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RESEARCH ARTICLE

OSTROWSKI'S FOURTH ORDER METHOD SYSTEM OF NONLINEAR EQUATIONS

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ABSTRACT

In this paper, we present many new fourth -order optimal families of Ostrowski's method for computing zeros of system of nonlinear equations numerically. In this paper, we extending the idea of the proposed families of Ostrowski's method to system of nonlinear equations .It is proved that the above said families have fourth order of convergence. Several numerical examples are also given to illustrate the efficiency and the performance of the presented families.

INTRODUCTION

Due to the fact that systems of nonlinear equations arise frequently in science and engineering they have attracted researcher's interest. For example, nonlinear systems of equations, after the necessary processing step of implicit discretization, are solved by finding the solutions of systems of equations. We consider here the problem of finding a real zero, $x^* = (x^*_1, x^*_2 \ldots\ldots; x^*_n)^T$, of a system of non linear equations

$$f_1(x_1, x_2, \ldots\ldots\ldots, x_n) = 0;$$
$$f_2(x_1, x_2, \ldots\ldots\ldots, x_n) = 0;$$
::;
::;
$$f_n(x_1, x_2, \ldots\ldots\ldots, x_n) = 0;$$

This system can referred in vector form by

$$F(X) = 0 \qquad (1.1)$$

Where $F = (f_1, f_2, \ldots\ldots, f_n)^T$ and $X = (x_1, x_2, \ldots\ldots, x_n)^T$

Let the mapping $F: D \subseteq R^n \rightarrow R^n$ assumed to satisfy the assumptions (1.1). F(X) is continuously differentiable in an open neighborhood D of X*. There exists a solution vector X* of (1.1) in D such that $F(X^*) = 0$ and $F'(X^*) \neq 0$. Then the standard method for finding the solution to equation (1.1) is the classical Newton's method [2-5] given by

$$X^{k+1} = X^k - F(X^k)/F'(X^k), \quad k = 0, 1, 2, \ldots\ldots\ldots \qquad (1.2)$$

$$\begin{cases} y_n = x_n - \dfrac{f(x_n)}{f'(x_n)} \\ x_{n+1} = x_n \\ -\dfrac{1 f(x_n)}{2 f'(x_n)}\left[\dfrac{(b_4{}^2 + b_{4b_5} - b_5{}^2) f(x_n) f(y_n) - b_4(b_4 - b_5) f^2(x_n)}{(b_4 f(x_n) - b_5 f(y_n))((2b_4 - b_5) f(y_n) - (b_4 - b_5) f(x_n)}\right] \end{cases} \qquad (1.1)$$

[10] Now extend this idea for system of equations, we have

$$\begin{cases} y_n = x_n - \dfrac{f(x_n)}{f'(x_n)} \\ x_{n+1} = x_n \\ -\dfrac{F(X_n)}{2F'(X_n)}\left[\dfrac{[(b_4{}^2 + b_{4b_5} - b_5{}^2) F(Y_n) - b_4(b_4 - b_5) F(X_n)]^T}{(b_4 F(X_n) - b_5 F(Y_n))^T((2b_4 - b_5) F(Y_n) - (b_4 - b_5) F(X_n)}\right] F(X_n) \end{cases} \qquad (1.2)$$

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COVNERGENCE ANALYSES

We shall present the mathematical proof for the order of convergence of formula (1.2).

Lemma 1: *Let $D \subseteq R^n \to R^n$ be P-times Frechet differentiable in a convex set $D \subseteq R^n$ then for any $X, H \in R^n$, the following expression holds:*
$F(X+H) = F(X) + F'(X) H + 1/2!\ F''(X) H^2 + 1/3!\ F'''(X) H^3 + \ldots + 1/(p-1)! F^{p-1}(X) H^{p-1} + R_p$. (3.1)
Where

$$\|R_p\| \le 1/p! \sup_{0 \le t \le 1} \|Fp(X + tH)\| \|H\|^p,$$

and $Hp = (h, h, \ldots\ldots^{p}, \ldots, h)$.
Now we analyze the behavior of (1.2) through the following theorem:

