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RESEARCH ARTICLE

SELECTION OF THREE STAGE CHAIN SAMPLING PLAN OF TYPE ChSP (0, 1, 2) INDEXED THROUGH MINIMUM ANGLE METHOD

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ABSTRACT

In this paper provides a procedure for designing the three stage chain sampling plan of type ChSP (0, 1, 2) indexed through minimum angle method. A Table and methods are given for the construction of plans indexed by minimum angle method.

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INTRODUCTION

Statistical Quality Control (SQC) is those aspects of quality control in which statistics are applied, in contrast to the broader scope of quality control which includes many other procedures, such as preventive maintenance, instrument function checks, and performance validation tests. Acceptance sampling is a major field of statistical quality control. It is a methodology dealing with procedures, which help, in making a decision as whether to accept or reject the lot based on the results of inspection of samples. Single sampling plan is a Sampling inspection in which the decision to accept or not to accept a lot is based on the inspection of a single sample of size 'n'. The usual practice to use single sampling plan with a small sample size and an acceptance number zero to base the decision to accept or reject the lot. Single sampling plan with acceptance number has zero has the following undesirable characteristics.

1. A single defect in the sample calls for rejection of the lots (or for classifying the lot as non-conforming), and
2. The OC curves of all such sampling plans have a uniquely poorer shape, in that the probability of acceptance starts to drop rapidly for the smallest values of percent defective.

Dodge (1955) treats this problem using a procedure, called chain sampling plan (ChSP – 1). These plans make use of the cumulative inspection results from several results, from one or more samples along with the results from the current sample, in

making a decision regarding acceptance or rejection of the current lot. The chain sampling plans are applicable for both small and large samples. Dodge and Stephens (1966) extended the concept of chain sampling plans and presented a set of two-stage chain sampling plans based on the concept of ChSP – 1 developed by Dodge (1955). They presented expressions for OC curves of certain two – stage chain sampling plans and made comparison with single and double sampling attributes inspection plans. The three stage chain sampling plan of type ChSP (0, 1, 2) developed by Soundararajan and Raju (1984) is a generalization of Dodge (1955) chain sampling plan ChSP – 1 and Dodge and Stephens (1966) chain sampling plan ChSP – (0, 1). Soundararajan and Raju (1984) gives the structure and operating procedure of generalized three – stage chain sampling plan and expressions for OC curve of certain three – stage plans are also given. ChSP (0, 1, 2) can be used for both small and large samples, but it is particularly useful when samples must necessarily be small (eg., when tests are costlier). The greater generality in the choice of parameters in the ChSP – (0, 1,2) plan allows for greater flexibility in matching these plans to other plans, and allows for improved discrimination between good and bad quality. A more complete discussion of chain sampling plan can be found in Schilling (1982). The three stage chain sampling plan has 7 parameters which are defined below:

n = sample size

k_1 = The maximum number of samples over which the cumulation of the defectives take place in the first stage of procedure.

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- k_2 = The maximum number of samples over which the cumulation of the defectives take place in the second stage of procedure.
- k_3 = The maximum number of samples over which the cumulation of the defectives take place in the first of procedure.
- c_1 = The allowable number of defectives in the cumulative results from k_1 or fewer sample of n . Thus c_1 is an acceptance number for cumulative results. It is the cumulative results criterion (CRC) that must be met by cumulative sampling results during the first stage of of the restart period in order to permit acceptance of a lot.
- c_2 = The allowable number of defectives in the cumulative results from $k_1 + 1$ to k_2 sample of n . Thus c_2 is also an acceptance number for cumulative results and the CRC that must be met by cumulative sampling results during the second stage of the restart period in order to permit acceptance of a lot.
- c_3 = The allowable number of defectives in the cumulative results from $k_2 + 1$ to k_3 sample of n . Thus c_3 is also an acceptance number for cumulative results and the CRC that must be met by cumulative sampling results during the third stage of the restart period in order to permit acceptance of a lot.

When the sample size is not more than one-tenth of the lot size, and when the quality is measured interms of defectives, the OC curve can be computed using the binomial model. In addition to the condition of sample size being not more than one-tenth of the lot size, if the lot quality p (measured interms of defectives) is less than or equal to 0.01, the OC curve can be based on the Poisson model. When the quality is measured in terms of defects, the appropriate model is also the Poisson one. Under the condition for application of the Poisson model the probability of accepting a lot given the proportion non-conforming under the ChSP-(0,1,2) plan with parameters $n, k_1, k_2, k_3, c_1, c_2,$ and c_3 was derived by Raju (1984) as

