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International Journal of Current Research
Vol. 5, Issue, 12, pp.3930-3936, December, 2013

INTERNATIONAL JOURNAL
OF CURRENT RESEARCH

ISSN: 0975-833X

RESEARCH ARTICLE

COMBINED EFFECT OF COSINUSOIDALLY FLUCTUATING TEMPERATURE AND CONCENTRATION FLOW THROUGH POROUS MEDIUM

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ARTICLE INFO

Article History:
Received 27th September, 2013
Received in revised form
26th October, 2013
Accepted 06th November, 2013
Published online 25th December, 2013

Key words:
Cosinusoidally Fluctuating Temperature and Concentration,
Skin-friction,
Heat Transfer,
Natural Convection Flow

ABSTRACT

The purpose of this work is to study a laminar free convection flow of viscous incompressible fluid through a saturated porous medium bounded by an infinite vertical permeable plate in the presence of cosinusoidally fluctuating temperature and concentration. Assuming constant suction at the porous plate, the approximate solution for velocity, temperature and concentration are obtained analytically using regular perturbation techniques. The effects of governing parameters on flow characteristic are discussed graphically.

INTRODUCTION

Flows of fluid through porous medium with the combined effects of thermal and mass diffusion are of principal interest. Such flows have attracted the attention of a number of scholar due to their applications in many branches of science and engineering, viz. in a cooling device, in solar energy collector, in the early stages of melting adjacent to a heated surface, in chemical engineering processes which are classified as mass transfer processes, in drying and moistening processes as the mass movement through a porous material, in the study of the structure of stars and planets. The flow through porous medium, a series of investigations have been made by Raptis *et al*. (1981, 1981, 1982) into the steady flow past a vertical wall. Ramanaiah *et al*. (1991) studied free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient. The free convection effects on the flow past a porous medium bounded by a vertical infinite surface with constant suction and constant heat flux II is studied by Sharma (1992). Somers (1956), Wilcox (1961) and Gill *et al*. (1965) studied the effects of mass transfer on free convection. The combined effect of buoyancy forces from thermal and mass diffusion on forced convection was studied by Chen *et al*. (1980). Hossain (1992) also investigated the effect of the transpiration along with the combined effect of buoyancy forces from thermal and mass diffusion on forced convective heat and mass transfer from a vertical plate. Gersten and Gross (1974) has been studied flow and heat transfer along a plane wall with periodic suction velocity. Effects of such a suction velocity on a various flows and heat transfer problem along flat and vertical porous plates have also been studied extensively by Singh *et al*. (1978). Siegel (1958) investigated the transient free convection from a vertical flat plate. Kelleher *et al*. (1968) also studied the heat transfer response of laminar free convection boundary layers along vertical heated plates to surface-temperature oscillations.

The natural convection flows adjacent to both vertical and horizontal surface, which result from the combined buoyancy effects of thermal and mass diffusion, was investigated by Gebhart and Pera (1971) and Pera and Gebhart (1972). In case of unsteady free convective flows Soundalgekar (1972) studied the effects of viscous dissipation on the flow past an infinite vertical porous plate. It was assumed that the plate temperature oscillates in such a way that its amplitude is small. Also, the free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient is studied by Hossain *et al*. (2001) studied the influence of fluctuating surface temperature and concentration on natural convection flow from a vertical flat plate. Sharma (2005) also studied Simultaneous thermal and mass diffusion on three dimensional mixed convection flows through a porous medium. Recently the effects of fluctuating surface temperature and concentration on unsteady convection flow past an infinite

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vertical plate with constant suction has been investigated by Sharma *et al*. (2009). Therefore the object of present paper is to investigate the effect of permeability with the combined effect of cosinusoidally fluctuating temperature and concentration on flow of viscous incompressible fluid past vertical flat porous plate with constant suction at the plate. The approximate solutions is obtained for various values of flow parameters, viz: Prandtl number (Pr = 0.71) for air, the Schmidt number Sc = 0.60 (for Carbon-dioxide, CO_2), Sc = 0.78 (for Ammonia, NH_3), Gr (Grashof number), Gc (modified Grashof based on concentration), ω (frequency of oscillation), Re (Reynold number) and k (permeability parameter) are selected arbitrarily.