Theorem 3.1. *Let $D \subseteq R^n \to R^n$ be four times Frecht differentiable in a convex set D containing the root r of $F(x) = 0$. Then, the sequence x_k, $k \le 0$ ($x^0 \in D$) obtained by using the iterative expression of method (2.10) converges to r with convergence order four if $b_4 \neq 0$ and $b_4 \neq b_5$.*
Proof: The Taylor's expansion (3.1) for F(x) about x^k is

$$F(x) = F(x^k) + F'(x^k)(x-x^k) + 1/2!\ F''(x^k)(x-x^k)^2 + 1/3!\ F'''(x^k)(x-x^k)^3 + 1/4!\ F^{iv}(x^k)(x-x^k)^4 + O(\|x-x^k\|^5). \quad (3.2)$$

Let $e^k = x^k - r$. Then, setting x = r and using F(r) = 0 in (3.2), we obtain

$$F(x^k) = F'(x^k)e^k - 1/2!\ F''(x^k)(e^k)^2 + 1/3!\ F'''(x^k)(e^k)^3 - 1/4!\ F^{iv}(x^k)(e^k)^4 + O(\|e^k\|^5). \quad (3.3)$$

Pre-multiplying by $F'(x^k)^{-1}$ to both sides of (3.3)

$$F'(x^k)^{-1}F(x^k) = e^k - 1/2!\ F'(x^k)^{-1}F''(x^k)(e^k)^2 + 1/3!\ F'(x^k)^{-1}F'''(x^k)(e^k)^3 - 1/4!\ F'(xk)^{-1} F^{iv}(x^k)(e^k)^4 + O(\|e^k\|^5). \quad (3.4)$$

Now from (2.10), we yields

$$y^k - x^k = -ek + 1/2!\ F'(x^k)^{-1}F''(x^k)(e^k)^2 - 1/6\ F'(x^k)^{-1}F'''(x^k)(e^k)^3 + (\|(e^k)\|)^4. \quad (3.5)$$

Also,

$$(y^k - x^k)^2 = (e^k)^2 - F'(x^k)^{-1}F''(x^k)(e^k)^3 + (\|(ek)\|)^4, \quad (3.6)$$
$$(y^k - x^k)^3 = -(e^k)^3 + (\|(e^k)\|)^4, \quad (3.7)$$
$$(y^k - x^k)^4 = (\|(e^k)\|)^4. \quad (3.8)$$

Here $e^m = (e, e, \ldots\ldots^{m}, \ldots, e)$, $e \in R^n$.

Taylor's expansion of $F(y^k)$ about x^k is.

$$F(y^k) = F(x^k) + F'(x^k)(y^k-x^k) + 1/2! F''(x^k)(y^k-x^k)^2 + 1/3!\ F'''(x^k)(y^k-x^k)^3 + O(\|y^k-x^k\|^4). \quad (3.9)$$

From (3.5)-(3.8) and (3.9), we obtain

$$F(y^k) = F(x^k) - F'(x^k)e^k + F''(x^k)(e^k)^2 - [1/3F'''(x^k) + 1/2F''(x^k)F'(x^k)^{-1}F''(x^k)(e^k)^3] + O(\|e^k\|^4). \quad (3.10)$$

Now from (2.10)

$$e^{k+1} = e^k - F'(x^k)^{-1}$$

$$F(x^k)\left[\frac{[(b_4{}^2 + b_4 b_5 - b_5{}^2)F(Y^k) - b_4(b_4-b_5)F(X^k)]^T}{(b_4F(X^k) - b_5F(Y^k))^T((2b_4-b_5)F(Y^k) - (b_4-b_5)F(X^k)}\right]F(x^k)$$