$$P_a(p) = \frac{P_0 + P_1 P_0^{k_2-1} + (k_3 - k_2 - 1) P_1^2 P_0^{k_3-2} + P_2 P_0^{k_3-1} + P_1 P_0^{k_1} \left[\frac{1 - P_0^{k_2-k_1-1}}{1 - P_0} \right] + (k_2 - k_1) P_1^2 P_0^{k_2-1} + P_1^2 P_0^{k_2} \left[\frac{1 - (k_3 - k_2 - 1) P_0^{k_3-k_2-2}}{1 - P_0} + \frac{P_0 (1 - P_0^{k_3-k_2-2})}{(1 - P_0)^2} \right] + P_2 P_0^{k_2} \left[\frac{1 - P_0^{k_3-k_2-1}}{1 - P_0} \right]}{1 + P_1 P_0^{k_1} \left[\frac{1 - P_0^{k_2-k_1-1}}{1 - P_0} \right] + (k_2 - k_1) P_1^2 P_0^{k_2-1} + P_1^2 P_0^{k_2} \left[\frac{1 - (k_3 - k_2 - 1) P_0^{k_3-k_2-2}}{1 - P_0} + \frac{P_0 (1 - P_0^{k_3-k_2-2})}{(1 - P_0)^2} \right] + P_2 P_0^{k_2} \left[\frac{1 - P_0^{k_3-k_2-1}}{1 - P_0} \right]} \quad \text{----- (1)}$$

Where,
 P_0 = Probability of getting exactly zero non- conforming in a sample of size n
 P_1 = Probability of getting exactly one non- conforming in a sample of size n
 P_2 = Probability of getting exactly two non- conforming in a sample of size n

A Review on Minimum Angle Method

The practical performance of any sampling plan is generally revealed through its operating characteristic curve. When producer and consumer are negotiating for quality limits and

designing sampling plans, it is important especially for the minimize the consumer risk. In order to minimize the consumer’s risk, the ideal OC curve could be made to pass as closely through (AQL, $1-\alpha$) was proposed by Norman Bush (1953) considering the tangent of the angle between the lines joining the points (AQL, $1-\alpha$), (AQL, β). Norman Bush *et al.* (1953) have considered two points on the OC curve as (AQL, $1-\alpha$) and (IOL, 0.50) for minimize the consumer’s risk. But Peach and Littauer (1946) have taken two points on the OC curves as ($p_1, 1-\alpha$) and (p_2, β) for ideal condition to minimize the consumers risks here another approach with minimization of angle between the lines joining the points (AQL, $1-\alpha$), (AQL, β) and (AQL, $1-\alpha$), (LQL, β) was proposed by Singaravelu (1993). Applying this method one can get a better plan which has an OC curve approaching to the ideal OC curve. Govindaraju.K (1990), Soundararajan.V (1981) and many others have studied AQL.

The formula for $\tan\theta$ is given as

$$\tan\theta = \frac{\text{oppositeside}}{\text{adjacentside}} \quad \text{----- (2)}$$

Tangent of angle made by AB and AC is

$$\tan\theta = (p_2-p_1) / (P_a(p_1)-P_a(p_2)) \quad \text{----- (3)}$$

Where $p_1 = \text{AQL}$ and $p_2 = \text{LQL}$. This may be expressed as,

$$n \tan\theta = (np_2 - np_1) / (1 - \alpha - \beta) \quad \text{----- (4)}$$

The smaller value of this $\tan\theta$ closer is the angle θ approaching zero, and the chord AB approaching AC, the ideal condition through (AQL, $1-\alpha$)

Now

$$\theta = \tan^{-1}\{(n \tan\theta/n)\} \quad \text{----- (5)}$$

Using this formula the minimum angle θ is obtained, for the

Construction of Tables

The binomial model for the OC curve will be exact in the case of fraction non-conforming. It can be satisfactorily approximated with the Poisson model where p is small, n is large, and $np < 5$ when the quality is measured in terms of non conformities, the Poisson model is the appropriate one. Under the Poisson assumption, the expression for

$$P_0 = e^{-np}, P_1 = np e^{-np}, P_2 = ((np)^2/2) e^{-np} \quad \text{----- (6)}$$

The equation cannot easily solve. The solutions for np for a given P_a have been found by Newton’s method of successive approximation and are tabulated in Table 1 for different values of k_1, k_2, k_3 .