Formulation of the problem

We consider the flow of viscous incompressible fluid through saturated porous medium past an infinite vertical, porous flat plate lying on x^*-z^* plane. The x^* - axis is oriented in the direction of the buoyancy force and y^*-axis is taken perpendicular to the plate. Let (u^*, v^*, w^*) be the components of velocity in the (x^*, y^*, z^*) directions respectively. The plate is being considered infinite in x^*-direction; hence all physical quantities will be independent of x^*. Further, since the plate is subjected to the constant suction at the plate i.e. $v^* = -V$, thus w^* is independent of z^* and so we assume $w^* = 0$ throughout.
The temperature and concentration of the plate is considered to vary cosinusoidally fluctuating with time and assumed to be of the form

$$T^*_w(z^*, t^*) = T^*_0 + \varepsilon(T^*_0 - T^*_\infty)\cos\left(\frac{\pi z^*}{\ell} - \omega^* t^*\right) \quad (1)$$

$$C^*_w(z^*, t^*) = C^*_0 + \varepsilon(C^*_0 - C^*_\infty)\cos\left(\frac{\pi z^*}{\ell} - \omega^* t^*\right) \quad (2)$$

where T^*_0, C^*_0 T^*_∞, C^*_∞, T^*_w and C^*_w are the mean temperature, mean concentration, ambient temperature, ambient concentration wall temperature of the plate and wall concentration at the plate respectively, ω^* is the frequency of oscillation, t^* is the time, ℓ is the wave length and ε is a small parameter i.e., $\varepsilon << 1$. Using the Boussiness and boundary layer approximation, the governing equation for this problem can be written as follows:

$$\frac{\partial v^*}{\partial y^*} = 0 \Rightarrow v^* = -V, \quad V > 0 \quad (3)$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = \nu\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) + g\beta(T^* - T^*_\infty) + g\beta_c(C^* - C^*_\infty) - \frac{\nu u^*}{k^*} \quad (4)$$

$$\rho C_p\left(\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = \kappa\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) \quad (5)$$

$$\frac{\partial C^*}{\partial t^*} + v^*\frac{\partial C^*}{\partial y^*} = D\left(\frac{\partial^2 C^*}{\partial y^{*2}} + \frac{\partial^2 C^*}{\partial z^{*2}}\right) \quad (6)$$

where g, T^* , C^* D, k^*, κ, C_p, ν, β, β_c, ρ are acceleration due to gravity, fluid temperature, species concentration, chemical molecular diffusivity, permeability parameter, thermal conductivity, specific heat at constant pressure, kinematic viscosity, coefficient of volume expansion for heat transfer, volumetric coefficient of expansion with species concentration and density. The boundary conditions of the problem are:

$$\left.\begin{array}{ll} \left.\begin{array}{l} u^* = 0,\ T^* = T^*_0 + \varepsilon(T^*_0 - T^*_\infty)\cos\left(\frac{\pi z^*}{\ell} - \omega^* t^*\right), \\ C^* = C^*_0 + \varepsilon(C^*_0 - C^*_\infty)\cos\left(\frac{\pi z^*}{\ell} - \omega^* t^*\right) \end{array}\right\} & \text{at} \quad y = 0 \\ u^* = 0,\ T^* = T^*_\infty,\ C^* = C^*_\infty & \text{as} \quad y \to \infty \end{array}\right\} \quad (7)$$

Introducing the following non-dimensional parameters

$$y = \frac{y^*}{\ell}, \quad z = \frac{z^*}{\ell}, \quad u = \frac{u^*}{V},$$

$$\theta = \frac{T^* - T^*_\infty}{T^*_0 - T^*_\infty} \ (\text{Dimensionless temperature}),$$

$$C = \frac{C^* - C^*_\infty}{C^*_0 - C^*_\infty} \ (\text{Dimensionless concentration}),$$

$$\left[t = \omega^* t^* \ (\textit{Dimensionless time}), \quad \omega = \frac{\omega^* \ell^2}{\nu} (\textit{Dimensionless frequency of oscillation})\right]$$

$$k = \frac{k^*}{\ell^2} \ (\text{Permeability parameter}),$$

$$Pr = \frac{\mu C_P}{\kappa} \ (\text{Prandtl Number}),$$

$$Re = \frac{V\ell}{\nu} \ (\text{Reynold Number}),$$

$$Sc = \frac{\nu}{D} \ (\text{Schmidt Number}),$$

$$Gr = \frac{g\beta\nu(T^*_0 - T^*_\infty)}{V^3} \ (\text{Grashoff Number},$$

$$Gc = \frac{g\beta_c\nu(C^*_0 - C^*_\infty)}{V^3} \ (\text{Modified Grashoff Number}),$$

into the equations (4), (5) and (6), we get

$$\omega\frac{\partial u}{\partial t} - Re\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + Re^2\, Gr\,\theta + Re^2\, Gc\, C - \frac{u}{k} \quad (8)$$

$$\omega\frac{\partial \theta}{\partial t} - Re\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) \quad (9)$$

$$\omega\frac{\partial C}{\partial t} - Re\frac{\partial C}{\partial y} = \frac{1}{Sc}\left(\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) \quad (10)$$

