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RESEARCH ARTICLE

R-NORMAL ELASTIC ON NON-NULL SURFACE IN MINKOWSKI 3-SPACE

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ABSTRACT

In this paper, we derive intrinsic equations for r-normal elastic on non-null surface in Minkowski 3-space.

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INTRODUCTION

Elastic curve problem have long research history. The mathematical theories of elastic curves go back to Bernoilli and Euler (Goldstine, 1980). Elastic problem has been reinvestigated with different ways during the last three decades (Langer and Singer, 1984), (Gürbüz, 2007), (Barros and Garay, 2012). Barros and Garay derived critical points of the total normal curvature functional $\int_0^l ©_n^r ds$ in 3 dimensional space forms. This problem have a lot applications. For example, the case r=2 is used in the self assembly analysis of thin films formed by block copolymers in a cylindrical phase (Santangelo, Vitalli, Kamien and Nelson, 2007). In this work, we obtain the critical points for the generalized total normal curvature on non-null surface in Minkowski 3-space. Our aim is to minimize the energy

$$\int_0^l ©_n^2 ds \qquad (1.1)$$

and to find equilibrium equations.

Let $\sim$ denote an admissible curve on a connected oriented surface M in Minkowski 3-space. Apart from the Frenet frame {t,n,b}, there also exist a second frame {T,Q,N} at every point of the curve $\sim$. $T(s) = \sim '(s)$ denote the unit tangent vector, N is the unit normal to M and $Q = £N \times T, £ = \pm 1$ and s is arc length.

The analogue of the Frenet-Serret formulas is

$$\tilde{\nabla}_T T = \nabla_T T + £_3 \langle ST, T \rangle N = £_2 ©_g Q + £_3 ©_n Q$$
$$\tilde{\nabla}_Q Q = \nabla_Q Q + £_3 \langle ST, Q \rangle N = -£_1 ©_g T + £_3 {}^2{}_g N$$

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where $ST = -\tilde{\nabla}_T N = £_1 ©_n T + £_2 ²_g Q$, $©_g$ is the geodesic curvature, $²_g$ is the geodesic torsion, $©_n$ is the normal curvature, $\tilde{\nabla}$ is Lorentzian Levi-Civita , ∇ is Riemannian Levi-Civita, S is Weingarten map. Also

$$\langle T,T\rangle = £_1, \langle Q,Q\rangle = £_2, \langle N,N\rangle = £_3 .$$

Intrinsic Method

Let $\cdot\ (w,t):(-¢,¢)\times[0,L]\to M$ be a variation of non-null (space like or time like) curve $\sim$. $\cdot\ (0,t) = \frac{\partial \cdot}{\partial w}(0,t)$. The velocity vector field of $\sim$ non-null curve. $V(w,t) = \frac{\partial \cdot}{\partial t}(w,t)$. $v = \left|\langle V,V\rangle\right|^{1/2}$ is speed of $\sim$. Non-null admissible curve is critical point if and only if

$$\frac{\partial}{\partial w}\int_0^{L(w)} ©_n^r ds\Big|_{w=0} = 0$$

Arc α is called r-normal elastic line if it is an extremal for the variational problem of minimizing the value of (1.1) within the family of all arcs of length L on a non-null surface in Minkowski 3-space.

$$\begin{aligned}
\frac{\partial}{\partial w}\int_0^{L(w)} ©_n^r ds\Big|_{w=0} &= \int_0^{L(w)} [W(©_n^r)v + W(v)©_n^r]dt \\
&= \int_0^L £_3 r ©_n^{r-1}\langle(\nabla_T S)T, W\rangle ds + 2£_3 r\int_0^L ©_n^{r-1}\langle\nabla_T W, ST\rangle ds + £_1 r\int_0^L ©_n^r\langle\nabla_T W, ST\rangle ds \\
&= \int_0^L £_3 r ©_n^{r-1}\langle(\nabla_T S)T, W\rangle ds - 2£_3 r\int_0^L \left\langle \frac{d}{ds}(©_n^{r-1}²_g)Q, W\right\rangle ds + 2£_1 £_3\int_0^L \langle(©_n^{r-1}²_g ©_g)T, W\rangle ds \qquad (2.1) \\
&\quad - £_1\int_0^L \left\langle \frac{d}{ds}(©_n^r)T, W\right\rangle ds - £_1 £_2\int_0^L \langle ©_g ©_n^r Q, W\rangle ds + 2£_3\langle r©_n^{r-1}²_g Q, W\rangle\Big|_0^L + £_1\langle ©_n^r T, W\rangle\Big|_0^L \\
&= \int_0^L \langle E(\sim), W\rangle ds + U(\sim, W)
\end{aligned}$$

where W(0,0)=W(0,L)=0, $\nabla_T W(0,0) = \nabla_T W(0,L) = 0$. L(w) is the arc length of $\sim$. Also,

$$W(©_n^r) = £_3 r ©_n^{r-1}\{\langle(\nabla_W S)T, T\rangle + 2\langle\nabla_W T, ST\rangle\}$$