Let $R(x^k) = F'(x^k)^{-1} F(x^k))$

$[(b_4{}^2 + b_4b_5 - b_5{}^2)F(Y^k) - b_4(b_4 - b_5)F(X^k)]^T F(X^k)$ and $w(x^k) = (b_4F(X^k) - b_5F(Y^k))^T((2b_4 - b_5)F(Y^k) - (b_4 - b_5)F(X^k))$
$R(x^k) = (b_4(b_5 - b_4)(F'(x^k))^T F'(x^k)(e^k)^3 + [(3/2b_5{}^2 - 2b_4b_5 - b_4{}^2)(F''(x^k))^T F'(x^k) + 1/2\ (b4^2 - b_4b_5)(F'(x^k))^T F'(x^k)$
$+1/2b_4(b_4 - b_5)\ F'(x^k)^{-1}F''(x^k)(F'(x^k))^T F'(x^k)](e^k)^4 + O(\|e^k\|^5)$ $w(x^k) = b_4(b_5 - b_4)(F'(x^k))^T F'(x^k)(e^k)^2 + [1/2\ (3b_4 - 2b_5)b_4(F'(x^k))^T F''(x^k)$
$-(b_4 + b_5)(b_4 - b_5)(F''(x^k))^T F'(x^k)](e^k)^3 + O(\|e^k\|^4)$. (3.12)

From (3.11) and (3.12), we obtain

$$w(x^k)e^k - R(x^k) = [1/2\ (5b_4{}^2 - 3b_4b_5)(F'(x^k))^T F''(x^k) + (2b_4{}^2 - (5/2)b_5{}^2 + 2b_4b_5)(F''(x^k))^T F'(x^k) - 1/2(b_4{}^2 - b_4b_5)F'(x^k)^{-1}F''(x^k)(F'(x^k))^T F'(x^k)](e^k)^4 + O(\|e^k\|^5). \quad (3.13)$$

From(2.10) and (3.13), we have

$$w(x^k)\, e^{k+1} = w(x^k)\, e^k - R(x^k) = [1/2\ (5b_4{}^2 - 3b_4b_5)(F'(x^k))^T F''(x^k) + (2\ b_4{}^2 - 5/2b_5{}^2 + 2b_4b_5)\ (F''(x^k))^T F'(x^k) - 1/2(b_4{}^2 - b_4b_5)F'(x^k)^{-1}F''(x^k)(F'(x^k))^T F'(x^k)]\ (e^k)^4 + O(\|e^k\|^5). \quad (3.14)$$

From the above equation it is clear that (2.10) is fourth order convergence if $b_4 \neq 0$ and $b_4 \neq b_5$.

Special cases of formula (2.10):
(a) For b4 = 10 and $b_5 = 1$, family (2.10) read as:

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)} \\ x_{n+1} = x_n \\ -\frac{F(X_n)}{F'(X_n)}\left[\frac{[90F(X_n)-109F(Y_n)]^T}{(10F(X_n)-F(Y_n))^T(-9F(X_n)+19F(Y_n))}\right]F(X_n) \end{cases} \qquad (3.15)$$

This is a new fourth-order method and satisfies the following error equation

$w(x^k)\, e^{k+1} = [235\ (F'(x^k))^T F''(x^k) + (435/2)\,(F''(x^k))^T\ \ F'(x^k) - 45F'(x^k)^{-1}F''(x^k)(F'(x^k))^T F'(x^k)]\,(e^k)^4 + O(\|e^k\|^5)$.

(b) For b4 = 1 and b5 = -1 , family (2.10) read as:

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)} \\ x_{n+1} = x_n \\ -\frac{F(X_n)}{F'(X_n)}\left[\frac{[2F(X_n)+F(Y_n)]^T}{(F(X_n)+F(Y_n))^T(2F(X_n)-3F(Y_n))}\right]F(X_n) \end{cases} \qquad (3.16)$$

This is a new fourth-order method and satisfies the following error equation

$w(x^k)\, e^{k+1} = [4\ (F'(x^k))^T F''(x^k) + (5/2)\, F''(x^k))^T\ \ F'(x^k) - 2F'(x^k)^{-1}F''(x^k)(F'(x^k))^T F'(x^k)]\,(e^k)^4 + O(\|e^k\|^5)$.