The boundary conditions (7) reduces to

$$\left.\begin{array}{lll} u = 0,\ \theta = 1 + \varepsilon\cos(\pi z - t),\ C = 1 + \varepsilon\cos(\pi z - t) & & \text{at} \quad y = 0 \\ u = 0,\ \theta = 0, & C = 0 & \text{as} \quad y \to \infty \end{array}\right\} \quad (11)$$

Solution of the problem

Since the amplitudes of the temperature and concentration variation $\varepsilon << 1$ is very small, we now assume the solutions of the following form:

$$\left.\begin{aligned} u(y,z,t) &= u_0(y) + \varepsilon\, u_1(y)\, e^{\iota(\pi z - t)} + O(\varepsilon^2)....... \\ \theta(y,z,t) &= \theta_0(y) + \varepsilon\, \theta_1(y)\, e^{\iota(\pi z - t)} + O(\varepsilon^2).......... \\ C(y,z,t) &= C_0(y) + \varepsilon\, C_1(y)\, e^{\iota(\pi z - t)} + O(\varepsilon^2).......... \end{aligned}\right\} \quad (12)$$

substituting (12) in (8), (9) and (10), equating the coefficient of harmonic and non harmonic terms, neglecting the coefficient of ε^2, we get

$$\frac{\partial^2 u_0}{\partial y^2} + \text{Re}\,\frac{\partial u_0}{\partial y} - \frac{u_0}{k} = -\text{Re}^2\,(\text{Gr}\,\theta_0 + \text{Gc}\,C_0) \quad (13)$$

$$\frac{\partial^2 u_1}{\partial y^2} + \text{Re}\,\frac{\partial u_1}{\partial y} + (\iota\omega - \pi^2 - \frac{1}{k})\,u_1 = -\text{Re}^2\,(\text{Gr}\,\theta_1 + \text{Gc}\,C_1) \quad (14)$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + \text{Re}\,\text{Pr}\,\frac{\partial \theta_0}{\partial y} = 0 \quad (15)$$

$$\frac{\partial^2 \theta_1}{\partial y^2} + \text{Re}\,\text{Pr}\,\frac{\partial \theta_1}{\partial y} + (\iota\omega\,\text{Pr} - \pi^2)\,\theta_1 = 0 \quad (16)$$

$$\frac{\partial^2 C_0}{\partial y^2} + \text{Re}\,\text{Sc}\,\frac{\partial C_0}{\partial y} = 0 \quad (17)$$

$$\frac{\partial^2 C_1}{\partial y^2} + \text{Re}\,\text{Sc}\,\frac{\partial C_1}{\partial y} + (\iota\omega\,\text{Sc} - \pi^2)\,C_1 = 0 \quad (18)$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &u_0 = 0,\; u_1 = 0,\; \theta_0 = 1,\; \theta_1 = 1,\; C_0 = 1,\; C_1 = 1, && \text{at } y = 0 \\ &u_0 = 0,\; u_1 = 0,\; \theta_0 = 0,\; \theta_1 = 0,\; C_0 = 0,\; C_1 = 0 && \text{as } y \to \infty \end{aligned}\right\} (19)$$

Solving equations (13) to (18) under the corresponding boundary conditions (19) we get:

$$\theta_0(y) = e^{-\text{Re}\,\text{Pr}\,y}, \quad (20)$$

$$C_0(y) = e^{-\text{Re}\,\text{Sc}\,y}, \quad (21)$$

$$\theta_1(y) = e^{-a_1 y}, \quad (22)$$

$$C_1(y) = e^{-a_2 y}, \quad (23)$$

$$u_0(y) = \frac{\text{Gr}}{\text{Pr}^2 - \text{Pr} - \frac{1}{k}}\left(e^{-b_1 y} - e^{-\text{Re}\,\text{Pr}\,y}\right) + \frac{\text{Gc}}{\text{Sc}^2 - \text{Sc} - \frac{1}{k}}\left(e^{-b_1 y} - e^{-\text{Re}\,\text{Sc}\,y}\right) \quad (24)$$

$$u_1(y) = a_4\left(e^{-a_1 y} - e^{-a_3 y}\right) + a_5\left(e^{-a_2 y} - e^{-a_3 y}\right) \quad (25)$$

