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RESEARCH ARTICLE

ON πgb-QUOTIENT MAPPINGS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce πgb-quotient maps, πgb^*-quotient map via πgb-closed sets.Further, several characterizations and properties are obtained.

Key words: Quotient, stronglyπgb-quotient, πgb^*-quotient, π^*-quotient, π-quotient: **AMS Subject Classification:** 54C10.

1. INTRODUCTION

Andrijevic [3] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. This type of sets was discussed by Ekici and Caldas [6]under the name of γ-open sets. Levine [9] introduced the concept of generalized closed sets in topological space and a class of topological spaces called $T_{½}$ spaces. Extensive research on generalizing closedness was done in recent years as the notions of a generalized closed, generalized semi-closed, α-generalized closed, generalized semi-pre-open closed sets were investigated in [2,4,7,9,12,13]. The finite union of regular open sets is said to be π-open. The complement of a π-open set is said to be π-closed. In this paper we introduce weaker class of quotient maps and study the relationship between weak and strong forms of πgb^*-quotient map.Further we introduce strongly πgb-quotient maps and study the relationship with weak and strong forms of open maps.

2. Preliminaries

Throughout this paper (X, τ) and (Y, τ) represent non empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), cl(A) and int(A) denote the closure of A and the interior of A respectively.(X, τ) will be replaced by X if there is no chance of confusion.

Definition 2.1: A subset A of a space (X, τ) is called

(1) a pre-open set [11] if A⊂int (cl (A)) and a preclosed set if cl (int (A))⊂ A;

(2) a semi-open set[8] if A⊂ cl(int(A)) and a semi-closed set if int (cl(A))⊂A;

(3) aα-open set[14] if A⊂int (cl(int (A))) and a α -closed set if cl (int(cl (A)))⊂ A;

(4) a semi-preopen set[1] if A⊂ cl (intcl(A)) and a semi-pre-closed set if int (cl (int(A)))⊂ A;

(5) a regular open set if A=int (cl(A)) and a regular closed set if A=cl(int (A));

(6) b-open [3] or sp-open [5], γ –open [6] if A ⊂cl(int(A))∪int (cl(A)).

The complement of a b-open set is said to be b-closed [3]. The intersection of all b-closed sets of X containing A is called the b-closure of A and is denoted by bCl(A). The union of all b-open sets of X contained in A is called b-interior of A and is denoted by bInt(A).

Definition 2.2. A subset A of a space (X,τ) is called:

(1) a $\hat{g}$ -closed set [23] if cl(A)⊆U whenever A⊆U and U is semi-open in (X,τ).The complement of $\hat{g}$ -closed set is called $\hat{g}$ -open set;

(2)a α-generalized semi-closed (briefly αgs-closed) set [15] if αcl(A)⊆U wheneverA⊆U and U is semi-open in (X,τ). The complement of αgs-closed set is calledαgs-open set.

(3) A subset A of (X,τ) is called πgb-closed[20] if bcl(A)⊂ U whenever A⊂U and U is π-open in (X, τ). The complement of πgb-closed set is called πgb-open set.

Remark 2.3: By πGBC(τ) we mean the family of all πgb-closed subsets of the space(X, τ).

Definition 2.4. A map f : (X,τ)→(Y,σ) is called

(1) αgs-continuous [16] if f^{-1}(V) is a αgs-closed set in (X,τ) for each closed set Vof (Y,σ).
(2) strongly αgs-continuous [16] if f^{-1}(V) is a closed set in (X,τ) for each αgsclosedset V of (Y,σ).
(3) α-continuous [22] if f^{-1}(V) is a α-closed set in (X,τ) for each closed set V of(Y,σ).
(4) $\hat{g}$ -continuous [23] if f^{-1}(V) is a ˆg-closed set in (X,τ) for each closed set V of(Y,σ).
(5) π- continuous[20] if every f^{-1}(V) is π-closed in (X, τ) for every closed set V of (Y,σ)
(6) πgb- continuous[20] if every f^{-1}(V) is πgb- closed in (X, τ) for every closed set V of (Y,σ).