NUMERICAL RESULTS

In this section, we shall check the performance of the present formula b1(3:15) and b2(3:16) the comparison is carried out with Newton's method and with HM and CM [8]. A mat lab program has been written to implement these methods. We use the following stopping criteria for computer programs:

1. $\epsilon = e^{-10}$.
2. $|F(X_n)| < \epsilon$

For every method, we analyze the number of iterations needed to converge to the required solution. The numerical results are reported in the Table 1.

Table 1. Numerical results of problems (a) to (g) using different methods

F(X)	X	NM	HM	CM	b1	b2
(a)	$(3,2)^T$	10	7	6	3	3
	$(1.6, 0)^T$	9	7	6	3	3
(b)	$(.7,1.3)^T$	9	6	5	3	3
	$(-1,-2)^T$	9	6	5	3	3
(c)	$(0.91, -2)^T$	9	7	6	3	3
	$(1.8, -2.1)^T$	9	6	5	3	3
(d)	$(.7, .9)^T$	9	6	5	4	4
	$(-0.1, 0.2)^T$	9	6	6	3	3
(e)	$(.99, .1)^T$	9	7	6	4	4
	$(1.9, 1.4)^T$	9	7	5	4	4
(f)	$(1.5, 2)^T$	10	7	6	3	3
	$(.3, .5)^T$	9	6	5	3	3
(g)	$(.2, .6,1.5)^T$	9	6	5	4	4
	$(3, .5, 2)^T$	9	6	5	3	3

We consider the following problems for a system of nonlinear equations.

Problem (a)

$$x_1^2 - 2x_1 - x_2 + 0.5 = 0$$
$$x_1^2 + 4x_2^2 - 4 = 0$$

Problem (b)

$$x_1^2 + x_2^2 - 1 = 0$$
$$x_1^2 - x_2^2 + 0.5 = 0$$

Problem (c

$$x_1^2 - x_2^2 + 3\log(x_1) = 0$$
$$2x_1^2 - x_1 x_2 - 5x_1 + 1 = 0$$

Problem (d)

$$e^{x1} + x_1 x_2 - x_2 - 0.5 = 0$$
$$\sin(x_1 x_2) + x_1 + x_2 - 1 = 0$$

Problem (e)

$$x_1 + 2x_2 - 3 = 0$$
$$2x_1^2 + x_2^2 - 5 = 0$$

Problem (f)

$$x_1 + e^{x}{}_2 - \cos(x_2) = 0$$
$$3x_1 - x_2 - \sin(x_2) = 0$$

Problem (g)

$$x_1^2 + x_2^2 + x_3^2 - 9 = 0$$
$$x_1 x_2 x_3 - 1 = 0$$
$$x_1 + x_2 - x_3^2 =$$ Solution (a)

$r = (1.9006767263670658, 0.31121856541929427)^T$ Solution (b)

$r = (0.5000000000000000, 0.8660254378443865)^T$ $r = (-0.5000000000000000, -0.8660254378443865)^T$ Solution (c)

$r = (1.3192058033298924,-1.6035565551874148)^T$ Solution (d)

$r = (0, 1)^T$ Solution (e)

$r = (1.4880338717125849, 0.75598306414370757)^T$ Solution (f) $r = (0, 0)^T$ Solution (g)

$r = (2.2242448288477843, 0.28388497407293814, 1.58370776128252723)^T$

$r = (0.28388497407293814, 2.2242448288477843, 1.58370776128252723)^T$

Conclusions

The presented formula (3.15),(3.16) and (3.17) is simple to understand, easy to program and has the fourth order of convergence. We contribute to the development of iteration processes and propose several families of Ostrowski's method. We now obtain a wide general class of Ostrowski's families which are without memory and have the same scaling factor of function as that Ostrowski's method. Numerical tests have been performed,which not only illustrate the method practically but also serve to check the validity of theoretical results we have derived. The performance is compared with Newton method, CM [8] and HM [8].

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