Where

$$a_1 = \frac{1}{2}\left[\text{Re}\,\text{Pr} + \sqrt{\text{Re}^2\,\text{Pr}^2 + 4\pi^2 - 4\iota\omega\,\text{Pr}}\right],$$

$$a_2 = \frac{1}{2}\left[\text{Re}\,\text{Sc} + \sqrt{\text{Re}^2\,\text{Sc}^2 + 4\pi^2 - 4\iota\omega\,\text{Sc}}\right],$$

$$b_1 = \frac{1}{2}\left[\text{Re} + \sqrt{\text{Re}^2 + \frac{4}{k}}\right],$$

$$a_3 = \frac{1}{2}\left[\text{Re} + \sqrt{\text{Re}^2 + 4(\pi^2 + \frac{1}{k}) - 4\iota\omega}\right],$$

$$a_4 = -\frac{\text{Re}^2\,\text{Gr}}{a_1^2 - \text{Re}\,a_1 + \iota\omega - \pi^2 - \frac{1}{k}}, \quad a_5 = -\frac{\text{Re}^2\,\text{Gc}}{a_2^2 - \text{Re}\,a_2 + \iota\omega - \pi^2 - \frac{1}{k}}$$

The solution obtained in equations (20) to (25) are in complex variable notations and only the real part of it will have the physical significance.

RESULTS AND DISCUSSION

In order to point out the effect of various parameters on the velocity, temperature and concentration, when the plate is subjected to cosinusoidally fluctuating temperature and concentration with time, the following discussion are set out. Numerical calculations are carried out for the different values of Sc =0.60 (CO_2), Sc = 0.78 (NH_3), in air (Pr = 0.71). The values of Gr, Gc, Re, k and ω are selected arbitrarily.

Mean flow

The mean velocity is given by equation (24). This velocity component is presented in Fig.1. It is observed that mean velocity decreases with increasing Re, while it increases with increasing Gr, Gc and k (permeability parameter). The mean velocity increases near the plate and attains maximum value then it decreases far away from plate. The values of mean velocity is higher for Sc=0.60 (CO_2) than that of Sc = 0.78 (NH_3). The mean temperature and concentration profiles are given in Fig. 2. It is found that mean temperature and mean concentration decrease exponentially. They decrease with increasing Re. The value of mean concentration is higher for CO_2 than NH_3. After knowing the velocity field we can now obtain the dimensionless shear stress in terms of skin-friction at the plate. It is given by

$$\tau^* = \mu\left(\frac{d u^*}{d y^*}\right)_{y^* = 0} \quad (26)$$

and in non-dimensional it is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left(\frac{\partial u_0}{\partial y}\right)_{y=0} + \varepsilon\left(\frac{\partial u_1}{\partial y}\right)_{y=0} e^{\iota(\pi z - t)} \quad (27)$$

Denoting the mean skin-friction by

$$\tau_m = \left(\frac{\partial u_0}{\partial y}\right)_{y=0} \quad (28)$$

Substituting equation (24) in equation (28), we have

$$\tau_m = \frac{Gr}{Pr^2 - Pr - \frac{1}{k}} (Re\,Pr - Re) + \frac{Gr}{Pr^2 - Pr - \frac{1}{k}} (Re\,Sc - Re) \qquad (29)$$

The mean skin friction for air (Pr = 0.71) is presented in Fig. 6. The graph reveals that skin friction increases with Re. Physically it is true, because increase in Re leads to increase in viscosity, hence more fluid drag in the vicinity to the plate. It is interesting to note that skin friction increases with increasing Gr, Gc and k. Due to increase in porosity the flow resist, hence more dragging the fluid in vicinity of the plate. The shear stress is lower in case of NH_3 than that of CO_2 for the same value Re, Gr, Gc and k.