Definition 2.5: A map f : (X,τ)→(Y,σ) is called

(1) π- irresolute [4] if f^{-1}(V) is π- closed in (X, τ) for every π-closed of (Y,σ).
(2) πgb- irresolute[20] if every f^{-1}(V) is πgb-closed in (X, τ) for every πgb- closed set V of (Y,σ).

Remark 2.6: [20]

Every continuous map, α-continuous map, ˆg-continuous map, αgs-continuous, π-continuous map, π^*-continuous map is πgb-continuous but not conversely.

Definition 2.7: A surjective map f : (X,τ)→(Y,σ) is said to be

(1) a quotient map [20], provided a subset U of (Y,σ) is open in (Y,σ) if and onlyif f^{-1}(U) is open in (X,τ).
(2)a α-quotient map [10] if f is α-continuous and f^{-1}(V) is open in (X,τ) impliesV is an α-open set in (Y,σ).
(3) a α∗-quotient map [22] if f is α-irresolute and f^{-1}(V) is α-open set in (X,τ)implies V is an open set in (Y,σ).
(4) a ˆg-quotient map [18] if f is ˆg-continuous and f^{-1}(V) is open in (X,τ) impliesV is a ˆg-open set in (Y,σ).
(5) a αgs-quotient map[19] if f is αgs-continuous and f^{-1}(V) is open in (X,τ) implies V is a αgs- open set in(Y,σ).

Remark 2.8:[20]Every closed, g-closed, α-closed, $\hat{g}$ -closed, αgs-closed set is πgb closed but not conversely.

Remark 2.9:[22] Every quotient map is α-quotient but not conversely.

Definition 2.10:[16,17] A map f : X→Y is called α-open if f(V) is α-open in Y foreach open set V of X.
Definition 2.11:A map f : (X,τ)→(Y,σ) is said to be πgb-open[21] if f(U) is πgb-open in (Y,σ) for each open set U in (X,τ).
Definition 2.12:A map f : (X,τ)→(Y,σ) is said to be M-πgb-open[21] if f(U) is πgb -open in (Y,σ) for each πgb-open set U in (X,τ).
Definition 2.13: A topological space X is a πgb- space[20] if every πgb- closed set is closed.

Remark 2.14: Every πgb- irresolute function is πgb- continuous but not conversely[20].

3. πgb-QUOTIENT MAPS

Definition 3.1:A surjective map f: (X,τ)→(Y,σ) is said to be a πgb-quotient mapif f is πgb-continuous and f^{-1}(V) is open in (X,τ) implies V is a πgb-open set in(Y,σ).
Example 3.2: Let X={a,b}, τ={Φ,X,{a},{b,c}}, Y={1,2} and σ={ Φ,Y,{1}}. We have πGBO(X) =P(X) andπGBO(Y) =P(Y).The map f defined as f (a)=1, f(b)=f(c)=2 is πgb-quotient.
Definition 3.3:A surjective map f:(X,τ)→(Y,σ) is said to be a πgb^*-quotient map if f is πgb-irresolute and f^{-1}(V) is πgb- open in (X,τ) implies V is a open set in (Y,σ).
Example 3.4: Let X={a,b,c,d},τ={Φ,X,{a},{a,b},{a,c,d}},Y={1,2,3} and σ={ Φ,Y,{1},{2},{3}{1,2},{1,3}{2,3}}.We have πGBO(X)=P(X) and πGBO(Y)=P(Y). The map f is defined as f(a)=1,f(b)=3, f(c)=f(d)=2. The map f is πgb^*-quotient.
Definition 3.5:A surjective map f:(X,τ)→(Y,σ) is said to be a π-quotient map if f is π-continuous and f^{-1}(V) is open in (X,τ) implies V is a π-open set in (Y,σ).
Definition 3.6: A surjective map f:(X,τ)→(Y,σ) is said to be a π^*-quotient map if f is π-irresolute and f^{-1}(V) is π-open in (X,τ) implies V is a open set in (Y,σ).
Definition 3.7: Let f : (X,τ)→(Y,σ) be a surjective map. Then f is called strongly πgb-quotient map provided a set U of (Y,σ) is open in Y if and only if f^{-1}(U) is a πgb-open in (X,τ).