Table 1. The parametric values for three stage chain sampling plan of type ChSP (0, 1, 2)

k ₁	k ₂	k ₃	P _a (p)						
			0.99	0.95	0.9	0.5	0.1	0.05	0.01
1	2	3	0.10121	0.24023	0.35012	1.09024	2.53024	3.14024	4.65022
1	2	5	0.20012	0.32012	0.41998	1.12012	2.53017	3.13996	4.65017
1	4	5	0.06110	0.15987	0.26010	0.98032	2.47987	3.12018	4.64019
2	3	4	0.07000	0.17231	0.26012	0.88012	2.33023	3.00123	4.60021
2	5	10	0.08101	0.17023	0.24987	0.83998	2.31995	3.00101	4.60010
2	9	10	0.04011	0.12012	0.20112	0.82022	2.32023	2.99989	4.59994
3	4	5	0.06023	0.13998	0.22112	0.78992	2.30010	2.99015	4.56018
4	5	6	0.05013	0.12987	0.20210	0.74023	2.27012	2.97900	4.52012
5	6	7	0.04014	0.12023	0.18799	0.72012	2.26743	2.97600	4.48021
6	7	8	0.03891	0.11023	0.17112	0.70023	2.26499	2.96994	4.44001
7	8	9	0.03782	0.10112	0.15865	0.69998	2.26233	2.96600	4.40101
8	9	10	0.03670	0.09850	0.15121	0.69023	2.25983	2.96150	4.36010
9	10	11	0.03560	0.09015	0.15023	0.69017	2.25733	2.95700	4.31987
9	10	20	0.05000	0.11000	0.16012	0.69010	2.30021	2.99012	4.59878
10	11	12	0.03450	0.08860	0.13987	0.67897	2.25483	2.95250	4.28021
11	12	13	0.03340	0.08720	0.14013	0.67198	2.25233	2.94800	4.24013
11	12	19	0.04001	0.10112	0.14986	0.68997	2.29987	2.99031	4.60010
11	17	20	0.03012	0.08012	0.14013	0.69023	2.29981	2.99022	4.60005
12	13	14	0.03230	0.08580	0.13023	0.66521	2.24983	2.94329	4.19998
13	14	15	0.03120	0.07852	0.13012	0.65818	2.24733	2.93871	4.16019
13	14	19	0.03011	0.08788	0.14010	0.68987	2.30019	2.99014	4.60023
13	14	22	0.03013	0.09012	0.14008	0.68861	2.30011	2.99008	4.60015
13	17	22	0.03132	0.08123	0.13028	0.68798	2.29989	2.99002	4.59987
13	20	22	0.02013	0.07945	0.13025	0.68775	2.24531	2.98997	4.59975
14	15	16	0.03012	0.07850	0.13018	0.65123	2.24483	2.93414	4.12021
14	15	19	0.03011	0.07980	0.13010	0.68995	2.30024	2.98995	4.60022
14	15	22	0.03014	0.08760	0.14021	0.69023	2.30018	2.98985	4.60017
14	17	22	0.03001	0.08010	0.13007	0.69018	2.30010	2.98976	4.60011
15	16	17	0.02980	0.07700	0.13002	0.64414	2.24233	2.92957	4.08012
15	18	19	0.02314	0.08001	0.12998	0.69010	2.30012	2.99002	4.60010
15	18	22	0.03021	0.08012	0.12987	0.69005	2.30232	2.98995	4.60002
15	20	22	0.02021	0.08032	0.12895	0.68995	2.30422	2.98965	4.59998
16	17	18	0.02897	0.07550	0.12021	0.63743	2.23983	2.92500	4.04017
16	18	20	0.02100	0.08012	0.12015	0.69022	2.30012	2.98997	4.60021
16	19	22	0.02300	0.08000	0.12992	0.69019	2.30200	2.98700	4.60201
16	21	22	0.02400	0.07000	0.12013	0.69015	2.30400	2.98400	4.60412
17	18	19	0.02827	0.07400	0.12010	0.63021	2.23733	2.92043	4.00012
18	19	20	0.02757	0.07250	0.12005	0.62301	2.23483	2.91586	3.96010
19	20	21	0.02687	0.07100	0.12001	0.61643	2.23233	2.91129	3.92020
19	20	22	0.02617	0.06950	0.11999	0.60899	2.22983	2.90671	3.88013
19	21	22	0.02547	0.06800	0.11986	0.60197	2.22733	2.90214	3.84023
20	21	22	0.02477	0.06650	0.11977	0.59500	2.22483	2.89757	3.80010

Table 2. Certain characteristic values for three stage chain sampling plan through minimum angle method