Fig. 1. Mean Velocity Profiles for Pr=0.71 (air)

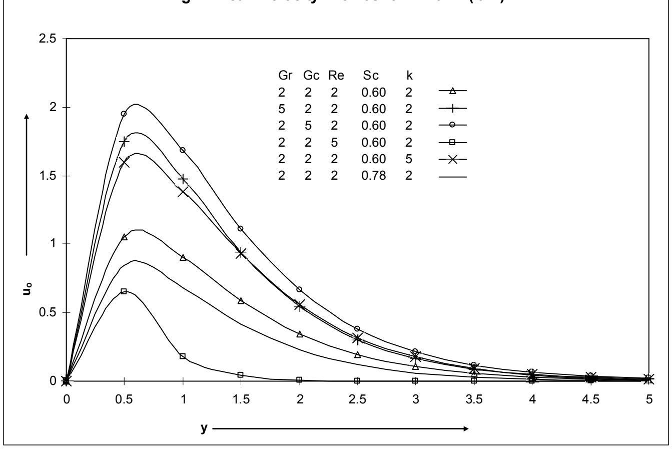
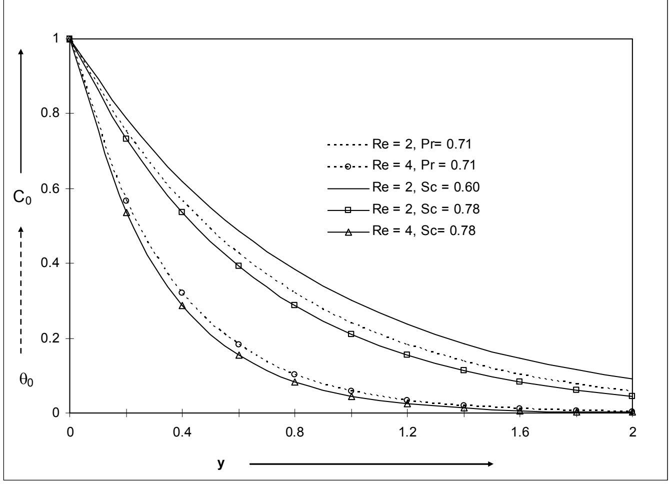


Fig.2. Mean Temperature (θ_0) and Concentration (C_0) Profiles



Unsteady flow

The velocity, temperature and concentration fields as given by equations (20) to (25) respectively can be expressed in terms of fluctuating parts as follows

$$f_1 (y) = f_r + \iota f_i \quad \& \quad f (y, z, t) = f_0(y) + \varepsilon [f_r \cos (\pi z - t) - f_i \sin (\pi z - t)] \qquad (30)$$

where f stands for u, θ and C.

Hence we can obtain the expression for the transient velocity, transient temperature and transient concentration profiles for z = 0 and t = π/ 2 as

$$u (y, 0, \frac{\pi}{2}) = u_0(y) + \varepsilon u_i \qquad (31)$$

$$\theta (y, 0, \frac{\pi}{2}) = \theta_0(y) + \varepsilon \theta_i \qquad (32)$$

$$C (y, 0, \frac{\pi}{2}) = C_0(y) + \varepsilon C_i \qquad (33)$$

where

$$u_i = n_{10} e^{-n_1 y} \cos n_2 y - n_9 e^{-n_1 y} \sin n_2 y - (n_{10} + n_{14}) e^{-n_5 y} \cos n_6 y + (n_9 + n_{13}) e^{-n_5 y} \sin n_6 y , \\ - n_{13} e^{-n_3 y} \sin n_4 y + n_{14} e^{-n_3 y} \cos n_4 y$$

$$\theta_i = - e^{-n_1 y} \sin n_2 y ,$$

$$C_i = - e^{-n_3 y} \sin n_4 y ,$$

$$a_1 = n_1 + \iota n_2 , \quad a_2 = n_3 \iota n_4 , \quad a_3 = n_5 + \iota n_6 , \quad a_4 = n_9 + \iota n_{10} , \quad a_5 = n_{13} \iota n_{14} ,$$

$$n_1 = \frac{1}{2}\left[Re\,Pr + \sqrt{\sqrt{(Re^2 Pr^2 + 4\pi^2)^2 + 16 \omega^2 Pr^2}} \cdot \frac{(Re^2 Pr^2 + 4\pi^2)^2 - 16\omega^2 Pr^2}{(Re^2 Pr^2 + 4\pi^2)^2 + 16\omega^2 Pr^2} \right] ,$$

$$n_2 = \frac{1}{2}\left[\sqrt{\sqrt{(Re^2 Pr^2 + 4\pi^2)^2 + 16 \omega^2 Pr^2}} \cdot \frac{8 \omega Pr (Re^2 Pr^2 + 4\pi^2)}{(Re^2 Pr^2 + 4\pi^2)^2 + 16\omega^2 Pr^2} \right] ,$$

$$n_3 = \frac{1}{2}\left[Re\,Sc + \sqrt{\sqrt{(Re^2 Sc^2 + 4\pi^2)^2 + 16 \omega^2 Sc^2}} \cdot \frac{(Re^2 Sc^2 + 4\pi^2)^2 - 16\omega^2 Sc^2}{(Re^2 Sc^2 + 4\pi^2)^2 + 16\omega^2 Sc^2} \right] ,$$