Example3.8: LetX={a,b,c,d},τ={Φ,X,{a},{b,c},{a,b,c}},Y={1,2,3}andσ={Φ,Y,{1},{2},{3},{1,2},{1,3),{2,3}}.We have πGBO (X) = P(X) and πGBO(Y)=P(Y).The map f defined as f(a)=1, f(b)=2=f(c), f(d)=3 is strongly πgb-quotient.

Proposition 3.9:

(1) Every quotient map is a πgb-quotient map.
(2) Every π-quotient map is a πgb-quotient map.
(3) Every strongly πgb-quotient map is πgb-quotient.
(4) Every π^*-quotient map is πgb^*-quotient map.
(5) Every πgb^*-quotient map is strongly πgb-quotient.
(6) Every πgb^*-quotient map is πgb-quotient.
(7) Every $\hat{g}$ -quotient map is πgb-quotient.
(8) Every α-quotient map is πgb-quotient.
(9) Every αgs-quotient map is πgb-quotient.
(10) Every π -quotient map is αgs-quotient.
(11) Every π^*-quotient map is αgs-quotient.

Proof: Straight Forward.Converses of the above need not be true as seen in the following examples.

Example 3.10: Let X={a,b,c,d}, τ={Φ,X,{a},{a,b},{a,c,d}}, Y={1,2,3} and σ={Φ,Y,{1}}. We have πGBO(X)=P(X) and πGBO(Y)=P(Y).The map f is defined as f(a)=1, f(b)=3, f(c)=f(d)=2. The map f is πgb-quotient but not quotient. Since for the πgb-open set {1,2} in (Y,σ) f^{-1}({1,2})={a,c,d} is open in (X,τ) but {1,2} is not open in (Y,σ).

Example 3.11: Let X={a,b,c,d}, τ={Φ,X,{a},{d},{a,d},{c,d},{a,c,d}}, Y={1,2,3} and σ={Φ,Y, {1},{2},{1,2}{1,3}}. We have πGBO(X)={Φ,X,{a},{c},{d},{a,b},{a,c},{a,d},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}} and πGBO(Y)={Φ,Y,{1},{2},{3},{1,2},{1,3},{2,3}}. The map f is defined as f(a)=1, f(b)=f(d)=2, f(c)=3. The map f is πgb-quotient but not π-quotient.

Example 3.12: Let X={a,b,c,d}, τ={ Φ,X,{a},{a,b},{a,c,d}}, Y={1,2,3} and σ={ Φ,Y,{1}}. We have πGBO(X)=P(X) and πGBO(Y)=P(Y). The map f is defined as f(a)=1, f(b)=3, f(c)=f(d)=2. The map f is πgb-quotient but not strongly πgb-quotient because f^{-1}({2})={c,d} is πgb-open in (X,τ) but {2} is not open in (Y,σ).

Example 3.13: Let X={a,b,c,d}, τ={Φ,X,{b,c},{b,c,d},{a,b,c}}, Y={1,2,3} and σ={Φ,Y,{1},{2},{3},{1,2},{1,3},{2,3}}.We have πGBO(X)=P(X) and πGBO(Y)=P(Y).The map f is defined as f(a)=1=f(d), f(b)=2, f(c)=3.The map f is πgb^*-quotient but not π^*-quotient because f^{-1}({1})={a,d} is not π-open in (X,τ). Hence f is not π-irresolute.

Example 3.14: Let X={a,b,c,d}, τ={ Φ,X,{a},{d},{a,d},{c,d},{a,c,d}}, Y={1,2,3} and σ={ Φ,Y, {1}, {2}, {3}, {2,3}, {1,2}, {1,3}}. We have πGBO(X)= {Φ,X, {a}, {c}, {d}, {a,b}, {a,c}, {a,d}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}} and πGO(Y)=P(Y). The map f is defined as f(a)=f(b)=1, f(c)=2, f(d)=3. The map f is strongly πgb-quotient but not πgb^*-quotient because f^{-1}({2})={c} is not πgb-open in (X,τ) but {2} is πgb-open in (Y,σ).Hence not πgb-irresolute.This implies f is not πgb^*-quotient.

