.99461}{n_3}+\dfrac{17.14591}{n_4} \\ +\dfrac{73.89185k_1}{n_1}+\dfrac{3.174731k_2}{n_2}+\dfrac{26.99192k_3}{n_3}+\dfrac{25.71886k_4}{n_4} \end{array}\right) \\ \text{subject to} & 6.51n_1+5.94n_2+5.36n_3+\dfrac{32.5n_1}{k_1}+\dfrac{26n_2}{k_2}+\dfrac{23.4n_3}{k_3}+\dfrac{32.5n_4}{k_4}\le 1700 \\ & 2\le n_1\le 306;\, 2\le n_2\le 205;\quad 2\le n_3\le 353;\quad 2\le n_4\le 186 \\ & k_h\ge 0 \\ \text{and} & n_h \text{ are integers; } h=1,2,\dots,4. \end{array}\right\}$$

Above problem is solved by an optimizing software LINGO-13 and obtain the optimum solution as:

$$n_1=30, k_1=1.802684, n_2=7, k_2=1.815039, n_3=20, k_3=1.687236, n_4=17,\ k_4=1.731488$$

with the corresponding value of variance V_1=14.01262.

For *j*=2 BONLPP takes the form:

$$\left.\begin{array}{ll} \text{Minimize} & \left(\begin{array}{l} \dfrac{15.19342}{n_1}+\dfrac{0.459236}{n_2}+\dfrac{5.806169}{n_3}+\dfrac{4.117129}{n_4} \\ +\dfrac{35.4513k_1}{n_1}+\dfrac{1.071551k_2}{n_2}+\dfrac{13.54773k_3}{n_3}+\dfrac{9.606634k_4}{n_4} \end{array}\right) \\ \text{subject to} & 6.51n_1+5.94n_2+5.36n_3+\dfrac{32.5n_1}{k_1}+\dfrac{26n_2}{k_2}+\dfrac{23.4n_3}{k_3}+\dfrac{32.5n_4}{k_4}\le 1700 \\ & 2\le n_1\le 306;\, 2\le n_2\le 205;\quad 2\le n_3\le 353;\quad 2\le n_4\le 186 \\ & k_h\ge 0 \\ \text{and} & n_h \text{ are integers; } h=1,2,\dots,4. \end{array}\right\} \quad (20)$$

Above problem is solved by an optimizing software LINGO-13 and obtain the optimum solution as:

$$n_1=27, k_1=1.476810, n_2=5, k_2=1.406972, n_3=18, k_3=1.351395, n_4=13,\ k_4=1.365949$$

with the corresponding value of variance V_1=5.560957.

To obtained the compromise allocations four methods used and according to the procedure discussed in the section 3 problems are formulated and solved by an optimization software LINGO (2013). LINGO is a user's friendly package for constrained optimization developed by LINDO System Inc. A user's guide-LINGO User's Guide (2013) is also available. For more information one can visit the site http://www.lindo.com. And the results are summarized in table 4 below:

Table 4. Compromise optimum solutions obtained by four different methods

	Allocations								Variances		
Approaches	n_1	k_1	n_2	k_2	n_3	k_3	n_4	k_4	V_1	V_2	Trace= V_1+V_2
Value function	29	1.695526	6	1.592187	20	1.629516	16	1.645804	14.03011	5.597118	19.627228
Fuzzy programming	29	1.657579	6	1.648854	19	1.502105	15	1.571902	14.0666	5.579078	19.645678
Goal programming	29	1.695526	6	1.592187	20	1.629516	16	1.645804	14.03011	5.597118	19.627228
Chebychev approximation	30	1.824339	7	1.708236	20	1.706003	17	1.764377	14.09261	5.674545	19.767155

6 Conclusion and Future work

In this article a Bi-objective nonlinear programming problem is formulated from bivariate stratified problem under partial responses. The problem of finding optimal allocations has been solved using four different methods viz. Value function, Fuzzy programming, Goal programming and Chebyshev approximation. Optimum compromise solution (Table 2) obtained by real life data shows that the value function and goal programming methods provides the most efficient solution. To check the efficiency of the results a simulation study has been carried out which also conclude that the value function and gaol programming methods give the most efficient solution. The comparison of the results has been graphically shown in the figure given below. In future one can check the efficiency of the methods for more than one population and can also try to formulate the problem under partial responses as a multi objective programming problem i.e $p \geq 2$.

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