$$n_4 = \frac{1}{2}\left[\sqrt{\sqrt{(Re^2 Sc^2 + 4\pi^2)^2 + 16 \omega^2 Sc^2}} \cdot \frac{8 \omega Sc (Re^2 Sc^2 + 4\pi^2)}{(Re^2 Sc^2 + 4\pi^2)^2 + 16\omega^2 Sc^2} \right] ,$$

$$n_5 = \frac{1}{2}\left[Re + \sqrt{\sqrt{(Re^2 + 4\pi^2 + \frac{4}{k})^2 + 16 \omega^2}} \cdot \frac{(Re^2 + 4\pi^2 + \frac{4}{k})^2 - 16\omega^2}{(Re^2 + 4\pi^2 + \frac{4}{k})^2 + 16\omega^2} \right] ,$$

$$n_6 = \frac{1}{2}\left[\sqrt{\sqrt{(Re^2 + 4\pi^2 + \frac{4}{k})^2 + 16 \omega^2}} \cdot \frac{8 \omega (Re^2 + 4\pi^2 + \frac{4}{k})}{(Re^2 + 4\pi^2 + \frac{4}{k})^2 + 16\omega^2} \right] ,$$

$$n_7 = n_1^2 - n_2^2 - Re\, n_1 - \pi^2 - \frac{1}{k},$$

$$n_8 = 2 n_1 n_2 - Re\, n_2 + \omega ,$$

$$n_9 = - \frac{Re^2\, Gr\, n_7}{n_7^2 + n_8^2},$$

$$n_{10} = \frac{Re^2\, Gr\, n_8}{n_7^2 + n_8^2},$$

$$n_{11} = n_3^2 - n_4^2 - Re\, n_3 - \pi^2 - \frac{1}{k},$$

$$n_{12} = 2 n_3 n_4 - Re\, n_4 + \omega ,$$

$$n_{13} = - \frac{Re^2\, Gc\, n_{11}}{n_{11}^2 + n_{12}^2},$$

$$n_{14} = \frac{Re^2\, Gc\, n_{12}}{n_{11}^2 + n_{12}^2}.$$

The transient velocity profile is shown in Fig.3 against perpendicular distance from plate for different parameter. It is observed that transient velocity decreases with an increase in Re, since an increase in Re fluid become more viscous, so decrease in velocity. The value of transient velocity is higher for CO_2 in comparison to NH_3. Furthermore it increases rapidly near the plate and attains maximum than it goes decreases far away from plate. It is interesting to note that it increases with increasing k. The transient temperature in reported in Fig. 4. It decreases exponentially with the perpendicular distance from the plate. It decreases with increasing Re (Reynold's number). The transient temperature decreases slightly with increasing ω. The transient concentration is given in Fig.5. It is observed that decreases with increasing Re and ω for same values of Sc.

Fig.3. Transient Velocity for z = 0, ε = 0.2, Gr =5, Gc = 5, Pr=0.71(air), ω = 5 and t = π/2