Example 3.15: Let X={a,b,c,d}, τ={ Φ,X,{a},{a,b},{a,c,d}}, Y={1,2,3} and σ={ Φ,Y,{1}}. We have πGBO(X)=P(X) and πGBO(Y)=P(Y).The map f is defined as f(a)=1, f(b)=3, f(c)=f(d)=2. The map f is πgb-quotient but not πgb^*-quotient because f^{-1}({2})={c,d} is πgb-open in (X,τ) but {2} is not open in (Y,σ).

Example 3.16: Let X={a,b,c,d}, τ={ Φ,X,{a},{a,b},{a,c,d}}, Y={1,2,3} and σ={ Φ,Y,{1}}. We have πGBO(X)=P(X) and πGBO(Y)=P(Y). The map f is defined as f(a)=1, f(b)=3, f(c)=f(d)=2. The map f is πgb-quotient but not ˆg-quotient because f^{-1}({1,2})={a,c,d} is ˆg-open in (X,τ) but {1,2} is not open in (Y,σ).

Example 3.17: Let X={a,b,c}, τ={Φ,X,{a},{b},{a,b}}, Y={1,2,3} and σ={ Φ,Y,{2},{2,3}}. We have πGBO(X)= {Φ,X,{a},{b},{a,b}{a,c},{b,c}}and πGBO(Y)=P(Y).The map f is defined as identity map. The map f is πgb-quotient but not α-quotient because f^{-1}({3})={c} is α-closed in (X,τ) but {3} is not closed in (Y,σ).

Example 3.18: Let X={a,b,c}, τ={Φ,X,{a},{a,b}}, Y={1,2,3} and σ={ Φ,Y,{3},{1,3}}. We have πGBO(X)= P(X) and πGBO(Y)=P(Y).The map f is defined as identity map. The map f is πgb-quotient but not αgs-quotient because f^{-1}({1,2})={a,b} is not αgs-closed in (X,τ) but {1,2} is closed in (Y,σ).Hence not αgs-continuous.

Example 3.19: Let X={a,b,c,d}, τ={ Φ,X,{a},{a,b},{a,c,d}}, Y={1,2,3} and σ={ Φ,Y,{1}}. We have αGO(X)={ Φ,X, {a}, {a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}} and αGO(Y)={ Φ,Y,{1},{1,2},{1,3}}. The map f is defined as f(a)=1, f(b)=3, f(c)=f(d)=2. The map f is αgs-quotient but not π-quotient. Since for the αgs-open set {1,2} in (Y,σ), f^{-1}({1,2})={a,c,d} is open in (X,τ) but {1,2} is not π-open in (Y,σ).

Example 3.20: Let X={a,b,c,d}, τ={ Φ,X,{a},{b},{a,b}}, Y={1,2,3} and σ={ Φ,Y,{1,2}}. We have αGO(X)={Φ,X, {a},{b},{a,b},{a,b,c},{a,b,d}} and αGO(Y)={ Φ,Y,{1},{2},{1,2}}. The map f is defined as f(a)=1, f(b)=2, f(c)=f(d)=3. The map f is αgs-quotient but not π^*-quotient because f^{-1}({1})={a} is π-open in (X,τ) but {1} is not open in (Y,σ).

Remark 3.21: The concepts of quotient maps and strongly πgb-quotient maps areindependent of each other.

Example3.22: Let X={a,b,c,d},τ={Φ,X,{a},{b,c},{a,b,c}},Y={1,2,3} and σ={ Φ,Y,{1},{2},{3},{1,2},{1,3),{2,3}}.We have πGBO(X)=P(X) and πGBO(Y)=P(Y).The map f defined as f(a)=1, f(b)=2=f(c), f(d)=3 is strongly πgb-quotient but not quotient because {3} is open in (Y,σ) but f^{1}({3}) ={d} is not open in (X,τ).