k ₁	k ₂	k ₃	np ₁	np ₂	P _a (p ₁)	P _a (p ₂)	Ntanθ
1	2	3	0.2402	2.5302	0.9505	0.1002	45.1946
1	2	5	0.3201	2.5302	0.9514	0.1005	42.582
1	4	5	0.1599	2.4799	0.9539	0.1009	42.3628
2	3	4	0.1723	2.3302	0.9526	0.1001	40.9229
2	5	10	0.1702	2.3200	0.9530	0.1005	40.1859
2	9	10	0.1201	2.3202	0.9502	0.1004	43.4549
3	4	5	0.1400	2.3001	0.9546	0.1006	39.1305
4	5	6	0.1299	2.2701	0.9503	0.1033	39.9000
5	6	7	0.1202	2.2674	0.9486	0.1036	41.1536
6	7	8	0.1102	2.265	0.9494	0.1038	40.5141
7	8	9	0.1011	2.2623	0.9511	0.1041	39.1367
8	9	10	0.0985	2.2598	0.9488	0.1044	40.6628
9	10	11	0.0902	2.2573	0.952	0.1046	38.2402
9	10	20	0.1100	2.3002	0.9495	0.1002	44.0167
10	11	12	0.0886	2.2548	0.9501	0.1049	39.3672
11	12	13	0.0872	2.2523	0.9485	0.1052	40.3775
11	12	19	0.1011	2.2999	0.9457	0.1003	47.8635
11	17	20	0.0801	2.2998	0.9534	0.1003	41.3271
12	13	14	0.0858	2.2498	0.9471	0.1054	41.1857
13	14	15	0.0785	2.2473	0.9517	0.1057	37.8095
13	14	19	0.0879	2.3002	0.9489	0.1002	45.0188
13	14	22	0.0901	2.3001	0.9496	0.1002	44.2987
13	17	22	0.0812	2.2999	0.9517	0.1003	42.7269
13	20	22	0.0795	2.2453	0.9486	0.1059	39.7342

Continue.....

14	15	16	0.0785	2.2448	0.9496	0.1059	38.9661
14	15	19	0.0798	2.3002	0.9527	0.1002	41.9285
14	15	22	0.0876	2.3002	0.9483	0.1002	45.5506
14	17	22	0.0801	2.3001	0.9515	0.1002	42.9233
15	16	17	0.0770	2.2423	0.9492	0.1062	39.0543
15	18	19	0.0800	2.3001	0.9453	0.1002	48.7132
15	18	22	0.0801	2.3023	0.9485	0.1000	45.7842
15	20	22	0.0803	2.3042	0.9453	0.0998	49.2973
16	17	18	0.0755	2.2398	0.949	0.1065	39.0098
16	18	20	0.0801	2.3001	0.9452	0.1002	48.845
16	19	22	0.0800	2.302	0.9458	0.1001	48.4046
16	21	22	0.0700	2.304	0.9529	0.0999	42.3553
17	18	19	0.0740	2.2373	0.9489	0.1067	38.8482
18	19	20	0.0725	2.2348	0.949	0.1070	38.5851
19	20	21	0.0710	2.2323	0.9492	0.1073	38.2355
19	20	22	0.0695	2.2298	0.9518	0.1075	36.3881
19	21	22	0.0680	2.2273	0.9520	0.1078	36.0871
20	21	22	0.0665	2.2248	0.9527	0.1081	35.5191

- Using np_1 and np_2 values from Table 1 the values of P_0 , P_1 and P_2 are calculated
- Then set k_1, k_2, k_3 values
- Compute $P_a(p_1)$ and $P_a(p_2)$, using Equations (1) and (5)
- Next substituting np_1, np_2 and $P_a(p_1), P_a(p_2)$ values in the below equation the $ntan\theta$ values are calculated as follows, $ntan\theta = (np_2 - np_1) / (P_a(p_1) - P_a(p_2))$
- Then Record minimum $ntan\theta$ value. ($tan\theta$ values are given in Table 2)

Example

1. Given $p_1 = 0.07$ and $p_2 = 2.24$, then $OR = p_2/p_1 = 2.24/0.07 = 32$. The Associated sets of values corresponding to the computed OR values from Table 2 is, $k_1=19, k_2=20, k_3=22, np_1 = 0.0695, np_2 = 2.22983$ and $ntan\theta = 36.3881$ from the above results, one can find,
 $n = np_1/p_1 = 0.0695/0.07 = 0.9929$
 $\theta = \tan^{-1}\{(ntan\theta/n)\}$
 $\theta = \tan^{-1}\{(36.3881)/0.9929\} = 0.9999$

Now the minimum angle is $\theta = 0.9999$. Hence the selected parameters for the three stage chain sampling plan of type ChSP (0, 1, 2) for given $p_1 = 0.07$ and $p_2 = 2.24$ with minimum angle $\theta = 0.9999$.

Conclusion

Acceptance sampling is the technique which deals with the procedures in which decision either to accept or reject lots or process which are based on the examination of samples. The work presented in this paper relates to the new procedure for the construction and selection of tables for designing sampling inspection plan through Minimum Angle Method. This procedure reduces the cost of inspection for the producer and the consumer, gets good items. In practice it is desirable to design any sampling plan with the associated quality levels which concern to producer and consumer. Tables provided in this paper are tailor – made which are handy and readymade, which are also well considered for comparison purposes. Tables are also useful for developing and under developing countries, which have limited resources to the Industrial shop floor- situations.

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