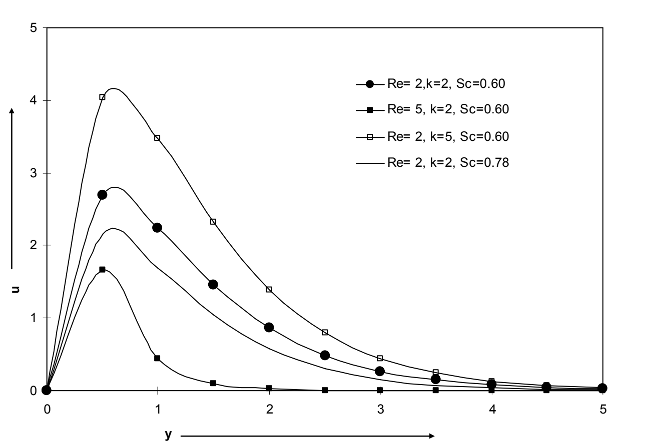


Fig.4. Transient temperature for z=0, Pr=0.71, ε = 0.2 and t = π/2

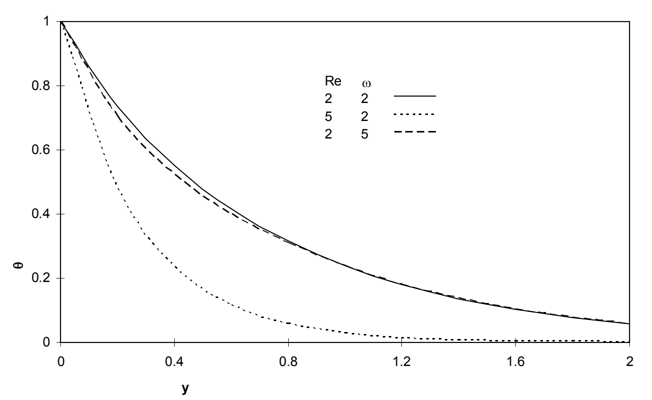


Fig.5. Transient Concentration for z=0, ε=0.2 and t=π/2

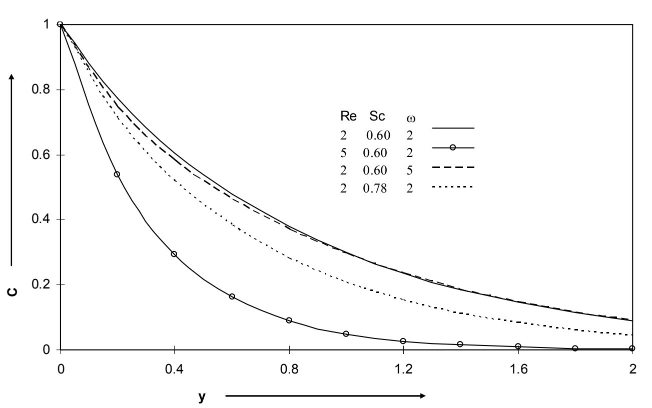
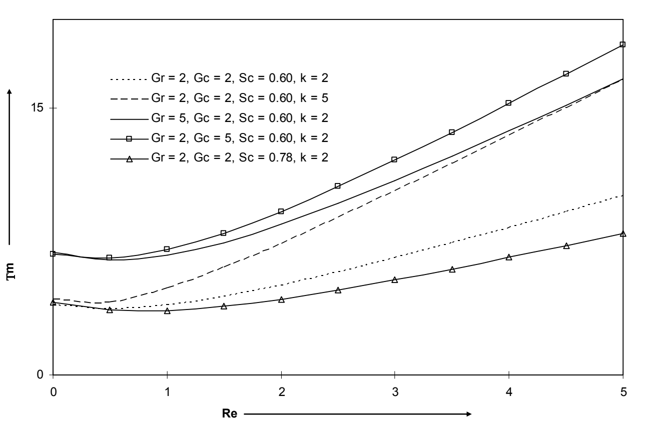


Fig.6. Mean Skin-Friction for Pr = 0.71 (air)



The values of concentration decay exponentially with the distance far away from the plate. It is interesting to note that concentration slightly decrease with increasing ω. It is now proposed to study the behaviour of amplitude and phase of skin-friction. From equation (25) and (27) we have

$$\tau = t_m + \varepsilon\, e^{\iota(\pi z - t)}\,[a_4\,(a_3 - a_1) + a_5\,(a_3 - a_2)] \qquad (34)$$

We can express equation (34) in terms of the amplitude and phase of skin-friction as

$$\tau = t_m + \varepsilon \left| M \right| \cos(\pi z - t + \phi) \qquad (35)$$

Where

$$M = M_r + \iota M_i = \text{coefficient of } \varepsilon\, e^{\iota(\pi z - t)} \text{ in equation (34)}$$

$$\left| M \right| = \sqrt{M_r^2 + M_i^2} \quad \text{and} \quad \tan\phi = \frac{M_i}{M_r}.$$

The amplitude of skin-friction for Gr = 2, and Gc = 2 in air (Pr = 0.71) is presented in Fig.8. It is observed that amplitude of skin-friction increases with Re and ω. It is interesting to note that amplitude of skin friction slightly increases with k. Also the magnitude of skin friction is less in case of NH_3 (Sc = 0.78) than that of CO_2 (Sc = 0.60). It increases slowly for small value of Re, while increases rapidly for higher values of Re. The phase of skin friction is given in Fig.7.