Example 3.23: Let X={a,b,c},τ={Φ,X,{a},{a,b}},Y={a,b} and σ={ Φ,Y,{a}}.We have πGBO(X)=P(X) and πGBO(Y)=P(Y).The map f defined as f(a)=f(b)=a, f(c)=b is quotient but not strongly πgb-quotient because f^{-1}({b}) ={c} is πgb-open in (X,τ) but {b} is not open in Y.

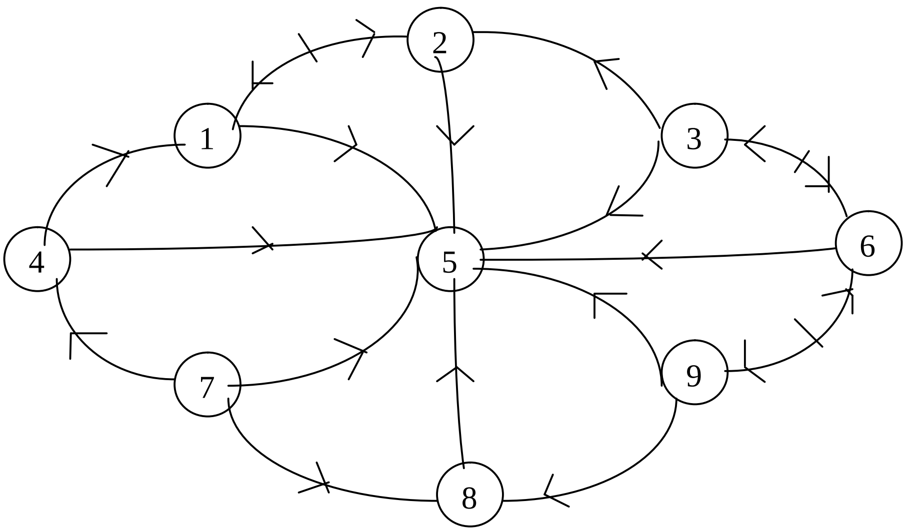
Remark 3.24 : The concepts of $\hat{g}$ -quotient and π-quotient maps are independent of each other.

Example3.25: Let X={a,b,c}, τ={Φ,X,{a},{b},{a,b}}, Y={1,2,3} and σ={Φ,Y,{1},{2},{1,2},{1,3}}.We have πGBO(X)={Φ,X,{a},{b},{a,b},{a,c},{b,c}}.The map f is defined as f(a)=1, f(b)=2, f(c)=3. The map f is $\hat{g}$ -quotient but not π-quotient because f is not π-continuous .

Example3.26: Let X={a,b,c}, τ={Φ,X,{a},{b},{a,b}}, Y={1,2,3} and σ={Φ,Y,{1},{3},{1,3}}.We have πGBO(X)={Φ,X,{a},{b},{a,b},{a,c},{b,c}}.The map f is defined as f(a)=1, f(b)=3, f(c)=2. The map f is π-quotient but not $\hat{g}$ -quotient because f^{-1}({2}) ={b} is open in X but {2} is not $\hat{g}$ open in Y .

Theorem 3.27: [7] The concepts of α-quotient maps and $\hat{g}$ -quotient maps are independent of each other.

The above discussions are summarized in the following diagram.



1.Stronglyπgb-quotient 2. quotient 3. α-quotient 4. πgb^*-quotient

5. πgb-quotient 6. $\hat{g}$ -quotient 7.π^*-quotient 8. αgs-quotient 9. π-quotient

Proposition 3.28: If a map f : (X,τ)→(Y,σ) is surjective, πgb-continuous and πgb-open,then f is a πgb-quotient map.

Proof: Let f^{-1}(V) be open in (X,τ). Then f(f^{-1}(V)) is a πgb-open set, since fis πgb-open. Hence V is a πgb-open set, as f is subjective, f(f^{-1}(V))=V. Thus, f is aπgb-quotient map.

Theorem 3.29: Every strongly πgb-quotient map is πgb-open.