And

$$\begin{aligned}
(\nabla_T S)T &= (£_1\frac{\partial ©_n^r}{\partial s} - £_1 £_2 ©_g ²_g - ©_g ²_g)T \\
&\quad + (£_1 £_2 ©_g ©_n + £_2\frac{\partial ²_g}{\partial s} - £_2 ©_g(2H - £_1 ©_n)Q
\end{aligned} \qquad (2.2)$$

Here H is mean curvature and

$$S = \begin{bmatrix} £_1 ©_n & £_2 ²_g \\ £_2 ²_g & 2H - £_1 ©_n \end{bmatrix} \qquad (2.3)$$

is Weingarten map non-null curves. Thus Euler- Lagrange equations is given by

$$E(\sim) = £_3 r ©_n^{r-1} (\nabla_T S)T - 2£_3 r \frac{d}{ds}(©_n^{r-1} 2_g)Q +$$
$$2£_1 £_3 (©_n^{r-1} 2_g ©_g)T - £_1 r \frac{d}{ds}(©_n^r)T - £_1 £_2 ©_g ©_n^r Q \qquad (2.4)$$

$$U(\sim, W) = 2£_3 \left\langle r ©_n^{r-1} 2_g Q, W \right\rangle \Big|_0^L + £_1 \left\langle ©_n^r T, W \right\rangle \Big|_0^L$$

Thus from (2.2), ((2.3) and (2.4), we have

$$E(\sim) = [£_1 £_3 r ©_n^{r-1} \frac{\partial ©_n^r}{\partial s} - £_1 £_2 £_3 r ©_n^{r-1} ©_g 2_g - £_3 r ©_n^{r-1} ©_g 2_g + 2£_1 £_3 ©_n^{r-1} ©_g 2_g - £_1 r \frac{d}{ds}(©_n^r)]T$$
$$+ [2£_1 £_2 £_3 r ©_g ©_n^r + £_2 £_3 r ©_n^{r-1} \frac{\partial 2_g}{\partial s} \qquad (2.5)$$
$$- 2£_2 £_3 r ©_g ©_n^{r-1} H - 2£_3 r \frac{\partial ©_n^{r-1}}{\partial s} 2_g - 2£_3 r ©_n^{r-1} \frac{\partial 2_g}{\partial s} - £_1 £_2 ©_n^r ©_g]Q$$

And

$$U(\sim, W) = 2£_3 \left\langle r ©_n^{r-1} 2_g Q, W \right\rangle \Big|_0^L + £_1 \left\langle ©_n^r T, W \right\rangle \Big|_0^L = 0 \qquad (2.6)$$

Theorem 2.1.

An non-null admissible curve is r-normal elastic if and only if,

$$£_1 £_3 r ©_n^{r-1} \frac{\partial ©_n^r}{\partial s} - £_1 £_2 £_3 r ©_n^{r-1} ©_g 2_g - £_3 r ©_n^{r-1} ©_g 2_g + 2£_1 £_3 ©_n^{r-1} ©_g 2_g - £_1 r \frac{d}{ds}(©_n^r) = 0$$

$$2£_1 £_2 £_3 r ©_g ©_n^r + £_2 £_3 r ©_n^{r-1} \frac{\partial 2_g}{\partial s} - 2£_2 £_3 r ©_g ©_n^{r-1} H - 2£_3 r \frac{\partial ©_n^{r-1}}{\partial s} 2_g - 2£_3 r ©_n^{r-1} \frac{\partial 2_g}{\partial s} - £_1 £_2 ©_n^r ©_g = 0$$

Corallary 2.1.

An non-null admissible geodesic are is r-normal elastic if and only if it satisfies the following equations

$$£_2 £_3 r ©_n^{r-1} \frac{\partial 2_g}{\partial s} - 2£_3 r \frac{\partial ©_n^{r-1}}{\partial s} 2_g - 2£_3 r ©_n^{r-1} \frac{\partial 2_g}{\partial s} = 0$$

Corallary 2.2.

An admissible non-null geodesic arc on non-null surface is r-normal elastic if and only if , it satisfies

$$©_n^{2(r-1)})2_g = 0$$

Proof. If surface is space like,

$£_1 = 1, \quad £_2 = 1, \quad £_3 = -1$. Thus we get

$$r\copyright_n^{r-1}\frac{\partial 2_g}{\partial s}+2r\frac{\partial \copyright_n^{r-1}}{\partial s}2_g=0 \qquad (2.7)$$

If surface is time like,

$\pounds_1=-1,\quad \pounds_2=1,\quad \pounds_3=1$. Thus we get

$$-r\copyright_n^{r-1}\frac{\partial 2_g}{\partial s}-2r\frac{\partial \copyright_n^{r-1}}{\partial s}2_g=0 \qquad (2.8)$$

From (2.5) and (2.6), the first integral

$\copyright_n^{2(r-1)}2_g$ =constant

From (2.5) and (2.6), constant must have zero.

Theorem 2.2.

An admissible curve on pseudo-sphere $S_1^2(1)$ is is r-normal elastic if and only if if it lies on a geodesic.

Proof.

For pseudo-sphere, geodesic torsion vanishes. Normal curvature $\copyright_n=-1$. From theorem 2.1, we obtain $\copyright_g=0$. Conversely any admissible geodesic arc on $S_1^2(1)$ satisfies theorem 2.1.

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