Fig.7. Phase of Skin_friction for Pr=0.71(air)

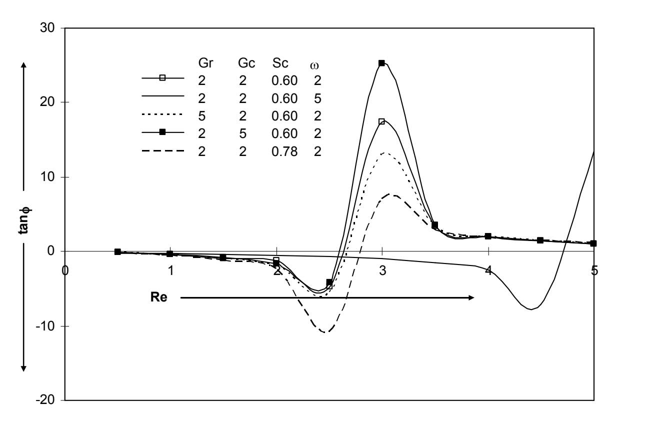
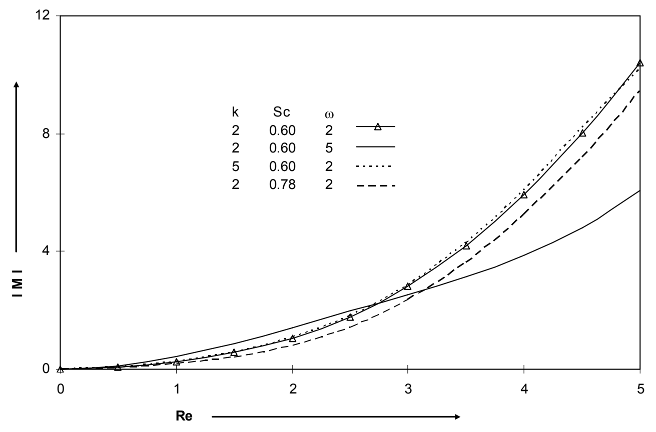


Fig.8. Amplitude of Skin-Friction for Pr = 0.71 (air), Gr = 2 and Gc = 2



It is observed that phase of skin friction increases with Gc while reverse effect is observed for Gr. There is always a phase lead with increasing ω. The magnitude of phase of skin friction in lower in case of Sc=0.78 (NH_3) than that of Sc=0.60 (CO_2). We now study the effect of various parameters on the rate of heat transfer. The rate of heat transfer in terms of the Nusselt number at the plate can be obtained as

$$Nu = -\frac{q_w^* \ell}{\kappa (T_0^* - T_\infty^*)} = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \left(\frac{\partial \theta_0}{\partial y}\right)_{y=0} + \varepsilon \left(\frac{\partial \theta_1}{\partial y}\right)_{y=0} e^{\imath(\pi z - t)} \quad (36)$$

$$Nu = -\,Re\,Pr \; + \varepsilon\,(-a_1)\,e^{\imath(\pi z - t)} \quad (37)$$

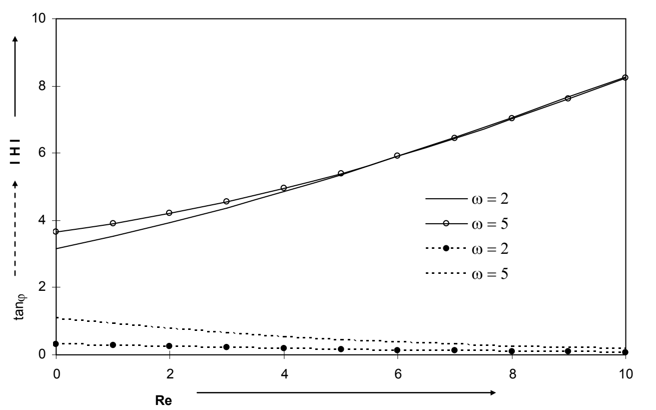
We can express (37) in terms of amplitude and phase of heat transfer as

$$Nu = -\,Re\,Pr + \varepsilon \left| H \right| \cos(\pi z - t + \psi) \quad (38)$$

$$H = H_r + \imath H_i = \text{coefficient of } \varepsilon\, e^{\imath(\pi z - t)} \text{ in equation (37)}$$

$$\left| H \right| = \sqrt{H_r^2 + H_i^2} \quad \text{and} \quad \tan\psi = \frac{H_i}{H_r}.$$

Fig.9. Amplitude (I H I) and Phase (tan ψ) of Rate of Heat Transfer



The amplitude and phase of heat transfer in shown in Fig. 9. It is observed that amplitude of heat transfer increases with increasing ω for small value of Re (i.e. increase in frequency of oscillation generate more heat due to increase in velocity of fluid hence more transfer of heat to the plate) and than coincide for higher value of Re > 5. The phase of heat transfer increases with an increase in ω while slightly decrease with increasing Re.

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