Proof: Let f: (X,τ)→(Y,σ) be a strongly πgb-quotient map. Let V be an open set in (X,τ). Since every open set is πgb-open, V is πgb-open in (X,τ). That is f^{-1} (f(V)) is πgb-open in (X,τ). Since f is strongly πgb-quotient, f(V) is open and hence πgb-open in (Y,σ). This shows that f is a πgb-open.

Remark 3.30: The converse of Theorem 3.29 need not be true.

Example 3.31: Let X={a,b,c,d}, τ={ Φ,X,{a},{d},{a,d},{c,d},{a,c,d}}, Y={1,2,3} and σ={ Φ,Y,{1},{3},{1,2},{1,3}}.Wehave πGBO(X)={Φ,X,{a},{c},{d},{a,b},{a,c},{a,d},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}} and πGBO(Y)=P(Y). The map f defined as f(a)=1=f(c), f(b)=3, f(d)=2 is πgb-open but not strongly πgb-quotient, since for open set {3} in (Y, σ), f^{-1}({3})={b} is not πgb-open in (X,τ).

Theorem 3.32: Every strongly πgb-quotient map is M- πgb-open.

Proof: Let f : (X,τ)→(Y,σ) be a strongly πgb-quotient map. Let V be a πgb-open set in (X,τ). That is f^{-1}(f(V)) is πgb-open in (X,τ). Since f is strongly πgb-quotient, f(V) is open and hence πgb-open in (Y,σ). This shows that f is strongly πgb-open.

Remark 3.33: The converse of Theorem 3.32 need not be true.

Example 3.34: Let X={a,b,c}, τ={ Φ,X,{a},{b},{a,b}}, Y={1,2,3} and σ={ Φ,Y,{1,2}}.We have πGBO(X)={Φ,X,{a},{b},{a,b},{b,c},{a,c}} and πGBO(Y)=P(Y). The map f is defined as f(a)=1, f(b)=2, f(c)=3. The map f is M-πgb-open but not strongly πgb-quotient because f^{-1}({1})={a} is πgb-open in (X,τ) but {1} is not open in (Y,σ).

Proposition 3.35: Every πgb*-quotient map is πgb-irresolute.
Proof: It follows from Definition 3.3.
Remark 3.36: The converse of proposition 3.35 need not be true.
Example3.37: Let X={a,b,c,d}, τ={Φ,X,{a},{a,b},{a,c,d}}, Y={1,2,3} and σ={ Φ,Y,{1},{1,2}}. We have πGBO(X)=P(X) and πGBO(Y)=P(Y). The map f is defined as f(a)=1, f(b)=3, f(c)=f(d)=2. The map f is πgb-irresolute but not πgb^*-quotient because f^{-1}({1,3})={a,b}is πgb-open in (X,τ) but {1,3} is not open in (Y,σ).
Theorem 3.38: Every πgb^*-quotient map is M- πgb-open.
Proof: Let f : (X,τ)→(Y,σ) be a πgb^*-quotient map. Let V be an πgb-open set in (X,τ). Then f^{-1}(f(V)) is πgb-open in (X,τ). Since f is πgb^*-quotient, this implies that f(V) is open in (Y,σ) and thus πgb-open in (Y,σ). Hence f is strongly πgb-open.
Remark 3.39: The converse of Theorem 3.38 need not be true.
Example 3.40: Let X={a,b,c}, τ={ Φ,X,{a},{b},{a, b}}, Y={1,2,3} and σ={ Φ,Y,{1},{1,2}}. We haveπGBO(X)={Φ,X,{a},{b},{a,b}{a,c},{b,c}}andπGBO(Y)=P(Y)The map f is defined as f(a)=1, f(b)=2, f(c)=3. The map f is M-πgb-open but not πgb^*-quotient because f^{-1}({2})={b} is πgb-open in (X, τ) but {2} is not open in (Y,σ).

4.APPLICATIONS

Proposition 4.1 Let f : (X,τ)→(Y,σ) be an open surjectiveπgb-irresolute map andg : (Y,σ)→(Z,η) be a πgb-quotient map. Then their composition gof : (X,τ)→(Z,η)is a πgb-quotient map.
Proof: Let V be any open set in (Z,η). Then g^{-1}(V) is a πgb-open set in (Y,σ) sinceg is a πgb-continuous map. Since f is πgb-irresolute, $f^{-1}(g^{-1}(V))=(gof)^{-1}(V)$ is aπgb-open set in (X,τ). This implies $(gof)^{-1}(V)$ is a πgb-open set in (X,τ). This showsthat gof is a πgb-continuous map. Also, assume that $(gof)^{-1}(V)$ is open in (X,τ) forV⊆Z, that is, $(f^{-1}(g^{-1}(V)))$ is open in (X,τ). Since f is open $f(f^{-1}(g^{-1}(V)))$ is openin (Y,σ). It follows that g^{-1}(V) is open in (Y,σ), because f is surjective. Since gis a πgb-quotient map, V is a πgb-open set in (Z,η). Thus gof : (X,τ)→(Z,η) is aπgb-quotient map.
Proposition 4.2: If h : (X,τ)→(Y,σ) is a πgb-quotient map and g : (X,τ)→(Z,η)is a continuous map that is constant on each set h^{-1}(y), for y∈Y, then g induces aπgb-quotient map f : (Y,σ)→(Z,η) such that foh=g.
Proof: Since g is constant on h^{-1}(y), for each y∈Y, the set $g(h^{-1}(y))$ is a one pointset in (Z,η). If f(y) denote this point, then it is clear that f is well defined andfor each x∈X, f(h(x))=g(x). We claim that f is πgb-continuous. For if we let Vbe any open set in (Z,η), then g^{-1}(V) is an open set in (X,τ) as g is continuous.But $g^{-1}(V)=h^{-1}(f^{-1}(V))$ is open in (X,τ). Since h is πgb-quotient map, f^{-1}(V) is aπgb-open set in (Y,σ). Hence f is πgb-continuous.
Proposition 4.3: Let f : (X,τ)→(Y,σ) be an strongly πgb-open surjective and πgb-irresolutemap and g : (Y,σ)→(Z,η) be a strongly πgb-quotient map then gof :(X,τ)→(Z,η) is a strongly πgb-quotient map.
Proof: Let V be any open set in (Z,η). Then g^{-1}(V) is a πgb-open set in (Y,σ) (sinceg is strongly πgb-quotient). Since f is πgb-irresolute, $f^{-1}(g^{-1}(V))$ is a πgb-open setin (X,τ). Conversely, assume that $(gof)^{-1}(V)$ is a πgb-open set in (X,τ) for V⊆Z.Then $f^{-1}(g^{-1}(V))$ is a πgb-open in (X,τ). Since f is strongly πgb-open, $f(f^{-1}(g^{-1}(V)))$is a πgb-open set in (Y,σ). It follows that g−1(V) is a πgb-open set in (Y,σ). Thisgives that V is an open set in (Z,η) (since g is strongly πgb-quotient). Thus gof is astrongly πgb-quotient map.
Theorem 4.5: Let g : (X,τ)→(Y,σ) be a πgb-quotient map where (X,τ) and (Y,σ) are πgb-spaces. Then f : (Y,σ)→(Z,η) is a strongly πgb-continuous if and only ifthe composite map fog : (X,τ)→(Z,η) is strongly πgb-continuous.
Proof: Let f be strongly πgb-continuous and U be any πgb-open set in (Z,η).Thenf^{-1}(U) is open in (Y,σ). Then $(fog)^{-1}(U)=g^{-1}(f^{-1}(U))$ is πgb-open (X,τ). Since (X,τ) is a πgb-space, $g^{-1}(f^{-1}(U))$ is open in (X,τ). Thus the composite map is stronglyπgb-continuous. Conversely let the composite map fop be strongly πgb-continuous.Then for any πgb-open set U in (Z,η), $g^{-1}(f^{-1}(U))$ is open in (X,τ). Since g isa πgb-quotient map, it implies that f^{-1}(U) is πgb-open in (Y,σ). Since (Y,σ) is a_πgb-spaces, f^{-1}(U) is open in (Y,σ). Hence f is strongly πgb-continuous.
Theorem 4.6: Let f : (X,τ)→(Y,σ) be a surjective strongly πgb-open and πgb-irresolutemap and g : (Y,σ)→(Z,η) be a πgb^*-quotient map then gof is πgb^*-quotientmap.
Proof: Let V be πgb-open set in (Z,η). Then g^{-1}(V) is a πgb-open set in (Y,σ) becauseg is a πgb^*-quotient map. Since f is πgb-irresolute, $f^{-1}(g^{-1}(V))$ is a πgb-open set in(X,τ). Then gof is a πgb-irresolute. Suppose (gof)−1(V) is a πgb-open set in (X,τ)for a subset V⊆Z. That is $(f^{-1}(g^{-1}(V)))$ is πgb-open in (X,τ). Since f is stronglyπgb-open, $f(f^{-1}(g^{-1}(V)))$ is πgb-open set in (Y,σ). Thus g^{-1}(V) is πgb-open set in(Y,σ). Since g is a πgb^*-quotient map, V is an open set in (Z,η). Hence gof is aπgb^*-quotient map.
Proposition 4.7: Let f : (X,τ)→(Y,σ) be an strongly πgb-quotient, πgb-irresolutemap and g : (Y,σ)→(Z,η) be a πgb^*-quotient map then gof is πgb^*-quotient map.
Proof: Let V be any πgb-open set in (Z,η). Then g^{-1}(V) is a πgb-open set in (Y,σ)(Since g is πgb^*-quotient map). We have $f^{-1}(g^{-1}(V))$ is also πgb-open set in (X,τ)(Since f is πgb-irresolute). Thus, $(gof)^{-1}(V)$ is πgb-open set in (X,τ). Hence gof is πgb-irresolute.Let $(gof)^{-1}(V)$ is a πgb-open set in (X,τ) for V⊆Z. That is, $(f^{-1}(g^{-1}(V)))$is πgb-open in (X,τ). Then g^{-1}(V) is open set in (Y,σ) because f is a stronglyπgb-quotient map. This means that g^{-1}(V) is a πgb-open set in (Y,σ). Since g isπgb^*-quotient map, V is an open set in (Z,η). Thus gof is a πgb^*-quotient map.
Composition of two πgb-quotient maps need not be aπgb-quotient map as shown in the following example.
Example 5.8: Let X = {a,b,c,d}, τ = { Φ,{b},{c},{b,c},X}, σ ={ Φ,{a,b,d},X} η = { Φ,{a,d},X}. Define f: (X, τ) → (X, σ) by f(a) = a, f(b) = c, f(c) = b ,f(d) =d. Define g: (X, σ) → (X, η) by g(a)= d, g(b) = c, g(c) = b, g(d)=a. Then f and g are πgb-quotient maps but g o f is not πgb-quotient map.
Theorem 4.9: The composition of two πgb^*-quotient maps is πgb^*-quotient.

Proof: Let f : $(X,\tau)\rightarrow(Y,\sigma)$ and g : $(Y,\sigma)\rightarrow(Z,\eta)$ be two πgb^*-quotient maps. Let Vbe a πgb-open set in (Z,η). Since g is πgb^*-quotient, $g^{-1}(V)$ is πgb-open set in (Y,σ).Since f is πgb^*-quotient, $f^{-1}(g^{-1}(V))$ is πgb-open set in (X,τ). That is $(gof)^{-1}(V)$ isπgb-open set in (X,τ). Hence gof is πgb-irresolute. Let $(gof)^{-1}(V)$ be πgb-open in(X,τ). Then $f^{-1}(g^{-1}(V))$ is πgb-open set in (X,τ). Since f is πgb^*-quotient, $g^{-1}(V)$ isan open in (Y,σ). Then $g^{-1}(V)$ is a πgb-open set in (Y,σ). Since g is πgb^*-quotient,V is open set in (Z,η). Thus gof is πgb^*-quotient.

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